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ERRATUM

Erratum to 'On Percolation and the Bunkbed Conjecture'

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Abstract

There was an incorrect argument in the proof of the main theorem in 'On percolation and the bunkbed conjecture', in *Combin. Probab. Comput.* (2011) **20** 103–117 doi:10.1017/S0963548309990666. I thus no longer claim to have a proof for the bunkbed conjecture for outerplanar graphs.

1. The error

In [1], a problem on edge percolation on product graphs $G \times K_2$ was studied. Here K_2 consists of two vertices $\{0, 1\}$ connected by an edge, and every edge in $G \times K_2$ is present with probability p independent of other edges. The bunkbed conjecture states that for any finite graph G and any $0 \leqslant p \leqslant 1$, the probability that $(u, 0)$ is in the same component as $(v, 0)$ is greater than or equal to the probability that $(u, 0)$ is in the same component as $(v, 1)$ for every pair of vertices $u, v \in G$.

Theorem 3.1 in [1] claimed that (a generalization of) the bunkbed conjecture was true for outerplanar graphs. The line of reasoning in the proof was that a minimal counter-example does not exist. This was done by reducing every possible configuration around u to a smaller graph. The error is in the second-to-last paragraph in the proof of Theorem 3.1, which deals with the case $xy \notin E(G)$ with the edge ux conditioned to be red. The problem occurs if $y \in T$. Then we cannot contract the edge uy as claimed since the formed vertex v_{uy} would be in T, which we have already ruled out by the mirror argument. But the mirror argument is not valid when we have conditioned on the colour of an edge. Thus we have reduced to a case not previously considered.

The same problem occurs also in the last paragraph of the proof if $y \in T$, *i.e.* a situation where $u \in T$ but one of the two outgoing edges from u is conditioned to be red. The problematic situation that the proof is not able to reduce to a smaller graph is depicted in Figure 1.

I see no easy way to resolve this problem with the methods used in the paper. For example, it does not help to apply the assumptions in a different order, *i.e.* conditioning on the edge xy before conditioning on the colour of the edges ux or uy. Also, one may not contract the edge ux instead, since $\deg(x) \geqslant 3$ could create new paths where, for example, uy could be used as well if blue.

There should also have been the following sentence on line 18, page 112 (but this is not the error in the proof): 'Since removing a blue edge at u, $u \notin T$, changes no probabilities of paths from u, a minimal counter-example with ux red will also be a minimal counter-example with no conditioning of the edges as long as $\deg(u) \leqslant 2$.'

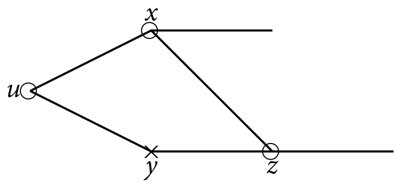

Figure 1. The problematic situation. Here $y \in T$, $u, x, z \notin T$. The edges ux, uy may (or may not) be conditioned to be red, since they could not be used if they were blue.

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References

[1] Linusson, S. (2011) On percolation and the bunkbed conjecture. *Combin. Probab. Comput.* **20** 103–117.

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