BULL. AUSTRAL. MATH. SOC. 58F14, 58F40, 93C55
VOL. 44 (1991) [171–174]

Study of a nonlinear control algorithm using dynamical systems theory

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This thesis presents a new control algorithm for nonlinear input-output systems and a mathematical framework for its analysis. The algorithm is referred to as nonlinear model-algorithmic control (nonlinear MAC). The initial attempt to analyse the stability of the control equation in nonlinear MAC led to some new and interesting investigations in discrete dynamical systems theory; namely, how to interpret the behaviour of multi-valued maps under iteration, and identifying singularity structure of families of maps where the number of independent parameters exceeds the order of the equation which describes the map. Chapters 3 and 4 develop the theory to handle these issues; the results from these chapters form the basis for the control analysis. Chapter 5 discusses the control algorithm and illustrates the main features of nonlinear MAC.

Nonlinear MAC is characterised by:

(1) its natural extension of the linear model-algorithmic control developed earlier by Mehra, Rouhani, Rault and Reid [1]; and
(2) its use of either the Volterra or the Wiener functional series representation to model the system.

Linear MAC used the impulse response convolution model of the system rather than the more common transfer function or the state-space representation as a basis for the control design. The equivalent representations for a nonlinear system are the multiple convolution expansions of nonlinear functionals investigated by Volterra and Wiener [2]. These expansions are analogous to the Taylor expansion and orthogonal polynomial expansions of functions.

Similar to the linear convolution model, the Volterra/Wiener representations do not require prior knowledge of the system's internal structure. They are particularly valuable in designing control for blackbox systems; that is, those systems for which the parametric model of the system is unknown. In many instances, the blackbox systems are approximated by linear models or nonlinear models of particular structure; whilst

Received 13 February 1991
Thesis submitted to Macquarie University, March 1990. Degree approved September 1990. Supervisor Dr J.V. Corbett.

this may suffice for certain operating ranges, the level of applicability decreases when the operating ranges are broadened. Thus, the strengths of the nonlinear MAC are that:

(1) the model can reflect the true nonlinearities of the system, and therefore remain applicable for a broad operating range; and
(2) the order of nonlinearity which is reflected by the model can be increased by adding another term of the Volterra/Wiener expansion; in particular, the Wiener model would not require the recalculation of the lower order terms when higher terms are added.

In Chapter 5, the control algorithm is developed, and its performance is demonstrated with simulation examples. The algorithm is examined for the discrete time case rather than for the continuous time case. The algorithm is restricted to models incorporating only up to the quadratic nonlinearity; however, it is shown that higher order nonlinear systems can be adequately controlled by a quadratic based control so long as the higher order terms do not dominate the system. The superiority of nonlinear MAC over linear MAC is also illustrated by some of the examples.

Studying the control equation for stability and robustness leads to some interesting mathematical problems. In particular, it is found that the control equation describes a multi-valued quadratic recurrence. In its simplest form, the recurrence is

$$(1) \qquad \zeta x_{n+1}^2 + x_{n+1} = x_n^2 + \mu.$$

The multi-valued property arises because of the nonlinearity on the left hand side of the equation. This contrasts with the form of more commonly studied recurrences, that is, those which can be expressed in the form:

$$x_{n+1} = f_\mu(x_n)$$

where f_μ is a single-valued map. In the control context, the multi-valued property presents questions of how many controls are generated at each time step, which one to choose, and whether stability is dependent on this choice, as well as the question of existence of at least one control at each time step.

It is found that the dynamical systems theory used to study the behaviour of maps on the interval and on the plane can be extended in order to study the control equations. Even though most of the results relate to maps which are single-valued, it is shown in Chapter 3 that much information about the multi-valued recurrences which arise in the nonlinear MAC scheme can be obtained from these results, or suitable extensions of them.

It is shown that the recurrences arising from the control algorithm define second order curves and surfaces. The dynamical behaviour of maps described by such curves and surfaces were found to be related to families of quadratic maps about which much knowledge exists. In particular, the behaviour of the family of quadratic maps on the interval

$$f_\mu(x) = x^2 + \mu \tag{2}$$

and the family of quadratic maps on the plane

$$F_{\mu,\varepsilon} = \left(\varepsilon y + x^2 + \mu,\ x\right) \tag{3}$$

provide insight into the behaviour of maps on second order curves and surfaces. These maps are indeed special examples of second order curves and surfaces where one of the parameters is zero; thus, the associated bifurcation diagram of these families of maps is a projection of the bifurcation diagram of the second order curves and surfaces in a lower dimension.

Chapter 2 reviews some known results in the dynamical theory of maps on the interval. These results form the framework for the study of recurrence (1) in Chapter 3. The main results are:

(1) the examination of recurrence (1) as a multi-valued 2-parameter map;
(2) the definition of selection maps which associate single-valued maps associated with recurrence (1);
(3) the derivation of a bifurcation diagram for this family of 2-parameter maps and identifying regions on the parameter plane where chaotic behaviour may occur.

Even though explicit calculation of the bifurcation curves for the 2-parameter family of maps is generally very complicated, it is shown that bounds for these curves can be found by recognising that the second order curves describing these maps are bounded by the family of quadratic maps (2) and a family of symmetric piecewise linear maps; information which exists about the behaviour of these maps is used to study the behaviour of maps governing (1).

In Chapter 4, the strategy used in Chapter 3 is extended to study the maps defined by second order surfaces. The map given by (3) provides the same sort of guidance here as the quadratic map (2) did in the previous case. Here, a 3-parameter family is examined, and lower order bifurcation surfaces are located. Bifurcations of higher orders are conjectured on the basis of identifying maps which bound the second order surfaces.

Locating bifurcation curves and surfaces for the family of maps associated with the control equation is useful in controllability, stability and robustness analyses. The bifurcation diagrams can identify regions where the control values can be generated, and model and control parameters which might lead the controller to instability and chaotic behaviour. When all the parameters lie within a stable region, the bifurcation diagrams can show how much allowance there is for parameter changes before they get too close to a bifurcation region. This is discussed with an example at the end of Chapter 5.

From these investigations, new avenues of study both at the fundamental and the application levels present themselves. One broad fundamental area is the development of a general framework for studying the dynamical behaviour of multi-valued maps. On the application side, algorithms to generate the bifurcation diagrams for general second order maps and surfaces need development. The analysis of nonlinear MAC for a model of higher than quadratic nonlinearity is an area which would require experimental study, but which would also require further fundamental mathematical work in understanding higher order multi-valued maps.

References

[1] R.K. Mehra, R. Rouhani, A. Rault and J.G. Reid, 'Model algorithmic control; theoretical results on robustness', *Proc. JACC* (1972), 387–392.

[2] K. Yasukawa, *The use of functionals in the analysis of non-linear physical systems*, B.A. Honours Thesis (Macquarie University, NSW, 1981).

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https://doi.org/10.1017/S0004972700029567 Published online by Cambridge University Press