Light curve analysis of rotating variable stars

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Abstract: A short description of a method for analysing light curves of rotating variable stars is given. This method is applied to light curves of HD 24712.

Introduction

According to the oblique rotator model, the typical brightness variations of magnetic CP stars are caused by an inhomogenous distribution of brightness over the stellar surface. Consequently, the light curves of spotted stars contain some information on large scale brightness distribution, which can be approximatively extracted from the light curves.

For this end we have to base the specific inverse problem (determination of some parameters from light curves) on a model of the type of the brightness distribution. The model has to be assumed in such a way that

i) the parameters to be determined should be interpretable by theory. For example, this model should allow a comparison with models of distributions of the magnetic field or of chemical elements.
ii) The model should not include a greater number of free parameters than can be derived from all the observed light curves of a given star.

The spot model

The model which serves as a basis is characterized by the following assumptions:

i) A very limited number of (more or less extended) spots on a spherical stellar surface may exist. (The light curves do not comprise any information on a possibly existing "small scale structure".)
ii) The fluxes derived from each spot and from the rest of the stellar surface should be described by a single parameter each of which depends on the wavelength λ.
iii) Each spot is to be of circular outline.
IV) We assume identical (linear) limb darkening laws inside and outside the spots. (Analogical to the analysis of light curves of eclipsing binaries we can expect that big errors in the limb darkening law - will result in only small errors of spot parameters - see Al Naimiy (1978)).

C. R. Cowley et al (eds.), Upper Main Sequence Stars with Anomalous Abundances, 171–174.

V) The values of the limb darkening coefficients u (λ) as well as the inclination angle i between the line of sight and the rotation axis have to be known.

The monochromatic flux L, integrated all over the visible hemisphere and going into the direction of the observer, will then be

$$L = L_o + \sum_{m=1}^{M} \frac{(q_m-1)\cdot L_o}{\pi \cdot R^2 \cdot (1-u/3)} \iint_{G_m} (1-u+u\cdot\cos\gamma)\cdot\cos\gamma\cdot ds \quad (1)$$

$$\cos\gamma = \cos i \cdot \cos\beta + \sin i \cdot \sin\beta \cdot \cos(\vartheta - \varphi)$$

$$ds = R^2 \sin\beta \, d\beta \, d\vartheta$$

where L_o - flux from the unspotted surface, q - ratio of flux densities inside and outside a spot, R - stellar radius, u - limb darkening coefficient, M - number of spots, G - area, occupied by a spot on the visible hemisphere, i - inclination angle, φ - rotation phase angle, β - polar distance and ϑ - stellar longitude.

To integrate numerically, we will devide the spot area into elements of selected extent ($D\vartheta$ x $D\beta$) and will test whether or not the given element is visible. Furthermore, we have to take into account that L and Lo are merely proportional to the observed values. In a simple way, we will replace L by the ratio of measured (and reduced) intensities of the variable and the comparison star, I/I_c. All the light curves of a given star (the observations I/I_c (φ, λ)) yield a system of equations, from which we have to estimate the following parameters:

I_o (λ) - the intensity level, which is observed in the case that the visible hemisphere is unspotted, and for each spot
q (λ) - the contrast between the spot and its surroundings,
ϑ_o - the longitude and
β_o - the angle to the rotation axis, of the spot center,
α - the spot radius.

With an approximative solution at α = DB/2, the spot radius will be expanded iteratively, and in each phase of iteration the other spot parameters will be calculated according to the method of differential correction of the parameters. In some special cases, β_o has also to be determined iteratively. As corresponding investigations have proved, this procedure secures a correct determination of even ambiguous solutions.

Light curve analysis of HD 24712

The rotational variability of this FOp star has been determined by Wolff and Morrison (1973) and also by Kurtz (1982). Kurtz determined the elements of the variability to be $JD_o = 2440578.0 + 12.458$ E.

Those light curves which have the greatest amplitudes were analysed, that is the light curve in v (Wolff and Morrison) as well as in B (Kurtz). Since each light curve is symmetrical to phase 0, a one spot model is capable to adapt the light curve completely in all colours. Our calculations have been based on an inclination angle of $i = 30°$. (According to Kurtz (1982) a small i isto be expected). Limb darkening coefficients were taken from Al-Naimiy (1978).
The results of the light curve analysis for both independently investigated (and independently observed) light curves are completely adequate as to the solutions for the geometrical parameters (location of spot and spot radius). We obtained a double solution. The geometrical parameters are the following:

	solution 1	solution 2
longitude	$\lambda_{01} = 2° \pm 1°$	$\lambda_{02} = 2° \pm 1°$
polar distance	$\beta_{01} = 47° \pm 1°$	$\beta_{02} = 110° \pm 40°$
spot radius	$\alpha_1 = 25^{\circ\,+20°}_{\;\;-15°}$	$\alpha_2 = 75^{\circ\,+10°}_{\;\;-15°}$

In both cases the spot is darker than its environment. The strongly differing error ranges in the two solutions are the result of the special correlation of spot radius and polar distance. With increasing α, the polar distance remains almost constant in the range of solution 1, while β_o rapidly grows in solution range 2.
Kurtz (1982) developed an oblique pulsator model of HD 24712. Starting from the observation of a pulsational variability (Kurtz, 1982) and from the variation of the effective magnetic field B_{eff} (Preston, 1972), he defined an axis of symmetry for the pulsation as well as for the field distribution. According to his particulars and to the value we used for i, this axis completely agrees with the spot center in solution 1. From this we conclude that the field distribution (as well as the geometry of pulsation) and the surface brightness distribution are strongly correlated.
However we have to take into consideration the ambiguous interpretations of the light variability and of the

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variation of B_{eff}. Oetken (1977) developed a magnetic field model with equatorial symmetry for this star. The equatorial surface brightness distribution (according to solution two) is described by the following parameters:

$\vartheta_0 = 2^\circ$, $\beta_0 = 90^\circ$, $\alpha = 66^\circ$, q (B) = 0.88, I_o/I_c(B) = 0.796

The comparison between this model and the observations is shown in fig. 1.

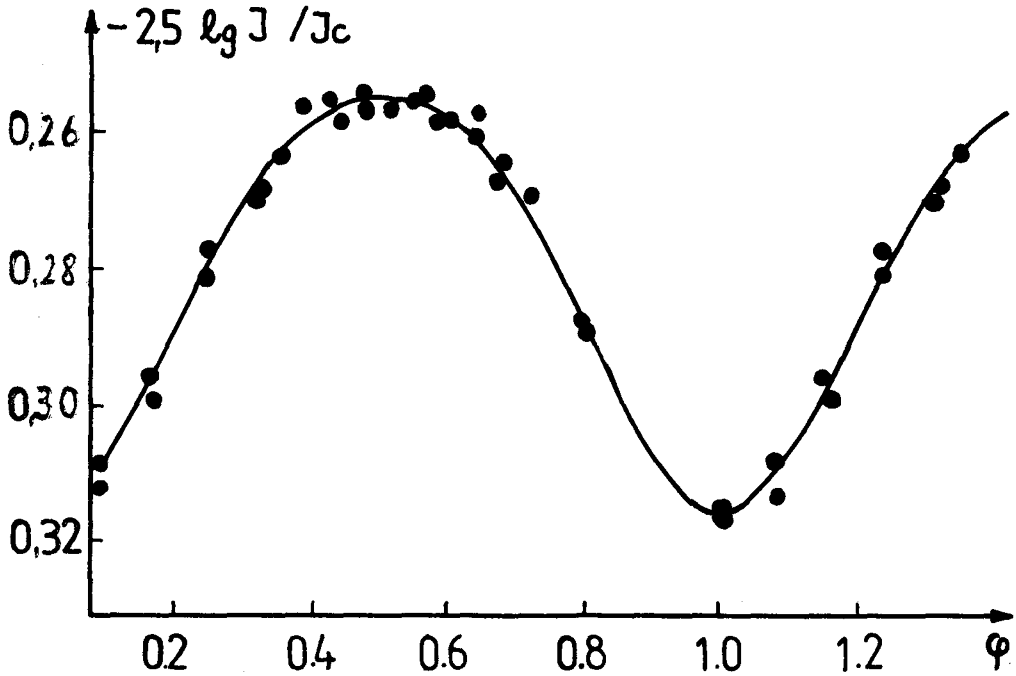

Fig. 1: Comparison between the observations (•) in B (Kurtz, 1982) and the one spot model of equatorial symmetry (—).

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