GALACTIC MASS MODELING

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1. INTRODUCTION

Galactic mass modeling has a long history. The first mass models were designed to represent the galactic attraction force in the radial direction. Considerable progress in galactic mass modeling was made during the fifties when Kuzmin (1952) introduced nonhomogeneous ellipsoids and Schmidt (1956) used a number of ellipsoids to represent various galactic populations. Further progress in galactic mass modeling has followed with the improvement of the system of the galactic constants and with the improvement of our knowledge of the structure of the galactic populations, in particular with the discovery of a massive corona around the Galaxy. In this report we present a new mass model of the Galaxy. It has been constructed using the most recent data available. A preliminary version of this model has been discussed earlier by Einasto, Jôeveer and Kaasik (1976).

2. THE METHOD OF MASS MODELING

By a model of the Galaxy we mean a set of functions and parameters which quantitatively describe the principal properties of the Galaxy and its populations. The main functions needed to describe the Galaxy are the gravitational potential, Φ, and its radial and vertical derivatives K_R, K_z. The structure of the galactic populations is given by the spatial density, ρ_i, the projected density, P_i, velocity dispersions in the cylindrical coordinates, σ_R, σ_θ, σ_z, and the centroid velocity V_i.

The galactic descriptive functions are interrelated by a set of formulae, the form of which depends on the principal properties of the Galaxy. On the basis of the existing data we may assume that the Galaxy is well relaxed, that its populations are physically homogeneous, and that equidensity contours of the galactic populations are similar concentric ellipsoids or can be represented in the form of sums of such ellipsoids. Under these assumptions simple relations hold between all the descriptive functions (Schmidt 1956, Einasto 1974).

W. B. Burton (ed.), The Large-Scale Characteristics of the Galaxy, 451–460.

The most convenient way of determining a model is to use a certain analytic expression for the density of the galactic populations. Schmidt (1956) and Innanen (1966, 1973) have used a polynomial expression for the density. Our expression has shown that a better representation can be obtained by the use of a modified exponential function (Einasto 1974)

$$\rho(a) = \rho(0)\ \exp\ \{x - [x^{2N} + a^2(ka_o)^{-2}]^{1/2N}\}, \tag{1}$$

where $a = R^2 + z^2/\varepsilon^2)^{\frac{1}{2}}$ is the major semiaxis of the equidensity ellipsoid, ε is the axial ratio of the ellipsoid, $\rho(0) = hM(4\pi\varepsilon\ a_o{}^3)^{-1}$ is the central density, M is the mass of the population, a_o is the harmonic mean radius of the population, x and N are structural parameters of the model, and h and k are dimensionless normalizing constants. The density distribution in the massive corona can be represented by a modified isothermal model (Einasto, Jôeveer and Kaasik 1976). The practical procedure of modeling consists of three steps: determination of the system of the galactic constants; determination of the parameters of the galactic populations; calculation of descriptive functions.

3. THE SYSTEM OF GALACTIC CONSTANTS

In the second Schmidt (1965) model the following principal galactic constants were adopted: $R_o = 10$ kpc, $V_o = 250$ km s^{-1}, A = 15 km s^{-1} kpc^{-1}, $\rho_o = 0.15\ M_\odot\ \mathrm{pc}^{-3}$. These values have been recommended by the IAU for general use. Recent observational data favor values of R_o, V_o, and ρ_o smaller than the conventional values. Table 1 summarizes the mean values of recent independent determinations of the galactic constants and their estimated rms errors. Here we use the designation

$$W = \frac{1}{2}\frac{dU}{dx} = AR_o, \tag{2}$$

where U is maximum relative radial velocity of rotation in the inner parts of the Galaxy (Fig. 1) and $x = R/R_o$;

$$C^2 = -\ (\partial K_z/\partial z)_{z=0} \tag{3}$$

and

$$k_z = \sigma_z^2/\sigma_R^2. \tag{4}$$

Some comments on the mean mass density, ρ_o, and the circular velocity, V_o. The galactic mass density adopted in this paper is close to Oort's (1932) first determination. The conventional IAU value is based on Hill's (1960) determination. As demonstrated by Eelsalu (1961), the method used by Hill is not very sensitive and thus his value is of little weight.

Recently Lynden-Bell and Lin (1977) have suggested a very high value, $V_o = 294$ km s^{-1}, for the circular velocity. This value is based on the mean velocity of the Sun with respect to the members of the Local Group. When discussing the dynamics of the Local Group, Lynden-Bell and Lin have treated together both the companions of the Galaxy and the distant members of the Group. To determine the solar velocity with

Table 1. Galactic Constants

Constant	Unit	Observed Value	Smoothed Value	Adopted Value	References
R_o	kpc	8.8 ± 0.7	8.5 ±0.3	8.5	1,2
V	$km\ s^{-1}$	220 ±10	221 ±5	225	3
W	$km\ s^{-1}$	120 ±15	133 ±4	131.8	4
A	$km\ s^{-1}\ kpc^{-1}$	16 ± 1	15.7 ±0.4	15.5	5-7
Ω	$km\ s^{-1}\ kpc^{-1}$	26. ± 2	26.0 ±0.7	26.5	8-10
k_z		0.282± 0.020	0.285±0.008	0.293	11
ρ	$M_\odot\ pc^{-3}$	0.1 ± 0.02		0.097	12,13
C	$km\ s^{-1}\ kpc^{-1}$	70 ± 5		74	14

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respect to the galactic center, one should use only objects moving in space together with the Galaxy. These objects form the galactic subgroup, our Hypergalaxy (Einasto 1977) in the Local Group. All members of the Hypergalaxy are located in a sphere with radius of 250 kpc around the galactic center. Using only these close companions of the Galaxy, we obtain $V_o = 220\ km\ s^{-1}$. If we omit close companions and consider only distant members of the Local Group, we obtain the sum of the solar and the galactic motion, $V_o + V_{Gal} = 380\ km\ s^{-1}$. We note that other methods, based on the inner dynamics of the Galaxy, yield also V_o = 215 - 225 $km\ s^{-1}$.

The observed values of the galactic constants are subject to random and undetected systematic errors. For this reason they do not exactly satisfy equations connecting individual galactic constants with each other. To remove the role of these errors to some extent, we have found by the method of least squares a smoothed and mutually concordant system of galactic constants where all equations are exactly fulfilled (for details see Einasto and Kutuzov 1964). This system of galactic constants, as well as the adopted system, is presented in Table 1. The last system differs from the previous one by the use of rounded values for principal constants.

4. GALACTIC POPULATIONS

The present model incorporates the following galactic populations: the nucleus, the bulge, the halo, the disk, the flat populations, and the massive corona. Parameters of galactic populations are given in Table 2. The structural parameters N and x of the galactic populations

Table 2. Parameters of galactic populations

Population	ε	a_0 (kpc)	M ($10^{10}M_\odot$)	N	x	h	k
Nucleus	0.6	0.005	0.009	1	0.5	2.871	0.48188
Bulge	0.6	0.21	0.442	1	0.5	2.871	0.48188
Halo	0.3	1.9	1.2	4	3.5	101.66	1.2304×10^{-4}
Disk	0.1	4.62	7.68	1	0.5	2.871	0.48188
	0.45	1.026	-0.379	1	0.5	2.871	0.48188
Flat	0.02	6.4	1.00	0.5	0	1.5708	1.1284
	0.025	5.12	-0.64	0.5	0	1.5708	1.1284
Corona*	1	75	110	0.5	25.12	8.3806	0.2575

* For the corona an isothermal model has been used. For the meaning of parameters N and x, see Einasto, Jôeveer and Kaasik (1976).

have been adopted mainly on the basis of the study of external galaxies. Other parameters and properties of populations will be discussed in the following sections.

(a) The nucleus. Its parameters have been adopted by analogy with the Andromeda galaxy.

(b) The bulge. According to Arp (1965), the radius of the bulge of the Galaxy is about half of the radius of the bulge of M31. The adopted mass and radius were found by the method of least squares from the first maximum of the rotation velocity curve (Fig. 1).

(c) The halo. The radius of the halo was determined from the spatial distribution of globular clusters (Kukarkin 1974, Woltjer 1975). The mass of the halo was estimated on the basis of the density determination of the halo population objects in the solar vicinity (Oort 1958, Schmidt 1975). Our halo represents all metal-deficient populations, both the extreme halo and the intermediate population II objects. For this reason a moderate value, $\varepsilon = 0.3$, has been adopted for the axial ratio of the halo.

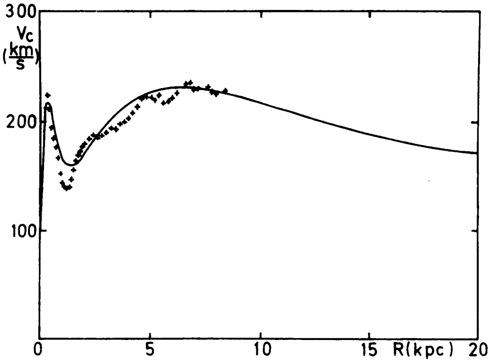


Fig. 1. Adopted curve of the circular velocity and of the observed rotation velocity (crossed).

Table 3. Galactic functions

R (kpc)	V (km s^{-1})	V_{esc}	A (km s^{-1} kpc^{-1})	-B	C	k_z	log ρ ($M_\odot$ pc^{-3})	log P ($M_\odot$ pc^{-2})
0.0	0	874	30168	32166	120740	0.500	5.438	5.476
0.1	150	756	247	1257	1776	0.455	2.059	4.355
0.2	202	727	335	657	1343	0.401	1.663	4.063
0.3	218	702	326	402	961	0.356	1.282	3.760
0.4	217	682	297	246	724	0.312	0.938	3.486
0.5	208	666	257	160	573	0.277	0.660	3.276
1.0	167	628	105	62	321	0.270	0.213	3.007
1.5	160	611	49	58	266	0.351	0.128	2.976
2.0	170	598	30	55	237	0.394	0.047	2.930
2.5	184	587	23	50	213	0.406	-0.038	2.870
3	196.3	575	20.9	44.5	193	0.405	-0.126	2.802
4	215.2	554	19.7	34.1	160	0.388	-0.301	2.656
5	225.8	534	19.1	26.1	133	0.366	-0.470	2.503
6	230.0	516	18.2	20.1	112	0.344	-0.631	2.347
7	229.9	500	17.2	15.6	95	0.322	-0.785	2.190
8.5	225.0	479	15.5	11.0	74	0.293	-1.011	1.951
10	217.1	462	13.8	7.9	57.8	0.268	-1.243	1.707
12	205.0	444	11.5	5.5	40.5	0.245	-1.576	1.372
14	193.6	430	9.6	4.3	27.7	0.236	-1.939	1.029
16	184.0	419	7.9	3.6	19.0	0.240	-2.310	0.686
18	176.6	410	6.5	3.3	13.6	0.252	-2.643	0.360
20	171.1	402	5.44	3.11	10.43	0.267	-2.894	0.063
30	159.4	373	2.91	2.40	5.46	0.311	-3.346	-0.913
50	155.6	336	1.59	1.53	3.17	0.329	-3.746	
75	154.4	304	1.06	1.00	2.09	0.328	-4.117	
100	152.6	283	0.81	0.72	1.54	0.321	-4.409	

(d) The disk. The mass and the radius of the disk were determined by the method of least squares from the rotation velocity curve. The disk represents galactic populations over a wide range of axial ratios between the flat and the intermediate population objects. Its axial ratio, $\varepsilon = 0.1$, is a compromise. There exists evidence indicating that the disk has a ring-like structure. Absence of interstellar hydrogen and young stars near the center of Sb galaxies is a well-known observational fact (Baade and Arp 1964, Roberts 1966, Burton et al. 1975). Stars are formed from the interstellar gas. Thus, if the interstellar gas had been absent in the central region also in the earlier period of Galaxy history, the whole disk should have a minimum in the density distribution near the center. The mass distribution of such a ring-like population can be represented as a sum of two components with identical structural parameters, but with different M, a_o, and ε. If the density of the disk at the center were zero, the parameters of both components should be chosen as follows: $M_2 = -\kappa^2 M_1$, $a_{o_2} = \kappa a_{o_1}$ $\varepsilon_2 = \kappa^{-1} \varepsilon_1$, where $\kappa < 1$ is a parameter which determines the extent of the "hole" in the center of the disk. By introducing the hole into the disk it is possible to represent the deep minimum in the rotation curve of the Galaxy at R = 1.5 kpc. The value of the parameter κ has been derived by trial-and-error to achieve the best possible representation of the rotation curve between the two maxima.

(e) The flat population. This population represents the interstellar gas and young stars. It has a ring-like structure (Fig. 2). The radius a_0, the axial ratio, and the parameter κ were derived from the data available on the distribution of gas and young stars. The mass of this population was determined on the basis of density estimates of gas and young population stars.

(f) The corona. The velocity dispersion of the companions of our Galaxy is only slightly smaller than the velocity dispersion of galactic globular clusters, thus the velocity dispersion remains practically constant over a wide range of distances. This suggests that our Galaxy is surrounded by a massive corona. The harmonic mean radius of the system of galactic companions is a_0 = 75 kpc; this value has been adopted for the corona. The mass of the corona has been calculated from the virial theorem, adopting σ_R = 85 km s^{-1} for the velocity dispersion of the coronal objects. Visible elements of the corona (galactic companions) form a flat disk. The form of the invisible corona is at present unknown; in the model we adopt a spherical corona.

5. DISCUSSION

Table 3 contains some descriptive functions. A number of the calculated descriptive functions are shown in Figures 1-6.

The present model differs from the Schmidt and Innanen models in two principal aspects: a new system of galactic constants has been adopted and a massive corona has been added. These changes have been made necessary by the body of available observational data. One consequence of the presence of a massive corona is a very high escape velocity which does not imply a high circular velocity (Table 3).

In our model the velocity dispersion of objects of a homogeneous population slightly decreases with increasing distance from the galactic plane (Figs. 3 and 4). This is a direct consequence of the use of the exponential density law. Schmidt and Innanen have used polynomial laws

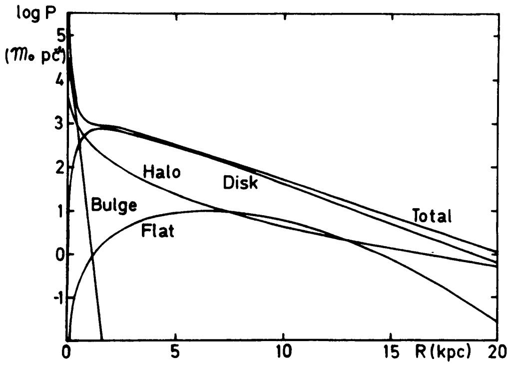
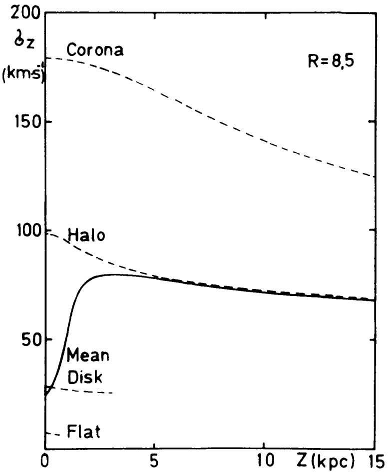


Fig. 2. The surface density of the Galaxy and of its components.



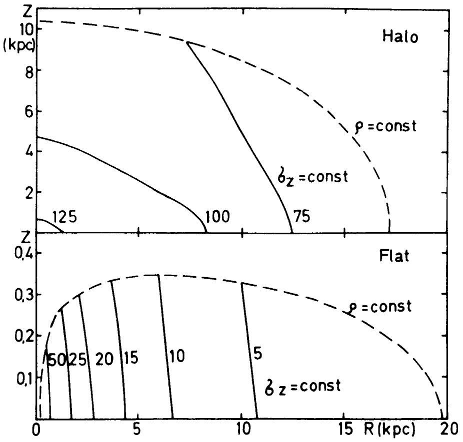


Fig. 3 (left). Velocity dispersion, σ_z, of galactic populations and the mean velocity dispersion versus distance from the galactic plane.

Fig. 4 (right). Meridional sections of the extreme halo ($\varepsilon = 0.6$) and flat ($\varepsilon = 0.02$) populations. Dashed line gives equidensity contours, corresponding to a density of 10^{-4} $M_\odot$ pc^{-3}, solid lines denote lines of constant velocity dispersion.

with a fixed boundary of the populations. In such a case the velocity dispersion would be zero at the boundary. Zero dispersion has not been observed at any distance from the galactic plane. On the contrary, real samples of stars are mixtures of stars of different populations; thus the mean dispersion may even increase with increasing z, as is indeed the case with the overall mean dispersion (Fig. 3).

The model has been checked for stability. As seen from Fig. 5, in general the mean velocity dispersion is larger than the critical Toomre (1964) dispersion, and thus the model is stable against small radial perturbations. However, between 2 and 8 kpc the Toomre dispersion is slightly higher than the calculated mean σ_R. Over a wide region the inner mass of the disk considerably exceeds the inner mass of the spheroidal component (Fig. 6). A similar picture has been found to exist also in other Sb galaxies.

ACKNOWLEDGMENTS

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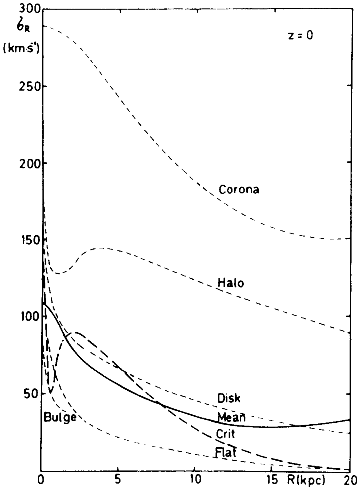

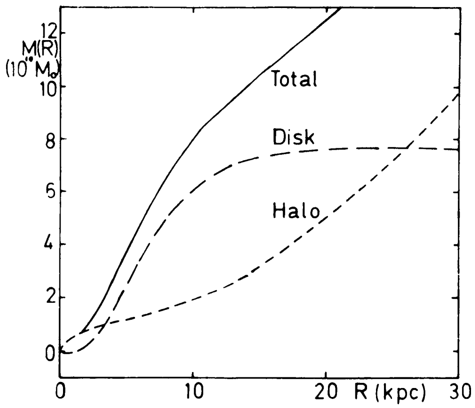

Fig. 5 (left). Velocity dispersions, σ_R, of galactic populations versus distance from the galactic center. The mean velocity dispersion as well as the critical Toomre (1964) dispersion have also been plotted.

Fig. 6 (right). Inner mass of the Galaxy, M(R), of the galactic spheroidal components (including the massive corona) and of disk components. The mass M(R) is defined as the mass of a spherical body which exercises the same attraction force as the model galaxy. Negative M(R) values indicate that the attraction force is directed away from the galactic center.

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DISCUSSION

Lynden-Bell: There are problems in the determinations of (B-A)/B from local stars. Not only is the vertex deviated but the Hyades group dominates one end--this may well disturb the ratio. Woolley et al. (MNRAS 179, 81) measured radial velocities of K stars at around 600 pc in the direction of galactic rotation and another group in the direction of the anticenter. From about 500 stars they found a ratio of velocity dispersion of $\sqrt{2}$, corresponding to B = -A.

Basu: Could you please comment on the uncertainties that might be introduced into your computed values by using the local values of Oort constants?

Ostriker: The overall uncertainties of the various model parameters are given in the paper. Because quantities enter into the determination with weight inversely proportional to the square of the uncertainty attributed to it, Oort's constant B is relatively unimportant and A is relatively important in contribution to the final result.

Rubin: A comment which is also a question. If the values of Oort's constants A and B are the local values, then the value of $\frac{dV}{dR}$ which they define may not be the correct value for your model. In particular, if the extended rotation curve is flat, then maybe A = -B would be a more appropriate choice. Is this correct? For the 11 rotation curves which we showed earlier, eight of them have their maximum velocity at R > 8 kpc; seven of them have their maximum beyond 15 kpc. In response to Dr. de Vaucouleurs, note that for $H \sim 100$ km s^{-1} Mpc^{-1}, this would be equivalent to rising curves at $R \sim 8$ kpc.

Einasto: The presently adopted Oort constants may indeed reflect only the local behavior of the rotation curve. In order to investigate the consequences of the totally flat rotation curve of the Galaxy at R > 5 kpc we will compute the appropriate version of the model.

Upgren: In response to Bok's comment a few minutes ago and in regard to the last two papers, I want to mention some pertinent observations which are being made. Jurgen Stock and I and others are using the new 1.5-meter Schmidt in Venezuela to get objective-prism radial velocities and spectral types of F stars to 2 kpc or farther. At latitude 9° N, we can reach the entire plane and both poles. We want to determine the force law in z from the F stars which would then not be dependent on the giants because they have been shown by Sturch and Helfer and others to have luminosity calibration problems. I also want to mention that Stock already has objective-prism radial velocities of about 6000 stars in a direction not far from that opposite the direction of galactic rotation. The percentage of high-velocity stars is very small, although a few have velocities of 400 km s^{-1} or more.

Trimble: Concerning your plot of velocity dispersion in the radial direction vs distance from the galactic center; the average stellar velocity distribution is less than Toomre's initial value in the region $\sim$ 4-7 kpc, which is just where we see all sorts of excitement in the form of lumpy molecular clouds, excess star formation, pulsars, and so forth; perhaps there really is an instability?

Einasto: This may be the case. Our attempt to avoid the instability by increasing the mass of the stellar halo and bulge failed because the masses which are needed are too large to be compatible with currently available data.

Burton: In view of the importance of the hole in the disk component at small galactic radii, and its dependence on the dip in the galactic rotation curve, it is worth emphasizing the uncertainty in this curve at $R \lesssim 3$ kpc. Although the [NeII] data probably give a good indication of the rotation in the inner few pc, at larger distances the pervasive expansion will have caused rotation curves derived on pure-rotation assumptions to be in error. The sense of this influence will exaggerate the appearance of the inner-Galaxy dip.

Einasto: Our adopted rotation curve is corrected for the effect of expansion. Corrected velocities (lower curve) are smaller than the observed ones; in particular the inner dip is considerably lower.