PROBLEMS FOR SOLUTION

P. 159. Let M be a metric space, M_0 a compact subset and $T: M \to M$ an isometry. Then if $TM_0 \subset M_0$ or $TM_0 \supset M_0$ we have $TM_0 = M_0$.

J.B. Wilker, Pahlavi University, Shiraz, Iran.

P. 160. Higman [Quart. J. Math. Oxford 10 (1959), 165-178] proves that a group satisfies the identical relation $[[x, y], [x, y^{-1}]] = 1$ if and only if all its two-generator subgroups are metabelian. Prove that the same conclusion holds for the relation $[[x, y], [x^{-1}, y^{-1}]] = 1$.

J. Gandhi, York University

P. 161. For any positive integer n and any n numbers $c_1, \ldots, c_n$, let further numbers $c_{n+1}, c_{n+2}, \ldots$ be defined as continued fractions

$$c_{n+1} = 1 - c_n/1 - c_{n-1}/1 - \ldots c_2/1 - c_1,$$

$$c_{n+2} = 1 - c_{n+1}/1 - c_n/1 - \ldots c_3/1 - c_2,$$

and so on. Prove that the sequence c_i is periodic with period $n + 3$; that is, $c_{n+4} = c_1$, $c_{n+5} = c_2$, and so on.

H.S.M. Coxeter, University of Toronto

SOLUTIONS

P. 149. Find all solutions, other than the trivial solution $(a, b, c) = (1, 1, c)$ of the simultaneous congruences:

$ab \equiv 1 \bmod c$, $bc \equiv 1 \bmod a$, $ca \equiv 1 \bmod b$ where a, b, c are positive integers with $a \leq b \leq c$.

G.K. White, University of British Columbia

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Solution by P. Smith, University of Victoria

Except in the trivial case, $1 < a < b < c$ and $ab = ck + 1$, where $0 < k < a$ and $(a, k) = 1$. Now $ack = a^2 b - a \equiv k \mod b$ whence $a + k \equiv 0 \mod b$ so $b \mid (a + k) < 2a < 2b$ and therefore $b = a + k$.

Similarly $a \mid (b + k) = a + 2k < 3a$ hence $a + 2k = 2a$, or $a = 2k$ and, since $(a, k) = 1$, $k = 1$.

Thus the only non-trivial solution is (2, 3, 5).

Also solved by W. J. Blundon, M. F. Collins, and the proposer.

P. 150. Let S be a set of commuting permutations acting transitively on a set Ω. Prove that S is a sharply transitive abelian group.

A. Bruen, University of Toronto

Solution by D. Ž. Djoković, University of Waterloo

The group G generated by S is abelian and transitive. G is sharply transitive by Proposition 4.3, in H. Wielandt's book "Finite Permutation Groups". Note that this proposition and its proof remain valid also when Ω is infinite. Since (i) $G \supset S$, (ii) G is sharply transitive, (iii) S is transitive, we must have $G = S$.

Also solved by the proposer.

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