GLOBAL REDUCTION OF FUNDAMENTAL ASTROMETRIC DATA

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ABSTRACT

Fundamental observational programs are often lengthened because of the sparsity of data during bad weather seasons and the need of having strong links all over the 24^h of right ascension. The global reduction procedure tries to minimize this problem by making more efficient use of the available information with full consideration of incomplete or isolated group observations. The whole set of observational material is treated as a single least squares problem, whose unknowns include corrections to the star positions. The problem is usually of huge dimensions, but we show that it can be reduced to quite tractable sizes. As an example, the method is applied to a two year series of astrolabe observations. The time and latitude curves are solved for under the form of cubic spline functions. The results are equivalent to those of conventional procedures, provided due account is given to the fact that, in the global reduction, long period components of image motion are fully included in the standard error estimates.

INTRODUCTION

Fundamental astrometric data, such as provided by transit circle and astrolabe observations, relate star coordinates to the instantaneous positions of the zenith. Since the rotation of the Earth has unpredictable components, the representation of the zenith motion poses a serious problem for obtaining final star positions. The classical way to face this difficulty consists of splitting the reduction procedure into two levels: in the lower one the observations are grouped in short duration observing tours, which are reduced individually, and, in the higher level, an over-all compensation among the observing tours is accomplished.

The chain method, which is employed in nearly all time and latitude services, is very simple and straighforward. However, in the course of its application to an observational series of the astrolabe at Valinhos ($\phi = -23^o00$, $\lambda = 46^o58'W$) we have encountered an important difficulty: because of weather conditions, many groups were incompletely observed

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and could not be used for the derivation of internal corrections. For the same reason, we often had isolated groups during the night and these could not contribute to the calculation of systematic differences between groups.

This difficulty is further aggravated by the fact that bad weather conditions are not evenly distributed along the seasons. The consequence of this is that the observing program has to be markedly lengthened to avoid in the chain links which are too weak.

From another point of view, we have been concerned with the final estimation of standard errors, both for star positions and time and latitude results. Neglecting of long period components of image motion, which are quite significant (Høg 1968) gives rise to a dispersion in the time and latitude curves which is usually greater than what could be inferred from formal internal standard errors. Also, in the case of astrolabe data, the final star coordinates corrections combine information from the chain adjustment with both (eastern and western) transits internal corrections and it is not clear whether, and how, eventual correlation among the data should be taken into account.

As an alternative to the chain method, we have devised the so called global reduction procedure. Its concept was inspired to a certain extent, by the overlap method of photographic astrometry, and is to consider each observed star transit as giving rise to an independent condition equation of a very extensive least squares problem, whose solution yields, in a single step, the time and latitude curves, as well as corrections to the stars positions.

A similar concept has been applied (Blaser 1959, Archinard 1970) to finding external group corrections, in a way which avoids the need for systematic differences of successive groups taken by pairs.

In the global reduction procedure every observation is useful and is considered on an equal footing with the remaining information, in a symmetrical way, so that poor observing seasons are most efficiently handled. Also, complete information on formal correlation among all the unknowns, including long period terms, can be derived from the least squares solution.

Of course, the global reduction procedure faces other difficulties, of which we may list at least four: 1. handling data of possibly non-uniform accuracy, 2. the personal equation problem, 3. adequate representation of the time and latitude curves, and 4., most important, its very practical feasibility, in view of the large dimension of the system of condition equations.

The main purpose of this paper is to demonstrate the practical feasibility of the global reduction procedure. It is show that the system of equations can be reduced to an equivalent system of much smaller di-

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mension and a practical application to a two year series of astrolabe observations is presented and discussed.

GLOBAL REDUCTION: THEORY

In the following we develop the theory for the astrolabe case, but the extension to other cases, such as transit circles, is quite obvious.

The condition equation relative to the transit of a star, identified by subscript $\underline{k}$, through the astrolabe locus of constant zenith distance may be written (Débarbat and Guinot 1970)

$$x_i \sin Z_k + y_i \cos Z_k + z + r_k = h_i \ , \tag{1}$$

where the meaning of the symbols is the following:

$x_i = \Delta t \cos \phi$,

Δt = clock correction,

ϕ = latitude,

y_i = correction to the assumed latitude,

z = correction to the assumed zenith distance,

Z_k = azimuth (from N to E) of star transit k,

r_k = correction to the star position in zenith distance,

h_i = observed time and other known terms.

Subscript $\underline{i}$ is the serial number of the transit, which increases monotonically with time. The clock correction and latitude are functions of time, so that $\underline{x}$ and $\underline{y}$ must receive the subscript $\underline{i}$ as well. The star transit identifier $\underline{k}$ is a function of $\underline{i}$, too.

Zenith distance variations, as a consequence of thermal deformations of the prism have been estimated (Benevides-Soares and Clauzet 1984) and were included in the independent term $\underline{h}$.

For those stars which are observed in both transits the corresponding $\underline{r}$'s may be combined to yield the correction in right ascension and, provided the transit is not too close to elongation, one can derive the correction in declination.

Suppose that the time functions $\underline{x}$ and $\underline{y}$ can be expanded in terms of suitable functions $f_j(t)$, so that

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$$x = x(t) = \sum_{j=1}^{p} a_j f_j(t) ,$$

$$y = y(t) = \sum_{j=1}^{p} b_j f_j(t) , \qquad (2)$$

where a_j and b_j, j=1 to p, are 2p unknown coefficientes.

Let the number of different transits be q; this is also the number of unknown corrections r_k. With z we have then a total of n unknowns, where n is given by

$$n = q + 2p + 1 \qquad (3)$$

Put

$$u = 2p + 1 \qquad (4)$$

and let c denote the vector of dimension u given by

$$c^T = | a_1 \ a_2 \dots a_p \quad b_1 \ b_2 \dots b_p \quad z | . \qquad (5)$$

Let r denote the q-vector whose components are r_k and let h denote the m-vector whose components are h_i, where m is the total number of observed transits.

The system of condition equations (1) may then be written

$$| E \,\vdots\, F | \begin{vmatrix} r \\ --- \\ c \end{vmatrix} = h , \qquad (6)$$

where we have introduced matrices E (dimension m x q) and F (dimension m x u) to represent the coefficients of r and c.

In practical cases system (6) is very large, as in the example below, where n ≅ 400 and m ≅ 4000, so that direct application od standard solution tecniques is out of question. One should remark, however, that only one of the r occurs in each of the condition equations, so that matrix E is very sparse. Its rows are series of zeroes, except for the k^{th} element, which is one. As a consequence, the columns of E are mutually orthogonal.

The orthogonal decomposition is

$$|E \,\vdots\, F| = |E \,\vdots\, Q| \; T , \qquad (7)$$

where Q is orthogonal (and orthogonal to E) m x u matrix and T is an upper triangular n x n matrix.

Matrix $\underline{T}$ may be partitioned into submatrices $\underline{I}$, $\underline{S}$, $\underline{O}$ and R in the following way

$$T = \left| \begin{array}{c|c} I & S \\ \hline O & R \end{array} \right| , \tag{8}$$

where $\underline{I}$ is the q x q identity matrix, $\underline{S}$ is a q x u rectangular matrix, $\underline{O}$ is the u x q rectangular null matrix and $\underline{T}$ is a u x u upper triangular matrix.

From (7) and (8) we arrive at

$$QR = F - ES . \tag{9}$$

But from (7) we see that E and Q are mutually orthogonal, so that

$$E^T QR = E^T (F-ES) = 0 ,$$

which can be solved for $\underline{S}$:

$$S = (E^T E)^{-1} E^T F . \tag{10}$$

With S in hand recast (9) in the form

$$QR = F - E(E^T E)^{-1} E^T F , \tag{11}$$

which means that $\underline{Q}$ and $\underline{R}$ may be obtained as an orthogonal decomposition of the matrix in the right hand term of (11). Notice that while the original problem was of dimensions m x n, with n = q + u, the reduced problem is of dimensions m x u only (4000 x 11 instead of the original 4000 x 400, in the example below).

The matrix operator $(E^T E)^{-1} E^T$ in (10) may be interpreted simply: when applied to an m-vector, it produces a q-vector whose elements are the average of the elements of the previous vector which correspond to the observations of each of the $\underline{q}$ different transits.

Once the decomposition (7) is accomplished, it is a trivial matter to solve the least squares problem (6) and obtain the covariance matrix of the unknowns.

There is still a problem left, due to the fact that system (6) is indeterminate, because the origins of right ascensions, declinations and zenith distance are not known. This indeterminacy is not essential and can be eliminated by adopting convenient conditions, which, in our case, are the following:

$$\sum_{k=1}^{q} r_k = \sum_{k=1}^{q} r_k \sin Z_k = \sum_{k=1}^{q} r_k \cos Z_k = 0 ;$$

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which is equivalent to adopting the corrections zero to the mean right ascension and declination of the program stars.

APLICATION TO A TWO YEAR ASTROLABE SERIES

The global reduction procedure was applied to an observation series extending a little over two years, from 1979 to 1981. The program comprised 367 different transits of FK4 and FK4Supp. stars, which were on the average observed 10 times, giving a total of 3914 observed transits. In spite of the markedly uneven distribution of clear nights, no transit was observed fewer than 7 times. Only two observers took part in the whole program.

The selection of the functions $f_j(t)$, with which we wish to represent the variations of clock corrections and latitude, is clearly a matter of importance. In the present context, where the main purpose was to test the method, we have not attempted to make an optimal selection. We tried, instead, to adopt a solution of least commitment. In this sense, cubic spline functions seem to be adequate, since they are known to interpolate curves of almost any shape very well and they display optimal properties of mean curvature (e.g., Stoer and Bulirsch 1980).

Next we had to decide on the number of nodes of the spline function. We tried four interval lengths, ranging from 120 to 210 days, that is, from 8 to 5 nodes, for the observing period of 840 days. As usual, we added the boundary conditions of zero curvature. The results proved to be quite independent of the interval lenght and we adopted the longest one, of 210 days (5 nodes), for reasons of parsimony.

The observations were timed in the UTC scale and the difference UT1-UTC, as published by the BIH, was added to the observed times, so that our "clock" correction Δt stands for

$$\Delta t = UT0 - UT1 , \qquad (12)$$

which, apart from an error in the adopted mean longitude and possible local effects, depends only on the polar motion.

The form the orthogonal decomposition QR of (11) we have used the modified Gram-Schmidt algorithm (c.f., e.g., Brosche 1966, or Lawson and Hanson 1974).

The resulting time and latitude curves are displayed in Figs. 1 and 2, respectively, where dots represent the usual solution by individual groups, with internal and external corrections. The dispersion of the points relatively to the curves is high, specially for latitude, where the r.m.s. of the residuals is larger than 0".1. However, in both cases, the spline function is as good an interpolating curve as one could wish.

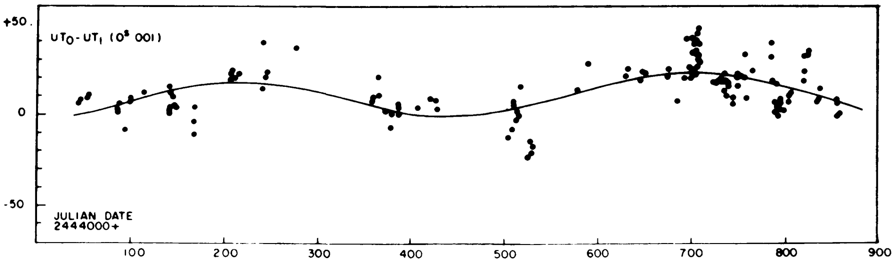

Fig. 1 - Time interpolating spline. Dots represent conventional group results.

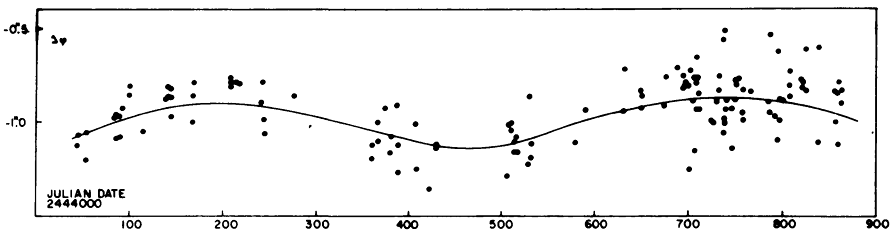

Fig. 2 - Latitude interpolating spline. Dots represent conventional group results.

The long period oscillation in both curves should be looked at with caution. They do resemble Chandler components, but they are in phase, while for true polar motion they should be in quadrature.

We have compared our results to the polar motion, as given by the BIH. The differences are smooth and do not display any significant structure. The r.m.s. of the differences is 6 ms for the time curve and 0"08 for the latitude curve. These values compare favourably with individual group results, giving due account to long period external errors.

The standard error for a single transit in the global reduction turned out to be 0"36, which is markedly higher than the value 0"24, typical of individual groups reduction. The reason is the influence of long period components of image motion, which displace individual groups as a whole and leave undisturbed their internal agreement. We believe that our result is a more realistic estimate of the overall accuracy attainable with this kind of observational technique.

For those stars observed in both transits or in elongation, the corrections in zenith distance yield corrections to right ascension and declination. For the computation of standard deviations of those corrections, the correlations involved where duly considered. The results will be published and discussed elsewhere. In Fig. 2 we just give the systematic error of the type $\Delta\alpha\delta$, in comparison with other sources. For

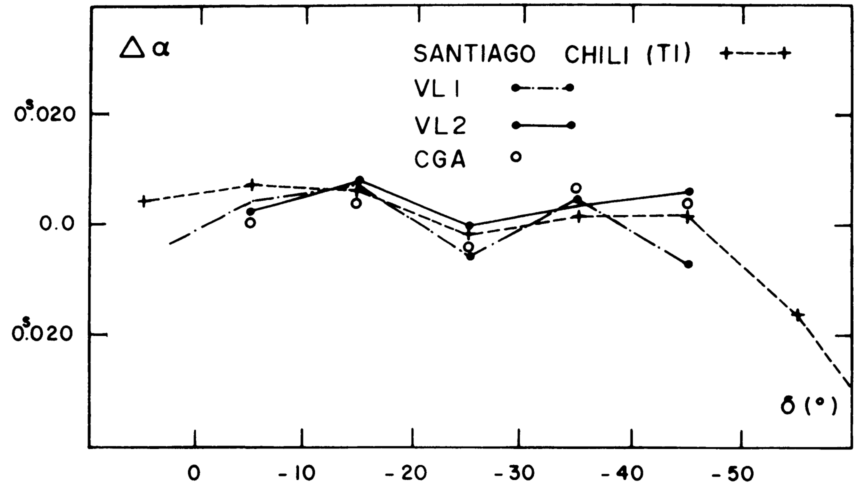

Fig. 3 - Systematic differences Δαδ, observed - FK4

the declination zone under consideration this error is not very important and all curves show good agreement.

CONCLUSIONS

The main conclusion is that the global reduction procedure is quite feasible for treating long series of fundamental astrometric data. The results are comparable to those of conventional methods, but the available data is more efficiently used. In spite of the apparently excessively large dimension of the numerical problem, the effort required by the global reduction procedure is quite moderate and stands a good comparison with the chain method.

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REFERENCES

Anguita C., Zverev H.S., Carrasco G., Loyola P., Naremova A., Nemiro A., Polojentsev D., Taibo R. and Shshikina V.: 1973, Inf. Bull. for the Southern Hemisphere, 22, pp28-34.
Archinard M.: 1970, Thèse, Université de Paris.
Benevides-Soares P. and Clauzet L.B.F.: 1984, Astron. Astrophys., 134, pp338-342.

Billaud G., Guallino G. and Vigoroux G.: 1978a, Astron. Astrophys. 63, pp87-95.

Billaud G., Guallino G. and Vigoroux G.: 1978b, Astron. Astrophys. Supp. Ser., 31, pp159-166.

Blaser J.P.: 1959, Pub. Observ. Neuchâtel, N. 5, 23pp.

Brosche P.: 1966, Veroff. Astron. Rechen-Inst. Heidelberg, Nr 17, 27pp.

Clauzet L.B.F.: 1983, Astron. Astrophys. Supp. Ser., 52, pp403-410.

Débarbat S. and Guinot B.: 1970, La méthode des hauteurs égales en astronomie, Gordon & Breach, Paris, 138pp.

Høg E.: 1968, Z. Astrophysik, 69, pp313-325.

Lawson C.L. and Hanson R.J.: 1974, Solving least squares Problems, Prentice-Hall, Englewood Cliffs, New Jersey, 340pp.

Stoer J. and Bulirsch R.: 1980, Introduction to numerical analysis, Springer Verlag, New York, 608pp.

Discussion:

GUINOT: I strongly agree with all you have said. The chain method is not a good one, because it loses informtion due to the continuity of the measured quantities. For instance, it assumes that the values of latitude during one day are totally independent of the latitude during the next 24 hours, which is evidently not true. Moreover, only global treatment can give correct estimations of the uncertainties, but the chain method cannot.