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A NOTE CONCERNING THE REAL INFLEXIONS OF REAL, PLANE, ALGEBRAIC CURVES

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THEOREM. "*If a crunode of a real, irreducible, plane, algebraic curve changes into an acnode via the intermediary stage of a real cusp, two real inflexions are introduced in a neighborhood of the double point.*"

This result is due to Klein [1].

We note that a crunode may change into an acnode without passing through the intermediary cuspidal stage. The objective of this note shall be to prove that unless the crunode becomes a cusp before becoming an acnode, the two associated real inflexions are not necessarily introduced.

We consider a pencil of real, elliptic, plane, quartic curves:

$$f_\varepsilon \equiv (x^2-2y^2+2z^2)^2-8x^2z^2+\varepsilon(x^2+y^2-z^2)^2 = 0.$$

It is clear that a curve of this pencil has double points at $(0, \pm 1, 1)$. The tangents at these points are real and distinct if $\varepsilon > -4$ and are pairs of complex conjugate lines if $\varepsilon < -4$. Therefore, if $\varepsilon > -4$, $(0, \pm 1, 1)$ are crunodes whilst if $\varepsilon < -4$, $(0, \pm 1, 1)$ are acnodes.

If $\varepsilon = -4$ the curve $f_{-4}=0$ degenerates into the repeated line $x=0$ together with the conic

$$\frac{x^2}{4/3}+\frac{y^2}{1/3} = z^2.$$

The associated hessian curve degenerates into the line $x=0$ with multiplicity four and the conic

$$\frac{y^2}{1/3}-\frac{x^2}{2/3} = z^2.$$

It is an elementary but lengthy procedure to show that:

(i) an acnodal curve of the pencil $f_\varepsilon=0$ has at most four real inflexions,

(ii) a crunodal curve corresponding to a value of ε such that $-4 < \varepsilon < -16/9$ has at least four real inflexions.

Therefore, no real inflexions are introduced as a result of the crunodes becoming acnodes. In fact, crunodal curves of this pencil possess exactly four whilst the

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acnodal curves possess no real inflexions. There is, therefore, in this example, a reduction rather than an increase in the number of real inflexions.

Reference

1. F. Klein, *Ueber eine neue Art von Riemann'schen Flächen*, Math. Ann. X (1876).

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