SPECTRAL ANALYSES OF THE CHANDLER WOBBLE

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Abstract. Spectral analyses of 70 yr of ILS-IPMS data on the variation of latitude are discussed from the point of view of the confidence limits that can be placed on statistical estimates. The available data cannot be used to support the existence of two resonant frequencies in the Chandler wobble, nor are the latter required to remove an anomaly in the value of Q for wobble as long as the oceans cannot be dismissed as an energy sink.

The 14-month period in the wobble component of the spectrum of changes in the Earth's rotation, known since its discovery by Chandler in 1891, still presents a number of fascinating unresolved problems, which have been reviewed recently by one of us (Rochester, 1970).

The period and damping time of the Chandler wobble can be estimated by carrying out a spectral analysis of the observational data on the geographical motion of the rotation pole. The least inhomogeneous data set is the 70-yr record of monthly mean pole positions produced by the ILS-IPMS (Walker and Young, 1957; Jeffreys, 1968; Yumi, 1968–70). Over most of its existence this organization has had five member stations measuring latitude variation. A longer record can be constructed by splicing earlier data sets from different observatories, but the fewer stations involved and the different observing programmes and methods of data reduction may largely offset any apparent gain in resolution.

We have computed the power spectrum of the polar motion from the 850 data points cited above (December 1899–October 1970) using a fast Fourier transform (FFT) technique (Cooley and Tukey, 1965). This not only saves computation time but also interpolates the spectrum between harmonics of the record length, and so shows greater detail than, for example, the well-known Fourier line spectrum obtained by Rudnick (1956). Figure 1 shows the raw spectrum in the vicinity of the Chandler band and the annual peak. The characteristic double-peak structure of the Chandler band was not revealed by Rudnick's analysis only because of the frequency spacing of the harmonics of the 54.4-yr record length of the data set he used.

It would be naive, however, to conclude from the raw spectrum that the real Earth exhibits two resonant frequencies in the Chandler region, differing by $\Delta f \simeq 0.02$ cpy. Even if there were no intrinsic errors of observation in the monthly means themselves (and we do not consider this factor), the data set is a filtered sample of the polar motion and must be treated statistically. The finite record length, and to a lesser extent the discreteness of the data points, inevitably make the raw spectrum a rather unreliable estimate. Thus the 80% confidence interval on the power at the highest spectral peak in Figure 1 is between 10 and 230 $(0\overset{''}{.}01)^2$/cpy.

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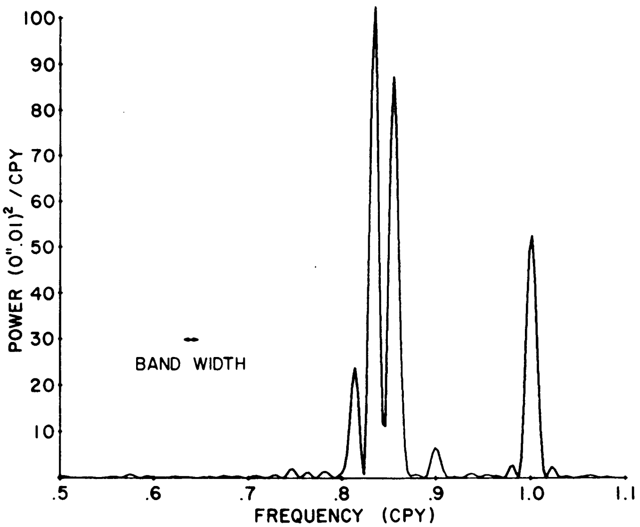

Fig. 1. Unsmoothed power spectrum of the polar motion, computed from ILS-IPMS monthly means, December 1899–October 1970.

To achieve a more reliable spectral estimate we smoothed the data, using a Parzen window (Jenkins and Watts, 1968, p. 244) to replace each data point by a weighted average. This is equivalent to subdividing the data into subsets, of record length equal to the width of the window, and averaging over the spectra obtained. In this way the confidence interval is narrowed and the reliability of the spectrum enhanced at the price of loss of detail. Figures 2–5 show the effects of using smoothing windows

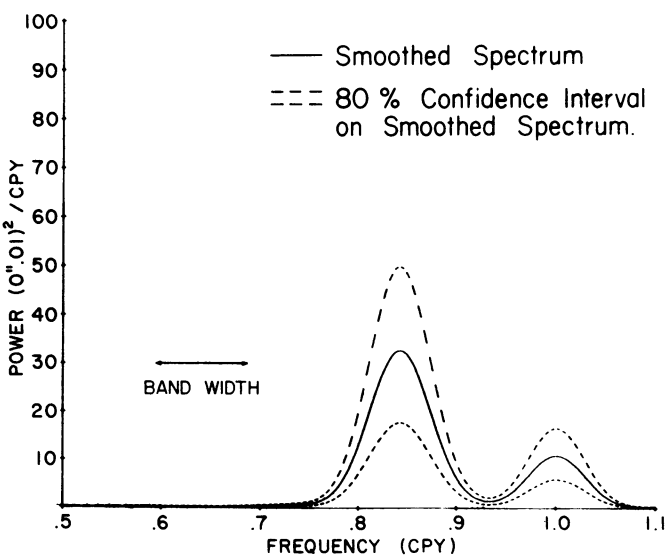

Fig. 2. Spectrum of the polar motion: 70-yr record smoothed with 20-yr Parzen window.

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of length 20, 32, 50 and 70 yr respectively. Only in the last does a trace of two peaks in the Chandler region remain, but the confidence interval is so wide as to lend little support to their reality.

Fedorov and Yatskiv (1965) have criticized an earlier interpretation (Yashkov, 1965) of the double peak in the raw spectrum as evidence for the Earth having two natural periods, instead of one, in the Chandler band. They showed how such bifurcation of a spectral peak could arise from a sudden large change in phase of the polar

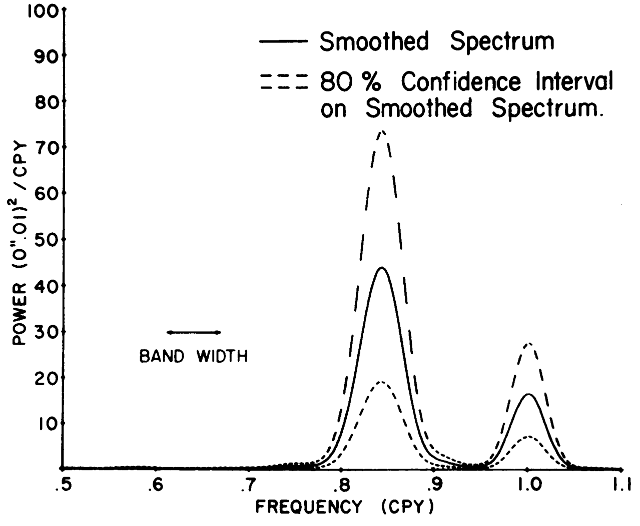

Fig. 3. Spectrum of the polar motion: 70-yr record smoothed with 32-yr Parzen window.

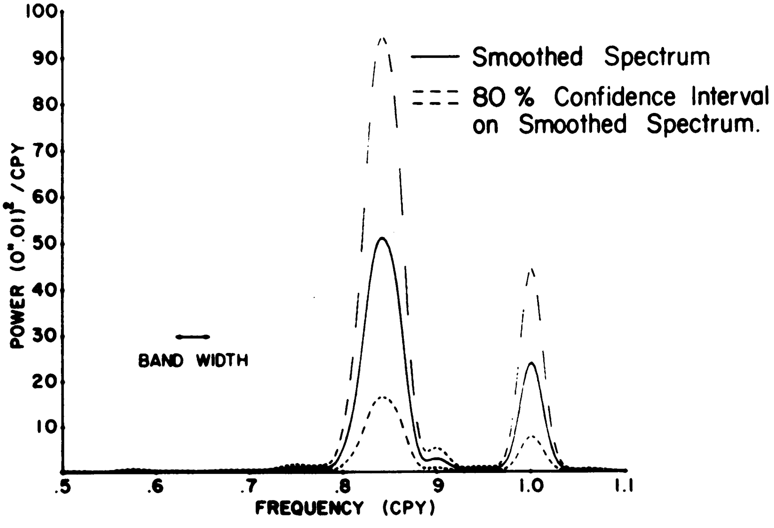

Fig. 4. Spectrum of the polar motion: 70-yr record smoothed with 50-yr Parzen window.

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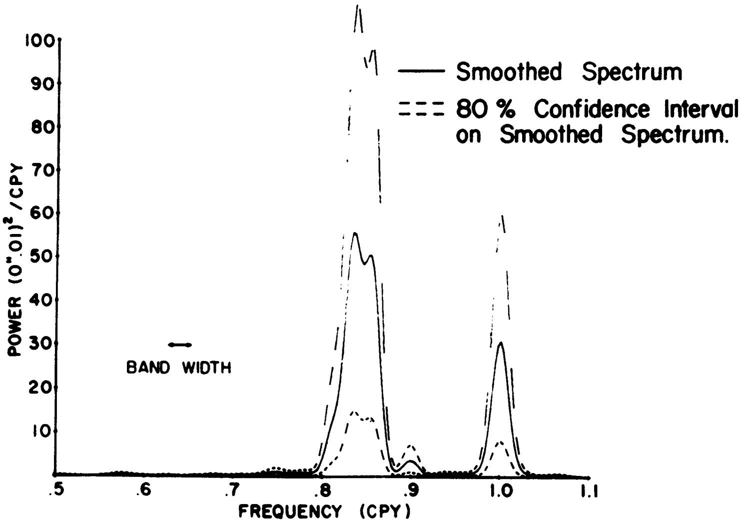

Fig. 5. Spectrum of the polar motion: 70-yr record smoothed with 70-yr Parzen window.

motion near the middle of the record, and attributed the double peak in the raw spectrum for the 70-yr period 1891.5–1962.0 to such a phenomenon having taken place in the mid-1920's.

More generally, we can say that the process by which the polar motion is randomly altered is not a stationary one, in the statistical sense. To illustrate the point, compare the raw spectra for four different records, each a 40-yr subset of the original 70-yr record (Figure 6). The annual peaks are quite consistent with one another, while the spectra in the Chandler region are very dissimilar. Their lack of resemblance to one

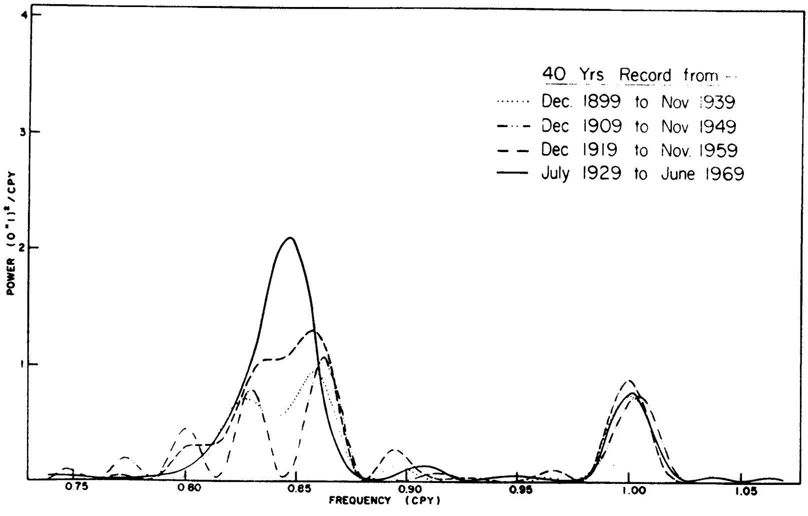

Fig. 6. Unsmoothed spectra of the polar motion, computed from 40-yr subsets of the 70-yr record.

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another is strong evidence that the random inputs to the free wobble constitute a highly non-stationary process.

Broadening of spectral peaks always suggests the presence of a damping mechanism, with relaxation time

$$\tau \simeq (\pi \Delta f_0)^{-1}$$

for a peak of width Δf_0 at the half-power points. The dimensionless measure of damping conventionally used by geophysicists is the dissipation factor

$$1/Q \simeq \Delta f_0/f_0 \simeq (\pi f_0 \tau)^{-1}$$

where f_0 is the Chandler frequency (0.84 cpy) in the present context. Rudnick (1956) finds $\tau \simeq 11$ yr; Jeffreys's (1968) best estimate is $\tau \simeq 23$ yr. These correspond to values of Q in the range 30 to 60. This is anomalously low if the wobble damping is to be attributed entirely to anelasticity of the mantle (where studies of the free oscillations yield Q's of several hundreds) and if, as seems to be the case, Q is nearly independent of frequency.

Colombo and Shapiro (1968) have argued that the variable amplitude of the Chandler wobble is strikingly suggestive of a beat phenomenon between two resonant periods within the Chandler band, separated by roughly 10 days, i.e. $\Delta f \simeq 0.02$ cpy. As they point out, the existence of two neighbouring sharp peaks in place of a single broad one would dispose of the 'problem' with Q. The 'problem', however, may not be real after all, because the oceans have by no means been eliminated as a possible sink of wobble energy (Munk and MacDonald, 1960, p. 172; Lagus and Anderson, 1968). Moreover, if there are actually two such peaks in the Chandler region, they will be reliably resolved in the spectrum only for a record of length at least $2/\Delta f \simeq 100$ yr, and consisting of reasonably homogeneous data.

We must conclude that the ILS-IPMS data so far available cannot be used to support the existence of more than one natural frequency in the Chandler wobble, and that the discrepancy between the values of Q for wobble and for the free oscillations does not yet present compelling physical grounds for suggesting a splitting of the Chandler peak.

Note added in proof: Since the work reported in this paper was done we have recomputed the spectrum using the revised coordinates of the pole referred to the CIO (Vicente and Yumi, 1969, 1972). There are no significant differences in the spectra and our conclusions remain unchanged.

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