The Jacobian Curve of a Net of Quadrics

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§ 1. Much has been written, from the algebraical as well as from the geometrical standpoint, on the subject of pencils of quadrics: algebraically the problem consists of the canonical reduction of a pencil of quadratic forms, and the classical paper on the subject is by Weierstrass.[1] But among the different kinds of pencils of quadratic forms there is the "singular pencil," in which the discriminant of every form belonging to the pencil is zero; interpreted geometrically this means that every quadric belonging to the pencil is a cone. This case was expressly excluded from consideration by Weierstrass, and the canonical reduction was only accomplished later by Kronecker.[2] But, although Weierstrass and Kronecker together solved completely the problem of the canonical reduction of a pencil of quadratic forms, a much clearer insight into the nature of the problem was gained when Segre gave the geometrical solution. He published two papers, one[3] dealing with the non-singular pencils and the other[4] with the singular pencil.

A new method of carrying out the canonical reduction of a singular pencil of quadratic forms has been given quite recently by Turnbull and Aitken,[5] although these authors make no reference to

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1 "Zur Theorie der bilinearen und quadratischen Formen," *Berliner Monatsberichte* (1868), 310-338.

2 "Ueber Schaaren von quadratischen Formen," *Berliner Monatsberichte* (1874), 59-76. Kronecker also wrote a later paper on the subject; see the *Berliner Sitzungsberichte* (1890), 1375, and the continuation of this paper, *ibid.* (1891), 9 and 33.

3 *Memorie Acc. Torino* (2), 36 (1884), 3-86.

4 "Ricerche sui fasci di coni quadrici in uno spazio lineare qualunque," *Atti Acc. Torino* 19 (1884), 878.

5 *An Introduction to the Theory of Canonical Matrices* (Blackie, 1932); Ch. 9. The authors solve the more general problem of the canonical reduction of a singular pencil of matrices; when the matrices are symmetric the problem reduces to that of the reduction of a singular pencil of quadratic forms.

Segre's work (nor, it seems, does Kronecker in his paper of 1890). They introduce the idea of the minimal order of dependence of the rows, and similarly of the columns, of a matrix upon one another; a matrix thus has associated with it two sets of *minimal indices*, one set arising from the rows, the other from the columns. In the case however of a symmetric matrix these two sets of minimal indices are the same, and the canonical form of a singular pencil of quadratic forms depends upon this set of minimal indices $m_1, m_2, \ldots, m_k$. These numbers are the same as those which occur in Segre's second paper. The number k of different minimal indices is one greater than the dimension of the space $[k-1]$ which is the vertex of a general quadric of the pencil. The sum $m_1 + m_2 + \ldots + m_k$ of the minimal indices is the order of the locus V_k of the vertices of the quadrics, while the indices $m_1, m_2, \ldots, m_k$ themselves are simply the orders of the minimum directrix curves on V_k.

It is rather surprising that, notwithstanding the great importance and wide publicity of the above writings, nothing seems yet to have been written about a net of quadrics, except for the case of a net of quadric surfaces in ordinary space; at any rate I have not succeeded in finding a paper, and others whom I have asked know of none. The three-dimensional case is of course very well known, and it derives great interest from Hesse's use of it in studying the configuration of the bitangents of a non-singular plane quartic.[1] A paper by Sturm is also fundamental[2]; he obtains several original results, but his work would have gained considerably had he made use of that of Hesse, to whose paper he never refers. But apart from the three-dimensional case there does not seem to be any work, either geometrical or algebraical, on the subject. It may well be that, in view of the papers on a pencil of quadratic forms, an algebraist could not approach the study of a net of quadratic forms without serious misgivings; but Segre's work does not afford any such excuse to a geometer. It seems as though the net of quadrics in [4] is in some ways the most interesting of all, and a paper on this is now in an advanced stage of preparation.

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[1] *Journal für Math.*, 49 (1855), 279-332.

[2] *Journal für Math.*, 70 (1869), 212-240.

The present note considers briefly the Jacobian curve of a net of quadrics in $[n]$; in particular the existence of certain *secant spaces* of the curve is established and a few properties of these spaces are obtained. These spaces, being of dimension $n-2$ and each meeting the curve in $\frac{1}{2}n(n-1)$ points, are analogous to the trisecants of the Jacobian curve of a net of quadric surfaces in [3]; but, whereas any twisted curve (of order greater than three) in [3] has a single infinity of trisecants, it is not usual for a curve in $[n]$ to have an infinity of spaces $[n-2]$ meeting it in $\frac{1}{2}n(n-1)$ points when $n > 3$.

§ 2. We consider a doubly-infinite linear system of quadric primals in n-dimensional space; the system is given algebraically by an equation

$$x\,Q_0 + y\,Q_1 + z\,Q_2 = 0,$$

where $x:y:z$ are variable parameters and Q_0, Q_1, Q_2 are three linearly independent homogeneous quadratic functions of $n+1$ coordinates ξ. Such a system of quadrics is called a *net* of quadrics; it will be denoted by **N**. All the quadrics of **N** have in common a *base locus* V^8_{n-3} of dimension $n-3$ and order 8; V^8_{n-3} is met by any solid in a set of eight associated points and by any [4] in a canonical curve of genus 5.

Of the ∞^2 quadrics belonging to **N**, ∞^1 are cones, and the locus of the vertices of these cones is a curve ϑ. When a quadric is a cone it has a double point at the vertex of the cone, so that the curve ϑ is given algebraically by the vanishing of all the three-rowed determinants of a matrix of three rows and $n+1$ columns, the elements of this matrix being the partial derivatives of the quadratic forms Q with respect to the coordinates ξ, and so being linear in the coordinates ξ. Thus ϑ is of order[1] $\frac{1}{2}n(n+1)$; it is called the *Jacobian curve* of **N**.

If we regard for the moment $x:y:z$ as the homogeneous coordinates of a point in a plane σ then the quadrics of **N** are *represented* by the points of σ; to each quadric of **N** there corresponds a definite set of values of the parameters $x:y:z$ and so a definite point of σ, and conversely. In this representation the cones of **N** will be represented by the points of some curve in σ, and the points of ϑ are clearly in (1, 1) correspondence with the points of this plane curve,

[1] Salmon, *Higher Algebra* (Dublin 1885), Lesson 19.

whose genus is therefore the same as the genus of ϑ. Now the condition that a quadric should be a cone is that its discriminant should vanish; hence the curve in σ whose points represent the cones of **N** is given algebraically by equating to zero a symmetrical determinant, of $n+1$ rows and columns, whose elements are homogeneous linear functions of x, y, z. The curve is therefore of order $n+1$ and, since it is actually without double points or cusps, its genus is $\frac{1}{2}n(n-1)$. Hence the genus of ϑ is also $\frac{1}{2}n(n-1)$.

§3. Those quadrics of **N** which pass through an arbitrary point O of $[n]$ are quadrics belonging to a pencil; they all pass through the same quartic locus V_{n-2}^4 and $n+1$ of them are cones, the vertices of these $n+1$ cones being the vertices of a simplex which is self-conjugate in regard to all the quadrics of the pencil. But the pencil of quadrics of **N** which pass through a point P on ϑ includes only $n-1$ cones whose vertices are at points of ϑ other than P; the cone whose vertex is P counts twice among the $n+1$ cones of the pencil. Moreover[1] the base locus V_{n-2}^4 of the pencil has a node at P, and all the quadrics of the pencil have the same tangent prime ϖ at P. This prime ϖ contains the vertices $A_1, A_2, \ldots, A_{n-1}$ of those cones of the pencil whose vertices are not at P. Every line which passes through P and lies in ϖ lies on a quadric of **N**.

Since ϑ is of order $\frac{1}{2}n(n+1)$ the prime ϖ meets ϑ in the n points $P, A_1, A_2, \ldots, A_{n-1}$ and in $\frac{1}{2}n(n-1)$ further points $X_1, X_2, \ldots, X_{\frac{1}{2}n(n-1)}$.

The cone of **N** whose vertex is at the point P of ϑ will be denoted by the symbol (P), and similarly for any other point of ϑ.

§4. In order that a quadric should contain a line it must satisfy three linear conditions, hence, since the quadrics of **N** have only freedom 2, there is no quadric of **N** containing an arbitrary line of $[n]$. But each quadric of **N** contains ∞^{2n-5} lines,[2] so that there are in all ∞^{2n-3} of the ∞^{2n-2} lines of $[n]$ which do lie on quadrics of **N**. The lines of $[n]$ which lie on quadrics of **N** therefore form a complex

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[1] For these statements *cf.* Segre: *Mem. Acc. Torino* (2), **36** (1884), 70. The pencil of quadrics has the characteristic [211 ...... 1], the number of 1's occurring in the symbol being $n-1$.

[2] *Cf.* Segre, *loc. cit.*, p. 36.

V. This may also be defined as the complex of lines which are cut in involution by the quadrics of **N**. The involution cut out on any line of V by the quadrics of **N** has two double points, and these double points are conjugate points in regard to every quadric of **N**; hence we have a third definition of V as the complex of lines which join pairs of points that are conjugate in regard to every quadric of **N**.

It is seen at once, for example by taking a section of the figure by an arbitrary [3], that V is a cubic complex; the lines of V which pass through a point O generate a cubic cone of $n-1$ dimensions. If however we consider those lines of V which pass through a point P of ϑ the cubic cone breaks up into the quadric cone (P) and the lines which pass through P and lie in ϖ. It follows that if a line of V meets ϑ in a point P and is not a generator of (P) then it must lie in the prime ϖ associated with P. Now there are $\frac{3}{2}n(n+1)$ lines passing through an arbitrary point O of $[n]$ which belong to V and meet ϑ; they are the lines common to the cubic cone generated by the lines of V which pass through O and the two-dimensional cone of order $\frac{1}{2}n(n+1)$ which projects ϑ from O. Of these $\frac{3}{2}n(n+1)$ lines $n+1$ are generators of cones of **N**, since the pencil of quadrics of **N** which pass through O includes $n+1$ cones. Hence the remaining $\frac{1}{2}(n+1)(3n-2)$ lines, although they meet ϑ, are not generators of cones of **N** and so lie in the primes ϖ associated with their respective intersections with ϑ. Clearly no other points of ϑ can give rise to primes ϖ which pass through O, hence *there are* $\frac{1}{2}(n+1)(3n-2)$ *of the primes* ϖ *passing through an arbitrary point of* $[n]$. We may say that the primes ϖ form a developable of class $\frac{1}{2}(n+1)(3n-2)$. The genus of the family of primes ϖ is of course $\frac{1}{2}n(n-1)$, the same as the genus of ϑ.

§5. The quadrics of **N** form a doubly-infinite linear system; hence the polar primes of an arbitrary point O in regard to them also form a doubly-infinite linear system and so have in common a space $[n-3]$. Every point of $[n-3]$ is conjugate to O in regard to every quadric of **N**, so that we may speak of the space $[n-3]$ itself as *the space conjugate to* O. This space $[n-3]$ conjugate to O is determined as the space common to the polar primes of O in regard to any three quadrics of **N** which do not belong to the same pencil. The space $[n-2]$ which joins O to its conjugate space is the tangent space at O of the base locus V_{n-2}^4 of the pencil of quadrics of **N** which pass

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through O. O lies in the space conjugate to it when and only when it lies on the base locus V_{n-3}^8 common to all the quadrics of **N**.

Consider now the polar primes of a point P of ϑ in regard to the quadrics of **N**. The polar prime of P in regard to any quadric of **N** which passes through P is ϖ ; hence the polar prime of P in regard to any quadric of **N** *not* passing through P meets ϖ in a space $[n-2]$ through which the polar primes of P in regard to *all* the quadrics of **N** must pass. The space conjugate to a point P of ϑ is therefore not an $[n-3]$ but an $[n-2]$. Each prime passing through the $[n-2]$ is the polar prime of P in regard to all the quadrics of a pencil.

Take now any one of the $\frac{1}{2}n(n-1)$ points X in which ϖ meets ϑ. Since the line PX passes through P and lies in ϖ it belongs to V, and is therefore cut in involution by the quadrics of **N**. The two double points of this involution are P and X, since the line is not a generator either of (P) or of (X). Wherefore each of the $\frac{1}{2}n(n-1)$ points X is conjugate to P in regard to all the quadrics of **N**, so that all these points X must lie in the $[n-2]$ conjugate to P. Hence *the* $[n-2]$ *conjugate to P meets ϑ in $\frac{1}{2}n(n-1)$ points*; it will therefore be called a *secant space* of ϑ, or, when its relation to P is relevant to the discussion, the secant space conjugate to P.

There is no point P of ϑ lying in its conjugate secant space; for such a point P would have to lie in its polar prime in regard to any quadric of **N**, and therefore on the base locus V_{n-3}^8. But, if the net **N** is a general net of quadrics, ϑ does not meet V_{n-3}^8.

It has been remarked that a prime passing through the secant space conjugate to P is the polar prime of P in regard to all the quadrics of a pencil belonging to **N**. Corresponding to two primes through the secant space conjugate to P there are two different pencils of quadrics; these two pencils, since they both belong to the same net, have a quadric in common, and the two primes are *both* polar primes of P in regard to this quadric. The quadric must therefore be the cone (P) itself. A prime through the secant space conjugate to P is therefore the polar prime of P in regard to the quadrics of a pencil, one of the $n+1$ cones belonging to the pencil being (P). The prime is therefore one of the bounding primes of the common self-conjugate simplex of all the quadrics of the pencil. Whence we have the following: *any prime passing through the secant space conjugate to P meets ϑ in n further points*; *these n points and P form a set of $n+1$ points which are vertices of cones of* **N** *belonging to the same pencil.* The converse is also true.

§6. The ∞^1 secant spaces, being in (1, 1) correspondence with the points of ϑ, form a family of genus $\frac{1}{2}n(n-1)$ and generate a primal R. If the secant space conjugate to the point P of ϑ meets ϑ in a point Q then P and Q are conjugate points in regard to all the quadrics of $\mathbf{N}$, and so the secant space conjugate to Q passes through P; conversely, if a secant space passes through P the point of ϑ to which it is conjugate lies in the secant space conjugate to P. Hence through any point of ϑ there pass $\frac{1}{2}n(n-1)$ of its secant spaces, or *the curve ϑ is of multiplicity $\frac{1}{2}n(n-1)$ on the primal R.*

The order of R is determined by an elementary application of the principle of correspondence. Suppose R is of order ν. There is a (1, 1) correspondence between the points of the curve ϑ of order $\frac{1}{2}n(n+1)$ and the $[n-2]$'s of the primal R of order ν; the primes which join the points of ϑ to the corresponding $[n-2]$'s of R are the primes ϖ, and these we have seen to form a developable of class $\frac{1}{2}(n+1)(3n-2)$. Hence, since no point of ϑ lies in the corresponding $[n-2]$ of R,

$$\tfrac{1}{2}n(n+1)+\nu=\tfrac{1}{2}(n+1)(3n-2),$$
$$\nu=n^2-1.$$

Hence *the secant spaces of ϑ generate a locus $R_{n-1}^{n^2-1}$.* In three-dimensional space this locus is the ruled surface R_2^8 generated by the trisecants of the Jacobian curve.

Any two secant spaces of ϑ meet in an $[n-4]$, so that, since there are ∞^2 pairs of secant spaces of ϑ, there is a locus of dimension $n-2$ of points of $R_{n-1}^{n^2-1}$ which are common to two secant spaces of ϑ; in other words $R_{n-1}^{n^2-1}$ has a double locus of dimension $n-2$. Similarly it has a triple locus of dimension $n-3$, a quadruple locus of dimension $n-4$, and so on. The section of $R_{n-1}^{n^2-1}$ by an arbitrary [3] is a ruled surface, the [3] meeting each secant space of ϑ in a line; the ruled surface is of order n^2-1 and genus $\frac{1}{2}n(n-1)$. This ruled surface has a double curve of order $\frac{1}{2}(n+1)(n-2)(n^2+n-3)$ on which there are $\frac{1}{6}(n^2-5)(n+1)(n^3-n^2-9n+12)$ triple points.[1] Hence the order of the double locus of $R_{n-1}^{n^2-1}$ is $\frac{1}{2}(n+1)(n-2)(n^2+n-3)$ and the order of the triple locus is $\frac{1}{6}(n^2-5)(n+1)(n^3-n^2-9n+12)$.

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[1] A ruled surface of order N and genus P in [3] has a double curve of order $\frac{1}{2}(N-1)(N-2)-P$, on which there are $(N-4)\{\frac{1}{6}(N-2)(N-3)-P\}$ triple points. See Edge, *The Theory of Ruled Surfaces* (Cambridge, 1931), 31, and the references there given.

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§7. Call the secant space conjugate to P, U_{n-2}; it meets ϑ in the $\frac{1}{2}n(n-1)$ points $X_1, X_2, \ldots. X_{\frac{1}{2}n(n-1)}$. In $[n]$ the polar space of an $[n-2]$ in regard to a quadric is a line, unless the quadric be a cone whose vertex lies in the $[n-2]$, in which case the polar space is a plane; suppose then that π_1 is the polar plane of U_{n-2} in regard to the cone (X_1). Since P is conjugate to U_{n-2} the polar prime of every point of U_{n-2} in regard to (X_1) passes through P, so that π_1 passes through P. Now the polar prime of any point X other than X_1, say of X_2, in regard to (X_1) contains both π_1 and the secant space conjugate to X_2; hence π_1 meets the secant space conjugate to X_2 in a line. Hence the secant spaces which are conjugate to the $\frac{1}{2}(n+1)(n-2)$ points $X_2, X_3, \ldots. X_{\frac{1}{2}n(n-1)}$ all meet the plane π_1 in lines through P. Wherefore we have the following. *Through any point P of ϑ there pass $\frac{1}{2}n(n-1)$ secant spaces. If one of these secant spaces is omitted the remaining $\frac{1}{2}(n+1)(n-2)$ are all met by the same plane in lines through P.* This plane is the polar plane of the secant space conjugate to P in regard to that cone whose vertex is the point of ϑ to which the omitted secant space through P is conjugate.

We have thus a configuration of $\frac{1}{2}n(n-1)$ secant spaces and $\frac{1}{2}n(n-1)$ planes passing through P. If we take a section of this by an arbitrary prime not passing through P we obtain in the prime a configuration of $\frac{1}{2}n(n-1)$ $[n-3]$'s and $\frac{1}{2}n(n-1)$ lines; each line meets all but one of the $[n-3]$'s. We have in fact the configuration known as the "double-$\frac{1}{2}n(n-1)$" of lines and $[n-3]$'s in $[n-1]$.

The plane π_1 meets ϑ in the points P and X_1 and only in these points. For suppose that π_1 contains a third point Y of ϑ. Since Y lies in π_1 the polar prime of Y in regard to (X_1) contains U_{n-2}; it also contains the secant space conjugate to Y. Hence U_{n-2}, the secant space conjugate to P, meets the secant space conjugate to Y in an $[n-3]$. But it is not in general true that there is a pair of secant spaces of ϑ with an $[n-3]$ in common.

We have therefore the following construction for obtaining the point of ϑ which is conjugate to a given secant space U. Take any intersection of U and ϑ; through this point, P say, there pass $\frac{1}{2}(n+1)(n-2)$ other secant spaces, and these are all met in lines by a plane π through P. π has one intersection, other than P, with ϑ, and this other intersection is the point of ϑ to which U is conjugate. Since U meets ϑ in $\frac{1}{2}n(n-1)$ points there are $\frac{1}{2}n(n-1)$ different ways of passing from U to the point of ϑ to which it is conjugate.

§ 8. When the quadrics of **N** are represented by the points of a plane σ the cones of **N** are represented by the points of a curve C^{n+1} of order $n+1$, without multiple points, and of genus $\frac{1}{2}n(n-1)$. The quadrics of any pencil belonging to **N** are represented by the points of a line in σ, the $n+1$ intersections of the line with C^{n+1} representing the $n+1$ cones which belong to the pencil. In particular, the quadrics of **N** which pass through a point P of ϑ are represented in σ by the points of the tangent of C^{n+1} at the point p which represents (P); *i.e.* at the point which corresponds to P in the (1, 1) correspondence between ϑ and C^{n+1}. The tangent of C^{n+1} at p meets it again in $n-1$ further points $a_1, a_2, \ldots, a_{n-1}$; these points correspond to the points $A_1, A_2, \ldots, A_{n-1}$ of ϑ which are vertices of cones passing through P. It will be remembered that the points $P, A_1, A_2, \ldots, A_{n-1}$ all lie in the prime ϖ, the common tangent prime at P of all the quadrics of the pencil.

The curve C^{n+1} has $3(n^2-1)$ points of inflection, the tangent of C^{n+1} at any one of these points only having $n-2$ further intersections with the curve. Correspondingly we have $3(n^2-1)$ points I on ϑ; the pencil of quadrics of **N** which pass through a point I only includes $n-2$ cones whose vertices are not at I. This may be regarded as a limiting case of the above, when one of the $n-1$ points A coincides with P. The quadrics of the pencil have *stationary contact* at I, and the common tangent prime ϖ of the quadrics at I contains the tangent of ϑ at I.

The number of tangents of ϑ which lie in the primes ϖ associated with their respective points of contact can be obtained by the principle of correspondence, if we remember that the order of the ruled surface generated by the tangents of ϑ is[1]

$$2 \cdot \tfrac{1}{2}n(n+1) + 2 \cdot \tfrac{1}{2}n(n-1) - 2 = 2(n^2-1).$$

For the primes ϖ, which form a developable of class $\frac{1}{2}(n+1)(3n-2)$, are in (1, 1) correspondence with the tangents of ϑ, which generate a ruled surface of order $2(n^2-1)$, and the locus of the points of intersection of corresponding primes and tangents is the curve ϑ, of order $\frac{1}{2}n(n+1)$. Hence, if i is the number of primes ϖ which contain the corresponding tangents of ϑ,

$$\tfrac{1}{2}(n+1)(3n-2) + 2(n^2-1) - i = \tfrac{1}{2}n(n+1),$$
$$i = 3(n^2-1).$$

[1] The order of the surface generated by the tangents of a curve of order ν and genus π, in space of any number of dimensions, is $2\nu + 2\pi - 2$.

But this is exactly the number of the points I. Hence *the* $3n^2-3$ *points of* ϑ *at which quadrics of* **N** *have stationary contact, and these points only, are such that the tangent of* ϑ *lies in the associated prime* ϖ.

§ 9. It follows from Plücker's equations that the number of bitangents of C^{n+1} is $\frac{1}{2}(n-1)(n-2)(n+1)(n+4)$. Suppose t_1 and t_2 are the two points of contact of a bitangent of C^{n+1}; then there will be two points T_1 and T_2 of ϑ corresponding to them. The points of the line $t_1 t_2$ represent the quadrics of **N** belonging to a pencil, and this pencil may be defined as consisting of the quadrics of **N** which pass through T_1 or else the quadrics of **N** which pass through T_2. All the quadrics of **N** which pass through either of T_1 or T_2 pass also through the other. Wherefore *two points of* ϑ *which correspond to the two points of contact of* C^{n+1} *with a bitangent are such that all the quadrics of* **N** *which pass through either of them pass also through the other.*

These pairs of points on ϑ, corresponding to the pairs of points of contact of the bitangents of C^{n+1}, can easily be identified. If ϑ is projected from an arbitrary $[n-3]$ on to a plane it becomes a plane curve of order $\frac{1}{2}n(n+1)$ and genus $\frac{1}{2}n(n-1)$; the number of double points of this plane curve is

$$\tfrac{1}{2}\{\tfrac{1}{2}n(n+1)-1\}\{\tfrac{1}{2}n(n+1)-2\}-\tfrac{1}{2}n(n-1)=\tfrac{1}{8}(n-1)(n-2)(n+1)(n+4).$$

This then is the number of chords of ϑ which meet an arbitrary $[n-3]$; or the chords of ϑ form a three-dimensional locus of order $\frac{1}{8}(n-1)(n-2)(n+1)(n+4)$. The number of points in which this locus meets the base locus V^8_{n-3} is therefore $(n-1)(n-2)(n+1)(n+4)$. Suppose now that $T_1 T_2$ is a chord of ϑ which meets V^8_{n-3}. Then this chord lies on both the cones (T_1) and (T_2) and therefore on all the quadrics of the pencil determined by these two cones. Any quadric which does not belong to this pencil and which yet belongs to **N** meets the line $T_1 T_2$ in two points which must be base points of **N**, and which therefore lie on V^8_{n-3}. Hence *any chord of* ϑ *which meets* V^8_{n-3} *is a chord of* V^8_{n-3}; the number of these chords of ϑ which are also chords of V^8_{n-3} is $\frac{1}{2}(n-1)(n-2)(n+1)(n+4)$. Since $T_1 T_2$ is a chord of V^8_{n-3} any quadric of **N** which contains either T_1 or T_2 must contain the whole of the line $T_1 T_2$ and therefore the other point as well. *The chord joining the pair of points of* ϑ *which correspond to the two points of contact of any bitangent of* C^{n+1} *is also a chord of* V^8_{n-3}, *and no other chords of* ϑ *can meet* V^8_{n-3}.