Convective Overshoot as a Source of Helicity

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Abstract: In order to understand what happens in the Sun when convection overshoots into the radiation zone an idealized model of penetrative convection with rotation is studied. Here we highlight two properties of the model which occur with parameters relevant to the Sun. Firstly rotation allows motions to persist far beneath the convection zone, and secondly the profile of helicity with depth is dominated by a local maxima just beneath the convection zone. This second result has consequences in dynamo theory.

1. Introduction

Overshoot at the base of the solar convection zone motivates this study of astrophysical penetrative convection, even though the simplest examples of rotating penetrative convection remain poorly understood. Stability analysis is used to determine preferred horizontal length scales, eigenfunctions and critical Rayleigh numbers, and we are particularly interested in isolating features arising in the two layer configuration that do not occur in a single layer of fluid. No magnetic fields are included here, but a helicity distribution is obtained from which an α-effect can be calculated, this effect being the cornerstone of mean-field dynamo theory (see, for example Krause and Rädler, 1980). A full account of the mathematical details appears in Jennings (1990).

2. The model

Many simplifications are adopted here, such as the Boussinesq approximation, a simple equilibrium, and a plane parallel geometry. Although formally there is initially no motion, we loosely think of there being a mean flow in the convection zone that is turbulent, while the lower layer's fluid is laminar. Thus we assign turbulent values to the thermal diffusivity and kinematic viscosity in the upper layer, and laminar values to these quantities in the lower fluid, and further consider the convective region to be only slightly superadiabatic in contrast to the lower fluid which is extremely subadiabatic. Ratios of these respective quantities yield dimensionless parameters:

$$K = \frac{\kappa_{laminar}}{\kappa_{turbulent}}, \qquad V = \frac{\nu_{laminar}}{\nu_{turbulent}}, \qquad B = \frac{\beta_{subcritical}}{\beta_{supercritical}} \,. \tag{1}$$

Both K and V are small and positive, while B is large and negative. Since the transition from a convectively unstable stratification to one that is stable to convection is probably very sharp in the Sun it is convenient to neglect the details of the transition and model using a step function (see Fig. 1).

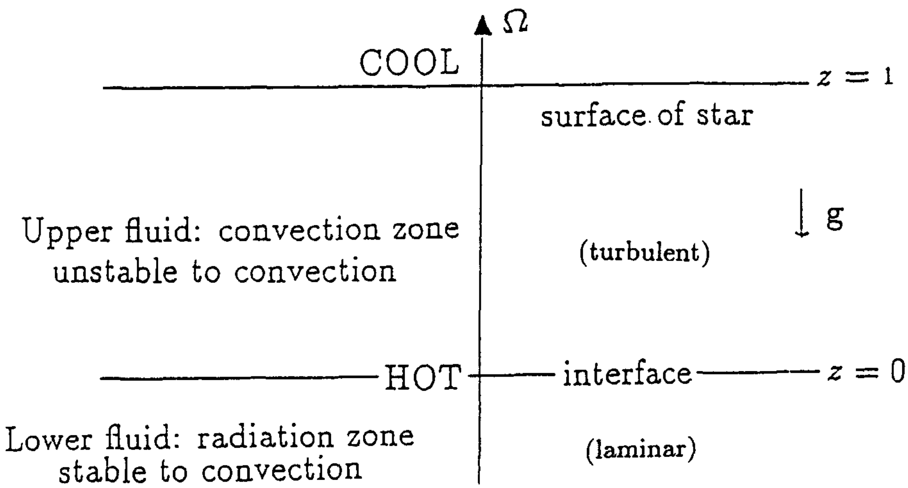

Fig. 1. The geometry; the stable layer is semi-infinite and $-\infty < x, y < \infty$.

2.1 Eigenvalues

Following the procedure in chap. 3 of Chandrasekhar (1961) we obtain equations for the vertical velocity perturbation $W(z)$ in each layer:

$$\left[\left(D^2 - a^2\right)^3 + TD^2 + Ra^2\right] W_{upper} = 0 \tag{2}$$

$$\left[\left(D^2 - a^2\right)^3 + \left(T/V^2\right) D^2 + \left(BRa^2/KV\right)\right] W_{lower} = 0 \,, \tag{3}$$

where $D = d/dz$, T is the Taylor number, R is the Rayleigh number, and $a^2 = a_x^2 + a_y^2$ is the horizontal wavenumber. Note that there is no difference here between x and y, and that overstable marginal solutions are not considered. Eqns. (2) and (3) together with the boundary conditions and conditions at the interface listed below define an eigenvalue problem, with non-trivial solutions for $W(z)$ only if R has a special value.

At $z = 1$ the boundary is impenetrable, stress free and isothermal, and as $z \to -\infty$ all perturbations vanish. At the interface ($z = 0$) we require the following quantities to be continuous:

- Vertical velocity;
- Horizontal velocity, (both x and y);
- Horizontal stress (both x and y);
- Normal stress;
- Temperature;
- Heat flux.

Analytic solutions for $W(z)$ in each layer which satisfy the boundary conditions at $z = 1$ and $z = -\infty$ are easily obtained. However, to satisfy the above conditions at the interface it is necessary to use numerical methods. Having fixed parameters T, V, K, B and a we seek the eigenvalue R. Varying only a, this is repeated until we find the critical value of a (a_c) at which R is minimized. This minimum value is the critical Rayleigh number R_c, and a_c gives the preferred horizontal length scale at the onset of convection.

3. Results

3.1 $V \ll 1$, an important limit

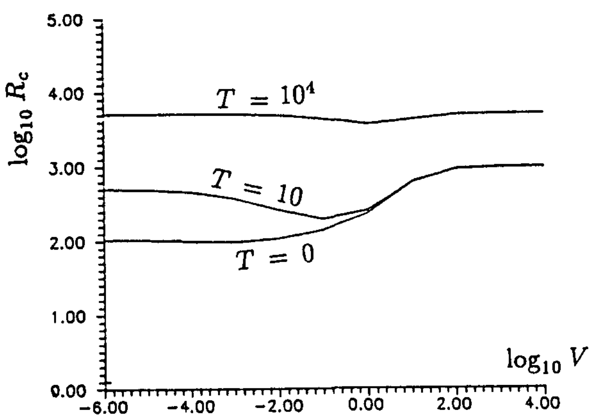

Fig. 2. Curves of $\log_{10} R_c$ versus $\log_{10} V$ at different values of T, with $K = -B = 1$ in each case.

From Fig. 2 it is seen that if $V \ll 1$ the results for no rotation ($T = 0$) are very different to those for small rotation ($T = 10$). In other words $T = 0$ is singular when V is small. To understand how this singularity arises for $V \ll 1$, consider Eq. (3) with $K = -B = 1$, which has solutions $W_{lower} \propto \exp(lz)$, where l is a root of the polynomial:

$$V^2 \left(l^2 - a^2\right)^3 + Tl^2 - VRa^2 = 0 . \qquad (4)$$

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In the case of no rotation ($T = 0$) the balance in Eq. (4) is:

$$Vl^6 - Ra^2 \approx 0\,, \tag{5}$$

from which we can deduce that there is no motion in the lower fluid except in thin viscous boundary layers of $O\left(V^{1/6}\right)$ thick. With rotation ($T \neq 0$), there are now two different balances:

$$V^2 l^4 + T \approx 0 \quad \text{and} \quad Tl^2 - VRa^2 \approx 0\,. \tag{6}$$

The first balance defines thin Ekman layers of $O\left(V^{1/2}\right)$, but the second shows that one wavenumber is very small, of $O\left(V^{1/2}\right)$, showing motions persist far beneath the interface to depths of $O\left(V^{-1/2}\right)$.

3.2 Helicity Profiles

Using the eigenfunctions we can compute the horizontally averaged helicity profile $\bar{h}(z)$;

$$\bar{h} = 2\left(\overline{WZ}\right) + a^{-2}\left[\overline{DZ\,DW} - \overline{Z\,D^2W}\right]\,, \tag{7}$$

where Z is the vertical vorticity, and bars denote horizontal averages. Eq. (7) shows that rotation is essential for the generation of helicity because without rotation Z decays to zero. An α-effect, is closely related to the flow's helicity; $\alpha \propto -h$, so the profiles $\bar{h}(z)$ provide qualitative information regarding the distribution of α in the Sun. Note however, that a formal derivation of α is via a two-scale analysis involving a large scale flow and small scale turbulence (Krause and Rädler, 1980), and therefore the relation here between $\bar{h}$ and α is not rigorous.

Estimating the parameter values which apply to the Sun in this model is not easy, but we took $K = 10^{-6}$, $B = -10^4$ and $T = 10^5$ after considering standard solar models discussed in Chaps. 1 and 2 of Priest (1982). There is considerable uncertainty with the value of V, but it is generally accepted that V is smaller than K, and we took $V = 10^{-8}$. For these parameters we find $R_c = 2 \times 10^4$ and $a_c = 8.5$ (tall thin cells), and the resulting helicity profile is shown in Fig. 3. Since helicity is a quadratic quantity its sign is unaffected by the normalization used for the eigenfunctions. The curve in Fig. 3 is normalized such that the maximum value is ± 1, and as can be seen this maximum is -1 and occurs just beneath the interface.

4. Conclusions

Two interesting results have come from this simple model which now need to be tested using better models. The first is that in the Sun motions can penetrate far below the convection zone as a consequence of rotation and V being small. Secondly, somewhat surprisingly overshoot leads to a large source of helicity just beneath the convection zone suggesting that a dynamo could work well there even

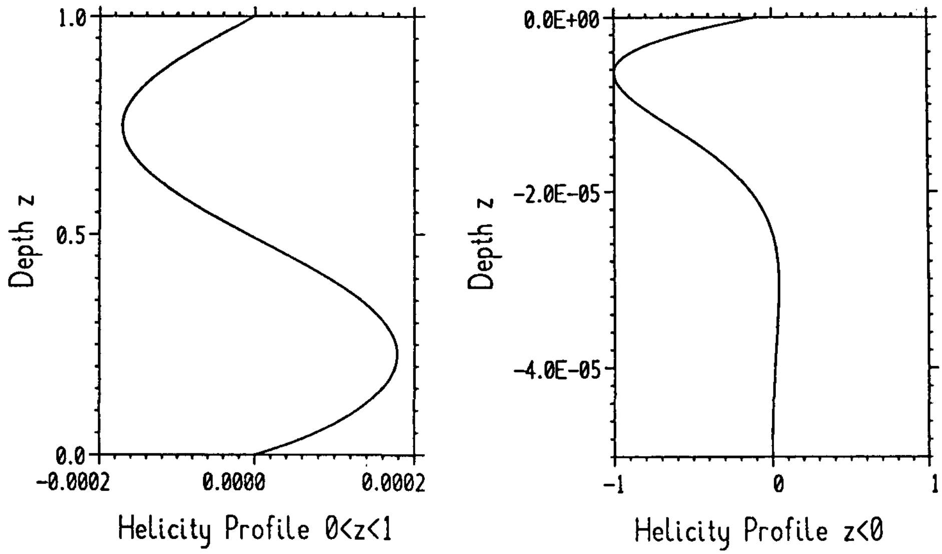

Fig. 3. Helicity profile for solar parameters. The scales of the two graphs are very different, and almost all the helicity occurs in a thin boundary layer beneath the interface. This helicity is negative so the local α-effect is positive. The shape of the profile in the upper layer is typical of the helicity that results in a single layer of convecting fluid.

though it is not the site of the most vigorous convection. This gives an α-effect which is positive in the northern hemisphere, which if correct implies that ω is negative because only when the product $\alpha\omega < 0$ do dynamo models yield the observed activity migrations from the poles to the equator.

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