

I N V I T E D D I S C O U R S E S

## PULSARS AND THEIR GENESIS

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### INTRODUCTION

The subject I have chosen for my talk today will come of age on the last day of this General Assembly. The first detection of the remarkable objects that we call pulsars, was on 28th November 1967 when Jocelyn Bell (now Bell-Burnell) who was working with Prof. Hewish at Cambridge discovered a new class of radio sources that put out pulses of radio radiation about once a second but with clock-like regularity. Two other major discoveries in the same decade using the radio spectrum were, of course, Quasars and the Cosmic microwave background. Those two took us to extremes in time and distance, and the amount of energy radiated, whereas the discovery of pulsars, situated near at hand by comparison, led us to extremes in the physical state of matter.

Against the background of what we *really* know about pulsars now, eighteen years after their discovery, it remains amazing how much of this was already appreciated within the first eighteen months. At the XIV General Assembly held in Brighton in 1970, there were two invited discourses on the same subject given by Professors Hewish and Ginzburg. The richness of the observational phenomena exhibited by pulsars and the extreme difficulties in understanding and explaining these phenomena were exemplified in these two discourses. They are recommended reading for anyone who wishes to see in perspective whatever progress we have made since then.

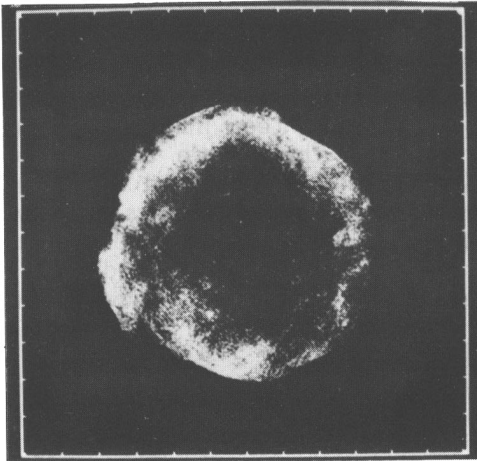
Within a year of their discovery, most people had come to accept that pulsars were neutron stars, one of the two possibilities suggested by Hewish et al (1968), the other being white dwarfs. So let me begin by saying something about these objects.

The stars we see when we look at the sky, shine like the Sun does by virtue of the energy produced in nuclear fusion. But this cannot go on forever and they must all finish up in some presumably cold end state. The commonest such final configuration is the white dwarf, which is a very compressed star about the size of the earth, but as massive as

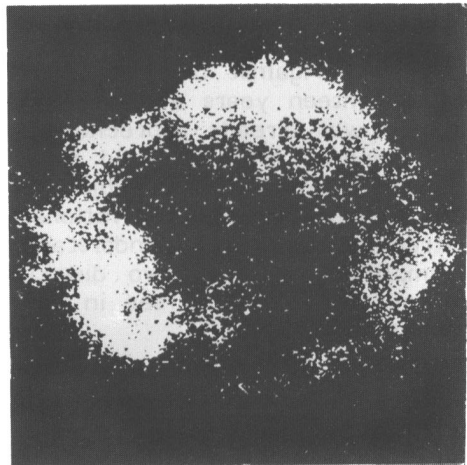
the Sun. The corresponding density is about a tonne per cubic cm and all the atoms have been stripped of their electrons. Such a star is called *degenerate* and one may think of it as a "giant molecule" in which the nuclei are immersed in a sea of electrons shared by all of them.

In the process of evolution of a normal not too massive star, the ash from the nuclear burning collects at its centre and forms such a degenerate core. In a famous paper written in 1931 by Chandrasekhar, he showed that quantum physics prevented such an object having a mass much greater than about 1-1/2 times the mass of the Sun, now called the Chandrasekhar limit. This raised a great controversy for many reasons, one of which was the difficulty of understanding what would happen to *much* more massive stars when they burn up all their fuel. We know now that stars which start off with up to 8 times as much mass as the Sun, manage to end up as white dwarfs by cleverly shedding the extra material in some way or other. The study of this phenomenon of mass loss has been an important field of investigation for many years now.

More massive stars build up their core much faster than they can get rid of their excess mass. At some stage, when the mass of the core exceeds the Chandrasekhar limit, there is a crisis and something drastic has to happen. What happens is a spectacular explosive event, i.e., a supernova, whose brightness for some weeks is that of a whole galaxy of stars. The outer mantle, or envelope of the star, is ejected with a velocity measured in thousands of km/sec, and the debris, usually in the form of an expanding shell, can be seen for some thousands of years afterwards by its radio, optical and X-ray emission. Fig. 1 is a picture of the radio emission from



**Fig. 1:** Radio photo of the remnant of Tycho's SN. From Green and Gull (1983).

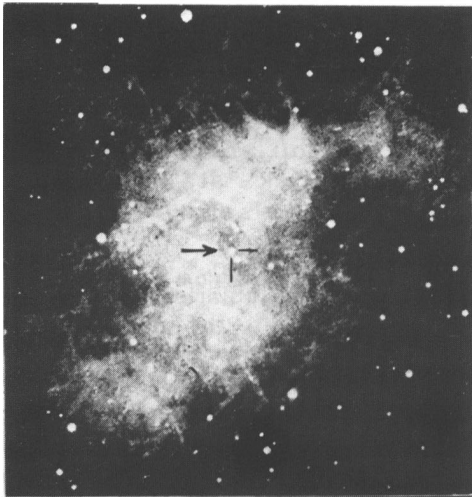


**Fig. 2:** X-ray picture of Cas-A from Murray et al. (1979).

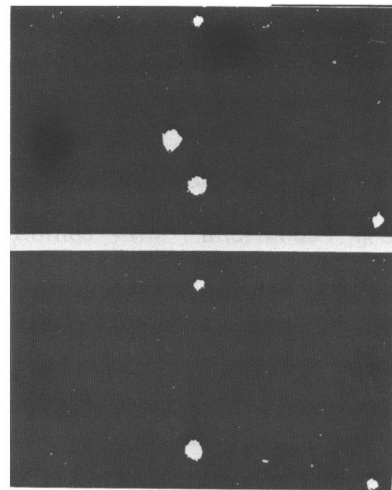
the remnant of the SN seen first by the great astronomer Tycho Brahe in 1572, and which shows the shell structure caused by the explosion. Fig. 2

is an X-ray picture of the SNR in the constellation of Cassiopeia. It is one of the strongest radio sources in the sky, and its X-ray emission is one of the many wonderful things discovered by the Einstein Observatory.

The most celebrated SNR in our Galaxy is, of course, the Crab Nebula, the site of the SN of 1054 observed and recorded by Chinese astronomers. Among the many who studied this fascinating object were Baade and Zwicky, two of the greatest astronomers of recent times. In a feat of intuition very hard for lesser mortals to comprehend, they predicted over 50 years ago that a SN explosion represented the collapse of a star to a tiny object just made of up neutrons. Five years later, Oppenheimer and Volkoff (1939) showed theoretically that such a stable configuration did indeed exist for stars, if they were roughly as massive as the Sun. The discovery of a pulsar in the Vela SNR (Large et. al., 1968) followed by that of another in the heart of the Crab Nebula (Staelin and Reifensten, 1968) not only put the seal on the identification of pulsars with neutron stars, but led to a dramatic vindication of the brilliant hypothesis of Baade and Zwicky (1934). The very star (Fig. 3) that was suspected to be the remnant of the Crab explosion (because of its unusually featureless spectrum) was found to be pulsing in the optical with the same incredibly rapid frequency



*Fig. 3: Photograph of the Crab Nebula and its pulsar. Taken from Hewish (1970).*



*Fig. 4: Television pictures taken through shutter rotating at the pulsar rate. Top: shutter opened in phase with the radio pulse; bottom: shutter closed in phase with the radio pulse.*

as the radio pulses,  $\sim 30$  Hz, and at the same phase. Fig. 4 shows television pictures taken through two shutters rotating at the speed of the radio pulsations, but in *antiphase*. That taken through the shutter which opened in synchronism with the radio pulse shows the pulsar shining like a star, while the other shows nothing (Miller and Wampler, 1969).

Even more of a surprise was the discovery of X-ray pulsations again at the same frequency and phase. But as we were soon to learn, this was unique to the Crab pulsar, less than a thousand years old, and the youngest one yet observed. All but 3 out of the more than 300 pulsars we know today emit radio pulses only. Most of them also have much longer periods.

To recapitulate the process of formation, the core of the normal star continues to accumulate matter till it exceeds the Chandrasekhar limit. At this point, it is about the size of the Moon and its density is about a 100 tonnes per cubic cm. After collapse, we have an object of only 10 km radius and a staggeringly high density, that of nuclear matter. Even before the discovery of pulsars, it had been surmised that neutron stars would be born spinning rapidly and with a very high magnetic field. Both guesses turned out to be right.

Let me mention in passing that besides white dwarfs and neutron stars, black holes are a third possibility as a final stage for stars at the end of their evolution. Stars which begin their career with masses more than about 30 times that of the Sun are believed to end up this way, and I shall return to this point for a moment at the end of my story.

The spiky, pulsing nature of the radio signals (Fig. 5) received

on earth not only inspired the name PULSAR, but also produced more theories than you could shake a stick at, that were based on *radial* pulsations of the star in question. A study of the polarisation pattern within the pulse showed however that it could not be radial motion that caused it, but only the rotational sweeping of a narrow beam past the observer, as in the case of a lighthouse. More importantly, it led to the identification of the magnetic polar region as the seat of the lighthouse beam and showed that the magnetic dipole was inclined to the rotation axis as shown in Fig. 6 (Radhakrishnan and

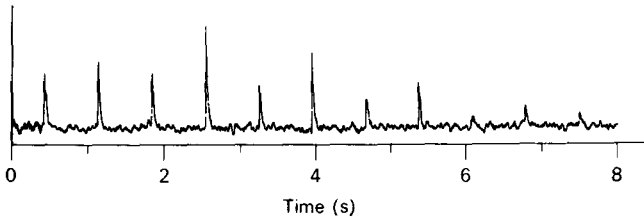


Fig. 5: Radio pulses from PSR 0329+54. Fig. taken from Manchester and Taylor (1977).

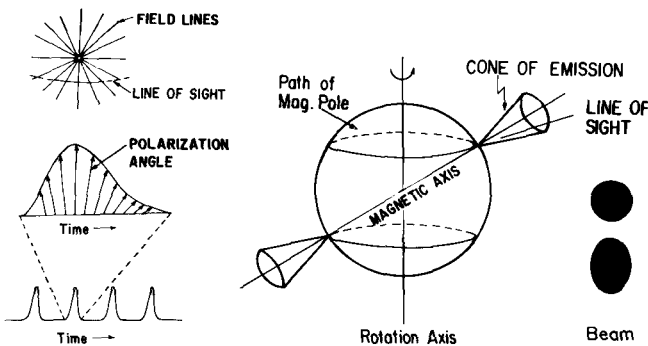
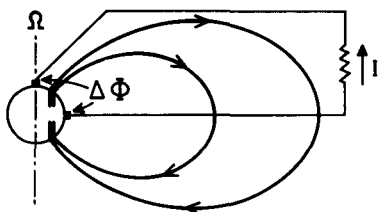


Fig. 6: The geometry of pulse emission.

Cooke, 1969). A much closer look at the details and statistics of these polarisation patterns, in more recent times, has led to the determinations of these inclination angles for several pulsars, and to a modification of our picture of the shape of the pulsar beam (Narayan and Vivekanand, 1983). It was assumed to be circular in the original model, but the indications now are that it is elongated in the latitudinal direction. This is depicted on the right hand side of Fig. 6. I shall say something a little later about the implications.

Another, and earlier indication that a rotational model was the right one was the lengthening with time of the periods of the Crab and Vela pulsars (Richards and Comella, 1969; Radhakrishnan et. al., 1969). A rotating magnetized neutron star will emit electromagnetic radiation and experience a braking torque which would slow it down gradually. By assuming a reasonable moment of inertia for the neutron star, the magnetic field of a pulsar can be estimated from the period and its derivate as  $B \propto \sqrt{P\dot{P}}$  (Ostriker and Gunn, 1969).

A spinning conducting magnet would also act as a dynamo as shown in Fig. 7. A ten centimeter IRON magnet of even 10 thousand Gauss



IRON MAGNET NEUTRON STAR

R (cm)	10	$10^6$
P (sec)	0.015	1
B (Gauss)	$10^4$	$10^{12}$
$\Delta\Phi$ (Volts)	5	$3 \times 10^{16}$

Fig. 7: Pulsar as a dynamo. From Goldreich (1972)

strength would need to be spun 60 times a second to develop 5 volts between the pole and equator. A neutron star going round only once a second can generate an incredible  $10^{16}$  volts, and this has led to the following picture of how pulsars radiate. The electric field is so great that it easily overcomes the enormous gravity of the neutron star, pulls out charges from its surface, and accelerates them to something like a million million volts. These particles immediately produce energetic gamma-rays, but the gamma rays have trouble in getting out because of something special which happens in the strong magnetic fields near a pulsar. The gamma-rays create electron-positron pairs (Sturrock, 1971), and each of these particles in turn produces fresh gamma-rays, which produce more  $e^-e^+$  pairs. This process of multiplication leads to a cascade resulting in many thousands of times more particles than came off the surface of the neutron star. It is bunches of such highly relativistic particles sliding along the "open" field lines which are believed to emit the radio signals we receive.

If you feel things are getting a bit complicated here and starting to sound hairy you are absolutely right. Models to explain even the major

features of the radio radiation from pulsars involve concepts from many branches of physics, nuclear, atomic, plasma, relativity and a considerable amount of electrodynamics, all in extreme conditions. It is not my purpose to discuss this aspect of pulsars today, which is a good thing because I would have great difficulty in doing so. I refer those who wish to learn about this subject to the papers by Ruderman and his colleagues (see Ruderman 1981, 1986 and references therein) which represent the best attempts yet by far to account for a very large variety of observational phenomena. The difficulty to be appreciated is that in terms of the energies involved, the radio radiation is just a squeak. It is like trying to figure out the working of a complicated giant machine inside a building just from the incidental noises heard outside, and also having to explain all the squeaks in detail. A further and a serious problem that Ruderman has been stressing for many years now is that more than one model is required to explain all the pulsars we know, depending on their rotation rate which declines with age.

Before I leave the magnetosphere of pulsars with a sigh of relief, I would like to make two important points. One is that the production of an adequate number of pairs - required by models to explain the radio radiation - depends both on the strength of the magnetic field and the speed at which the pulsar is spinning. For a given field or spin rate, if the other is not high enough, the pulsar will not function as such. The other point is that the energetics of the Crab Nebula, including its field and relativistic particle content, which posed such a mystery for many years, is now understood in terms of the efflux from the rapid pulsar inside.

So much for an introduction. I take up now the main theme of my talk, which is the relationship of pulsars to the rest of astronomy, that is to other stars. As a consequence of what I described earlier, the notion that pulsars were born in supernova explosions became gospel. It was therefore natural to expect an association, i.e., to find a pulsar in every supernova remnant, as the lifetime of these remnants measured in thousands of years, is far shorter than that estimated for pulsars, i.e., millions of years. But the number of good pulsar-supernova associations remained static at two while the total number of pulsars observed grew into the hundreds. We now have three associations out of 300 pulsars and 150 remnants.

Various selection effects were invoked to account for this discrepancy, and one began to look instead for an agreement between the pulsar birth rate and the rate of supernova production in the galaxy. This was done either by estimating the birth rate of the radio remnants which the supernovae left behind, or by looking at external galaxies similar to our own, in which supernovae could be seen going off. As far as the birth rate of pulsars is concerned, it is a more difficult exercise, since we only see pulsars relatively close to us, as opposed to SNRs for example, which can be seen all over the galaxy. The result of several such early exercises seemed invariably to indicate that we were seeing far more pulsars than we should.

It is my personal opinion that this problem has now largely disappeared not least due to contributions from several of my young colleagues.



The studies carried out by Vivekanand and Narayan over several years involved the determination from observations, of the luminosity function of pulsars, the selection effects involved in searches, the galactic electron density distribution needed to derive distances, and the angular extent of pulsar beams in the unseen coordinate which I mentioned earlier (Vivekanand and Narayan, 1981, 1982; Vivekanand et. al., 1982; Narayan and Vivekanand, 1983). This last governs the beaming factor or statistical observability of pulsars. The estimate for the birth rate of pulsars that they obtained by putting all these things together is no longer discordant with the best available estimates for the occurrence rate of SN as established from external galaxies.

The most remarkable result, however, that seemed to follow almost rigorously from their analysis was something called 'injection', namely that the initial rotation periods of pulsars must be very much longer than the minimum allowed period for neutron stars which is of the order of a millisecond. Let me remind you that it was often assumed that conservation of angular momentum during collapse would lead to an initial rotation speed for pulsars near the limit. The conclusion of Vivekanand and Narayan on the other hand was that most pulsars become observable as such, with periods in the *hundreds* of milliseconds and not five or ten or fifteen. This idea was received with general scepticism, and the absence of more short-period pulsars in the sample was attributed to selection effects in the searches. Support however, was provided by the theoretical investigations of Srinivasan, Bhattacharya and Dwarakanath (1984) on the number and intensity of Crab-like nebulae that we *should* see in the galaxy for any reasonable supernova occurrence rate, but do not. This meant that most pulsars were born spinning slowly. While these notions were received with scepticism at first, I am pleased to report that things are now looking up. An extensive new survey sensitive to pulsar periods as short as four milliseconds was recently carried out specifically to test the selection effect hypothesis, and led to the conclusion "that many pulsars must be 'injected' into the population with  $P \geq 0.5$  second" (Stokes et. al., 1985). An independent analysis by Proszynski and Przybycien (1984) led to a dependence of pulsar luminosity on  $P$  and  $\dot{P}$ , very similar to that obtained by Vivekanand and Narayan (1981). And finally, a theoretical study of pulsar evolution by Chevalier and Emmering (1985) adopting the luminosity function referred to above, led to the conclusion that most pulsars are injected with periods not much less than half a second, and also accounted for the lack of pulsar supernova associations.

Before turning to the next topic, I would just like to point out that the implications of injection are of fundamental importance in the understanding of the supernova process. If angular momentum conservation during core collapse could spin it up to a very short period of some milliseconds, but the pulsar seen when the debris clears away has a period of several hundred milliseconds, this would be observational evidence that rotational energy has been transferred from the core to the envelope *right at the beginning*. I don't wish to say more about this because I know that it will be talked about in the Joint Discussion on Supernovae on the last day of this General Assembly.



I shall turn now to binary stars, which I have come to believe are central to the understanding of pulsars. It has long been known that most stars are formed in binary or multiple systems, but it was only after the discovery of about 160 single pulsars that one was found to be in a binary system. This was the famous binary pulsar 1913+16 discovered by Hulse and Taylor (1974) and seen immediately to be in a tight highly eccentric orbit with a period of about 8 hours and a separation of  $\sim 1 R_{\odot}$ . Meticulous observations over the next several years by Taylor and his colleagues led uncompromisingly to the conclusion that its companion is also a neutron star with a mass about  $1.4 M_{\odot}$  (Taylor and Weisberg, 1982). Because neutron stars are so compact, they behave like point masses; the system is so dynamically clean that it became, and still is, the best laboratory by far for the successful tests of a number of General Relativistic effects. The most spectacular result was of course the measurement of the gradual change in the period of the orbit, consistent to very high accuracy with that expected for gravitational quadrupole radiation according to Einstein's theory (Taylor and Weisberg, 1982).

This is illustrated in Fig. 8 which also points out that this state of affairs cannot last forever. In 300 million years or so, there will be a

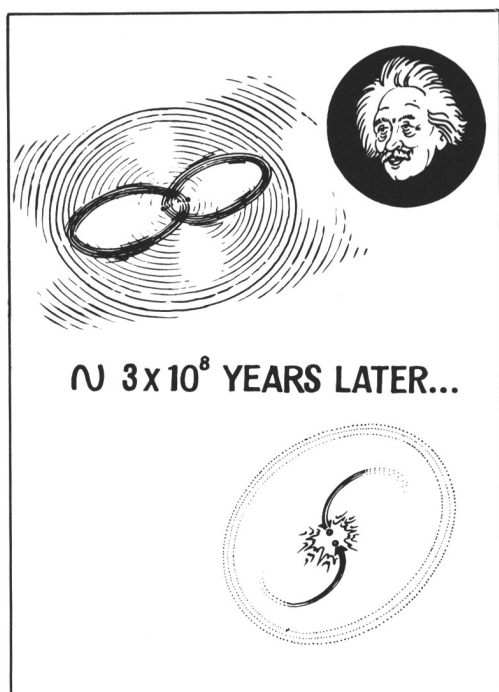


Fig. 8: Gravitational radiation from the Hulse-Taylor binary.

coalescence, the only certain outcome of which is a burst of intense gravitational radiation with a duration of a few seconds. After this dies down, whether there will be a black hole or some extra-massive neutron star no one can really tell yet. But considering that 300 million years is short compared with the age of the galaxy, and that there must be other binaries like this one, such events should occur periodically. They may be of interest for gravitational wave detection, but certainly are from the point of view of the formation of the particular end product. Let me return to the origin of this system with which I am more concerned today.

When this object was discovered, its period of 59 milliseconds was the second shortest known, lying in between the Crab with 33 milliseconds and the Vela pulsar with 89 milliseconds. These two pulsars represented the only known associations with supernova remnants, and the reason for their short periods was thought to be simply the result of our having caught them very young. Proof of this picture was the rapid slowing down of both these pulsars. By amazing contrast 1913+16 seemed hardly to be slowing down at all,

and there was no trace of any SNR around it. The rapid period of this pulsar taken together with the very small value of its period derivative indicated a surface magnetic field two orders of magnitude below that for the Crab and Vela pulsars, and something that needed to be explained. One could of course say, as many did, that it just happened to be born with a very low value of magnetic field, since we are really not sure exactly how such magnetic fields are generated; but it would have been strange that the only pulsar out of 160 to be in a binary system accidentally happens to have the lowest magnetic field. The connection clearly needed to be understood.

To understand the period evolution of neutron stars we could make an analogy with ordinary stars whose evolution is governed by a single parameter, the mass. The greater the mass, the faster the evolution, and given the initial mass the future history will be a track in a luminosity vs. colour diagram. If the star is single, it is master of its own fate; but if it is a member of a close binary, there can be large scale mass transfers to or from the star in question, which will change its career and move it to a new track. In the case of neutron stars, we may think of the magnetic field as analogous to mass for ordinary stars, and that which governs the period evolution. Given any value of field and period, the subsequent

evolution is predictable. The evolutionary tracks can be represented on a field-period diagram such as shown in Fig. 9. If the fields are constant the evolutionary tracks will be horizontal, but if the fields decay, as we believe they do, the tracks will curve down depending on the decay time, and can be calculated.

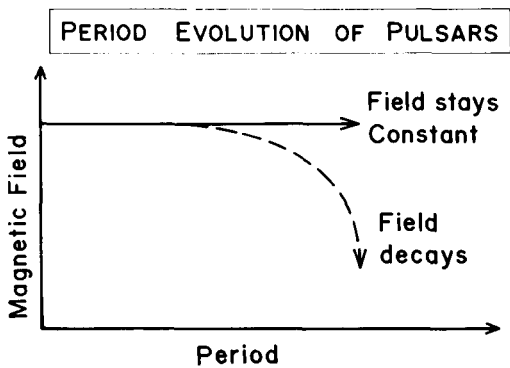


Fig. 9

Now, if neutron stars are created with initial fields lying in a small range, as the evidence seems to suggest, their evolutionary tracks will be as shown in Fig. 10. The position of the majority of observed pulsars is also indicated. In spite of our poor understanding of the emission mechanism which makes pulsars observable, there is both observational and theoretical justification for the existence of a cut-off or death line, as I mentioned earlier. Old neutron stars which evolve past this line will cease to be

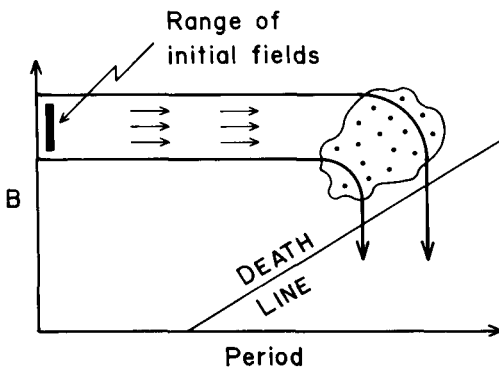


Fig. 10

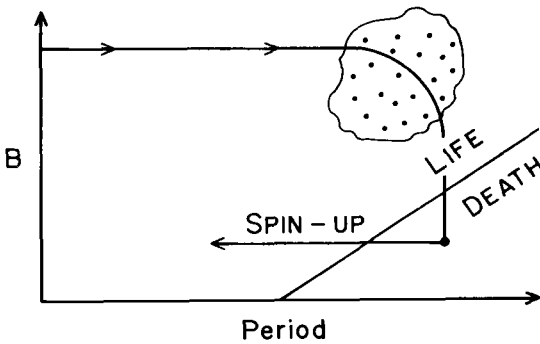


Fig. 11

observable as radio pulsars. The combination of weakened field and lengthened period kills the cascade machine. But if its rotation rate could be speeded up somehow, it will move sideways in the diagram *back* into the region of living pulsars (Fig. 11). The weak magnetic field will be compensated by the high spin rate, and the machine can function again. But it will be conspicuous because of the anomalous combination of low field and short period.

A mechanism by which a neutron star in a binary system could have its rotational speed altered drastically is by the accretion of matter which also deposits angular momentum on to it. I would like to digress briefly here and recall that some years before the discovery of pulsars Zeldovich (1964) and other Soviet astrophysicists had drawn attention to the importance of accretion onto neutron stars. Because of their very high surface gravity, energy equivalent to one-tenth the rest mass of infalling matter would be liberated and was predicted to produce copious X-radiation. Just a few months before pulsars were found, Shklovsky (1968) proposed a binary model with accretion on to a neutron star to explain the X-ray emission from Scorpius X-1 discovered by Giacconi et. al. (1962), using a rocket-borne detector. But it was only some time after the detection of pulsating X-ray sources by the UHURU satellite launched in 1970, that it could be established that there were close binary systems in which a neutron star was accreting matter from a normal star companion. One cannot help feeling that the important observational discovery of neutron stars should have belonged legitimately to X-ray astronomy as would have happened inevitably. It was a quirk of fate that just a few years earlier, pulsars were accidentally discovered doing what no one in his right mind would have expected them to do. Personally, I am of course delighted that things happened the way they did. So many innocent but eager radio astronomers had such fun discovering strange phenomena in the radiation from pulsars that theoreticians could never have predicted, and are still struggling to explain.

To return to binaries, the special problem in connection with a magnetised neutron star became of great interest in connection with the details of X-ray emission from these systems. In an important paper on this subject, Pringle and Rees (1972) noted that the accretion would speed up the rotation of the neutron star, and in another Davidson and Ostriker (1973) recognised that there would be an *equilibrium* period to which the neutron star could be spun up. As the equilibrium was between the magnetic pressure, and that of the infalling matter at the corotation radius, a weaker field would lead to a higher spin rate, and Smarr and Blandford (1976) suggested such a spin-up as the cause of the anomalously

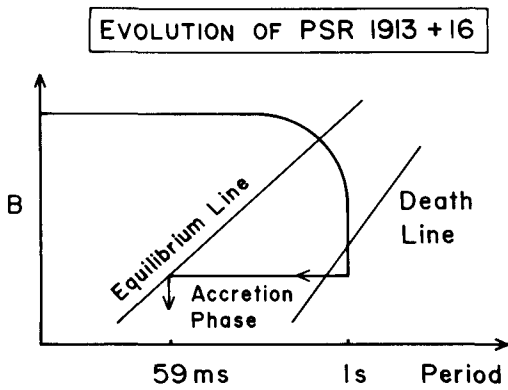


Fig. 12

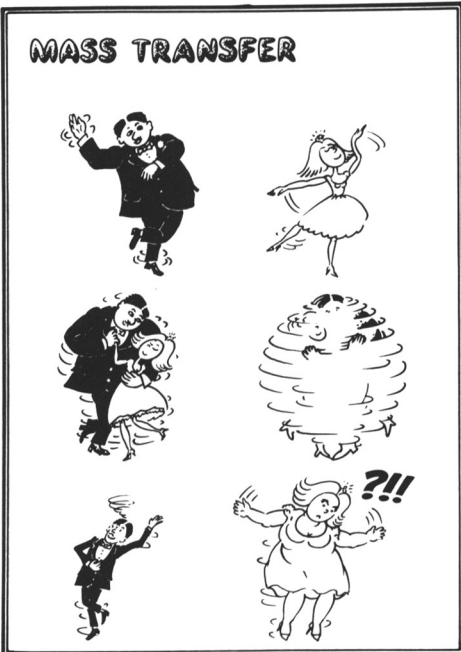
short period of the binary pulsar. Fig. 12 illustrates the 'period' evolution of this pulsar as worked out later by Srinivasan and van den Heuvel (1982).

I have dwelt at some length on this one pulsar, because I believe that the questions it raised, and the answers that came along - sooner or later - were a turning point in our understanding of the origins of pulsars. Let me quickly summarize them.

The reason there is no supernova remnant around this pulsar (unlike Crab and Vela) is simply because it is very old. The remnants created by both the first and second supernovae in the system have long since dispersed. Its period is so short simply because it was spun up in a binary, as we have just discussed. This also makes the lifetime of such a pulsar very long because of the slope of the death line as can be seen in Fig. 12. We don't see the other pulsar either because its beam is missing us, or because it is already dead.

Binary pulsars are so rare even though binary stars are so common

simply because the neutron star will not be detectable as a pulsar unless its companion is also a neutron star, as in this case, or a white dwarf in a large orbit. The sky is full of binary systems with one neutron star, some fraction of which are accreting enough matter from their companions to become X-ray sources. But even in the remaining cases the weak stellar wind from the companion is enough to smother the pulsar, as it is optically thick to radio radiation.



An all-important role is played by the phenomenon of mass transfer in close binaries. The cartoon on left illustrates several of its aspects following from angular momentum conservation; the shrinking and speeding up of the orbits while the donor is the more massive object, the common envelope phase, and

the subsequent increase in separation when the donor becomes the less massive. The most important point is that the first SN explosion will *not* disrupt the system because most of the mass is now in the companion. Subsequent evolution (on her part) will lead to a shrinking of the orbit as before, but because the tiny radius of the neutron star prevents rapid accretion, most of 'her' mass will be lost from the system. If she loses enough before exploding as a supernova, the system will remain bound and have two neutron stars in it. The small orbit of 1913+16 is proof of the fact that it went through a common envelope phase, just as predicted by theory; and its eccentricity shows that the system just barely survived disruption when matter was ejected in the second supernova explosion.

All this was appreciated at least a decade ago and modelled in papers by van den Heuvel and colleagues and Tutukov and colleagues among others (see van den Heuvel, 1981; Tutukov, 1981 and references therein). It was already clear then that for every one such rare system like 1913+16, there must be many cases where the system became disrupted. These would correspond to a higher mass for the hydrogen - or main sequence - star and therefore to a shorter evolution time between the birth of the first neutron star and when it starts accreting matter from its companion. The decay of its magnetic field will be correspondingly less and the equilibrium period to which it is spun up correspondingly longer. When such a system disrupts, the older or 'recycled' pulsar, now single, will have a period *longer* than that of PSR 1913+16 and lie somewhere above and to the right of it in a field period diagram. It was on this basis that Srinivasan and I identified the two pulsars shown in Fig. 13 some

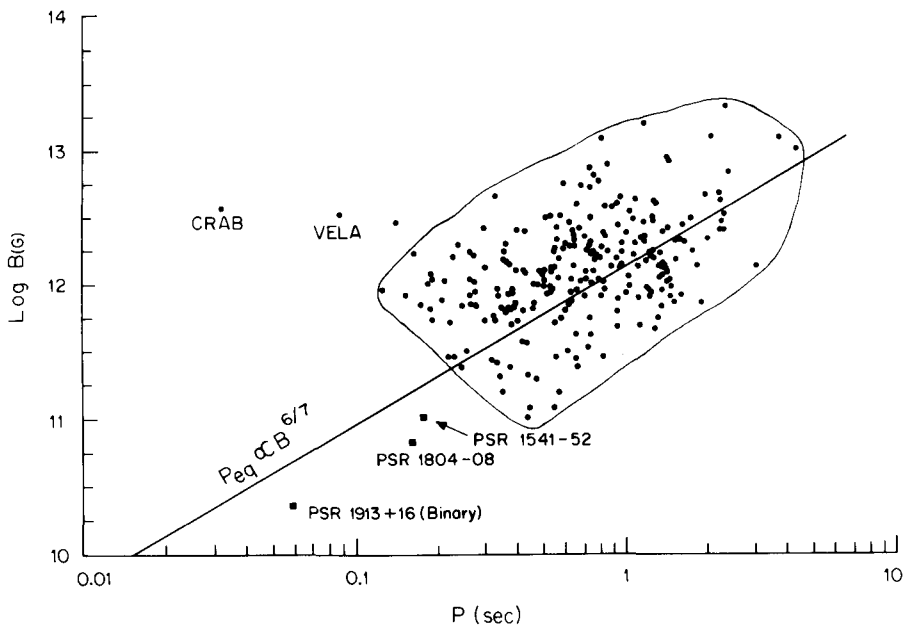


Fig. 13: The distribution of pulsars in the  $B$ - $P$  plane showing the Hulse-Taylor binary and two single but also recycled pulsars (filled squares). From Radhakrishnan and Srinivasan (1984).

years ago, as recycled ones when discussing this class of objects which clearly must exist. Note that if the main sequence mass is higher than some value, the field of the first born neutron star will hardly have decayed appreciably, and it cannot be spun-up to be noticeably different and therefore recognisable as the product of the first SNE. As in the case of injection, recycled pulsars were not very popular at first but now have many fans. What turned the tide is the next topic I shall address.

### MILLISECOND PULSARS

A fantastic morale booster for both pulsar watchers and pulsar worriers was the detection in late 1982 of a pulsar with a period of one and half milliseconds by Backer and coworkers (1982). This discovery was no accident, but the result of a long, hard search following a number of clues which went back several years and in which several people contributed to point the way. To those who haven't heard the story, it may be of interest to know that three years before it was found, a paper submitted by Backer suggesting that there could be such a pulsar at that position was turned down, because the referee felt that it was too speculative, and also badly written.

This pulsar has two radio beams like the Crab pulsar, but they sweep past us 20 times more rapidly. Most pulsars we know have periods somewhat less than a second and if we were to listen to their signals amplified and played through a loud speaker, they would sound like a pendulum clock from which you hear two ticks per second. Fast pulsars like the Crab and Vela would sound like the drone of a motor-cycle engine (in different gears), but the millisecond pulsar has now climbed to E flat in the musical scale. Courtesy of Don Backer, I have a tape recording of the sound of this pulsar made at the 1000' telescope of the Arecibo Observatory where it was discovered, and I shall play it for you (TAPE). Whether we are comfortable with the idea or not, that is the speed at which a star one and half times as massive as the Sun is spinning around, and in some ways it is the most astonishing object found so far in astronomy.

The issues raised by this discovery and that of a six millisecond pulsar some months later (Boriakoff et. al., 1983) were the subject of a workshop held at Green Bank last year for three whole days. I shall touch only upon two aspects here, one because of its extraordinary potential importance, and the other because it is a part of the general theme of my talk. Immediately after the discovery, the very special pulsar timing equipment that Taylor and colleagues had developed for the binary pulsar was put on the job of timing this object. Before long, its period

PSR 1937 + 21

$$P = 0.001\ 557\ 806\ 448\ 8724 \pm 2s$$

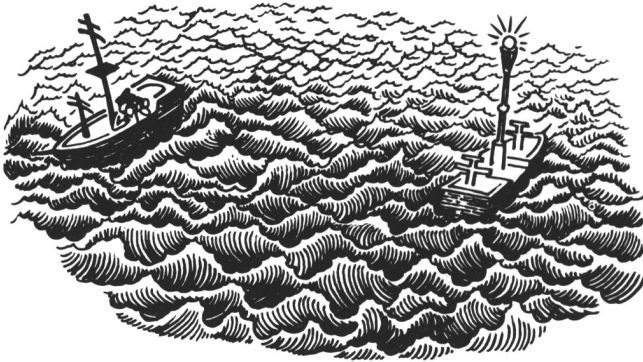
$$\dot{P} = (1.05110 \pm 0.00008) \times 10^{-19} s s^{-1}$$

and period derivative had been determined to the incredible accuracy indicated by the numbers in the box. This feat of precision unequalled in

any other astronomical measurement, revealed the fact that the time-keeping



ability of this pulsar is competitive in accuracy with a whole bank of cesium clocks. With such an accurate clock, one can hope to detect a cosmological background of gravitational waves expected to have been created at around



the time of the Big Bang, and filling the universe. Such waves would manifest their presence by differential buffeting of the pulsar and the earth, like the independent motions of the two ships in the cartoon on the left and this would show up as variations in the time of arrival of the pulses on earth.

The detection of such waves would rival the discovery of the three degree cosmic microwave background, which is the strongest of the few pillars on which our present notions of cosmology stand. Even without more millisecond pulsars of the same stability, Mike Davis and his collaborators feel that a very severe constraint on the amount of energy in long wave gravitational radiation can be established over the next few years (Davis et. al., 1984).

The other aspect I shall touch upon is not unrelated to what I have just said. The stability of the 'clock' is so high, because it is hardly slowing down. The rotational  $Q$  of this oscillator is about  $10^{20}$ , which means that in spite of its very rapid rotation, the pulsar is radiating away very little energy and angular momentum. Considering the energy put out by the Crab pulsar rotating 20 times slower, it follows that the magnetic braking of this object is far far less than that of other pulsars, or in other words, that its magnetic field must be very much smaller. The value of the field thus derived is five times  $10^8$  G, about ten thousand times less than that for the Crab and Vela pulsars and the smallest yet observed. That the field had to be so small could be predicted even before an accurate measurement of its slow-down rate, for the simple reason that at the position of this pulsar in the sky there is no blazing bubble like the Crab Nebula, as there would have been if its magnetic field were not so terribly weak (Radhakrishnan and Srinivasan, 1982).

Once again we have this extraordinary combination of an ultra-low field and an ultra-high spin rate. But we have been through this story once before in connection with the binary pulsar, and the thing to do is to look at the position of this pulsar on the field period diagram (Fig. 14). It would be an extraordinary coincidence indeed, if with the field and the period each many orders below the average, the combination should end up as it does, right on the spin-up line, given all that empty space in the diagram. It seems more than reasonable to conclude that this pulsar too was spun up like several of the others which are still in binaries, and indicated by circles on the diagram.



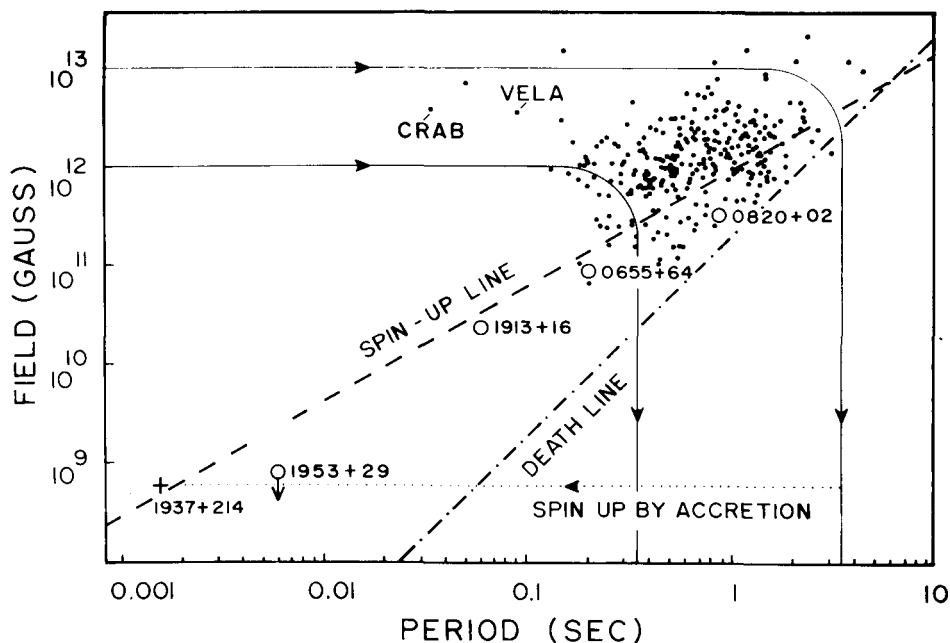


Fig. 14: The spin-up line and recycled pulsars.

Timing measurements have shown that this pulsar cannot have even a tiny object in orbit around it now. As Backer puts it, there is no rock around this clock. It would therefore have had to be liberated from its companion at some stage. While a position on the B-P diagram close to the spin-up line only testifies to spinning up by accretion from a companion or a disk, it leaves many other questions to be answered. For example, in the case of this pulsar, it can be shown that at least a tenth of a solar mass of material needed to be dumped on it, to give it the fantastic rotational energy it has now. Since the accretion would be limited by the Eddington rate, this would have required steady accretion for ten million years or more, which calls for a very different scenario from the massive X-ray binaries I mentioned earlier. The other important point is that this object is still in the galactic plane, in spite of its age. Its velocity must therefore also be small, and this further constrains its history.

Generally speaking, this is true for all pulsars which are spun-up, since their position relative to the spin-up line tells us about both the time for the companion to evolve, and for the pulsar field to decay. In other words, the present single or binary status, period, and velocity (or distance from the galactic plane) of a pulsar must all find a satisfactory explanation in terms of its history.

There is an incredible amount of work on stellar evolution in binaries pioneered by Paczynski (1971) and carried out over the last decade or so by many people in many places to explain all this. And I believe

it really does. I refer you to reviews by van den Heuvel (1983), and by Iben and Tutukov (1985), and the numerous references therein for the details of all this difficult and painstaking work by these authors, their colleagues, and others. The evolution of a binary system depends not only on the total mass, but also on the mass ratio and the separation of the components. But as far as this talk is concerned, I shall give in the next few examples, with apologies to the experts present here, a much simplified version of the outcomes of various initial conditions as if total mass of the system were the only determining factor. This is not a bad approximation at all for close binary systems, since the stars indulge in what I call mass-sharing. Most of the excess mass of the first star is re-used by the second, and you can get two neutron stars for the price of one and a half.

Case I: We have already touched upon this case which is at the massive

### CASE I

**SYSTEM : VERY MASSIVE BINARY**

**OUTCOME: TWO SINGLE PULSARS**

**BOTH HIGH FIELDS & HIGH VELOCITIES. CAN TRAVEL TO LARGE Z.**

end of the spectrum. The mass of the companion just before the (second) supernova is well over four solar masses, and therefore enough to disrupt the binary system. I will come back to the pulsars which might have been produced this way.

Case II: The difference between this category and the first is not so much in the masses of the stars as in their separation. When the secondary has

### CASE II

**SYSTEM : VERY MASSIVE BINARY. NEUTRON STAR SPIRALS INTO COMPANION WHICH DISRUPTS.**

**OUTCOME: ONE LOW-FIELD LOW-VELOCITY PULSAR.**

evolved and starts to transfer mass onto the first born neutron star, the orbit shrinks and the system invariably goes through a common envelope phase where the neutron star is *inside* its companion. If this happens before sharp density gradients have set in, complete spiral-in takes place and the companion

is eventually totally disrupted. The outcome will be a moderately low-field pulsar, with the velocity of the centre of mass of the binary system. I will come back to this case also.

Case III: This was the case discussed in connection with the binary pulsar

### CASE III

**SYSTEM : MASSIVE BINARY**

**OUTCOME: TWO PULSARS IN ECCENTRIC ORBIT**

**ONE RECYCLED LOW FIELD. OTHER NORMAL. ONE OR OTHER [OR BOTH] MAY BE SEEN. EXAMPLES ARE PSR 1913+16 & PSR 2303+46 .**

1913+16, where the system remains bound even after the second SN. In that case it was the old recycled pulsar we were seeing, but now we have another such system in PSR 2303+46 (Stokes et al., 1985) where we see the new pulsar, i.e., the second born neutron star.

Case IV: As we get to lower masses, there will be situations in which the

#### CASE IV

**SYSTEM : LESS MASSIVE BINARY**

**OUTCOME: RECYCLED LOW FIELD  
PULSAR + HIGH MASS  
WD IN CIRCULAR ORBIT**

**EXAMPLE PSR 0655+64  
CAN LEAD TO CASE V**

system is not massive enough to produce two neutron stars. This will lead to a neutron star - white dwarf combination. A nice example we have is the PSR 0655+64 system where the neutron star was formed first, and spun up later by accretion from its companion.

Case V: This case is a development of Case IV. Given a really tight orbit

#### CASE V

**SYSTEM : NS + MASSIVE WD IN  
CLOSE ORBIT  $P \sim 1$  HR.  
GRAVITATIONAL RADIATION  $\Rightarrow$  COALESCENCE  
 $\Rightarrow$  WD DISRUPTED.**

**OUTCOME: ONE VERY LOW FIELD  
HIGHLY SPUN-UP PULSAR.**

**EXAMPLE PSR 1937+21**

to begin with, gravitational radiation will eventually cause coalescence, and in the process disruption of the white dwarf into a disk. Subsequent spin-up can lead to all of the observed characteristics of the 1.5 milli-second pulsar. This is the best scenario yet for this object and was advanced by van den Heuvel and Bonsema (1984).

Case VI: We come now to those low mass cases in which even the primary

#### CASE VI

**SYSTEM : LOW MASS BINARY**

**OUTCOME: WD + LOW MASS COMPANION.  
ACCRETION LEADS TO N.S. + LOW MASS WD  
IN WIDE CIRC. ORBIT**

**IF SHORT ACCRETION HIGH FIELD,  
e.g. 0820+02. IF LONG ACCRETION  
LOW FIELD, e.g., 6 ms PULSAR.**

can only leave a white dwarf, but which accretes matter from its companion to become a neutron star. The difference between this kind of collapse and that of the core of a massive star which I described at the beginning, is that the last straw that broke the camel's back was just added after a long interval.

Let me dwell for a moment on this particular category, as there are several very important points to take note of.

1. Although the possibility of producing pulsars this way has been talked about for a long time, the identification of PSR 1953+29 and PSR 0820+02 as examples, was the first convincing demonstration of its reality. It provided an explanation both for the orbital configuration, and for the magnetic fields. If the accretion takes a very long time, then the pulsar field will decay considerably and lead to spinning up to a very short period, e.g., the 6ms pulsar. On the other hand, if the neutron star was formed just before accretion stopped, the field would still have been high and the period

hardly different from other normal pulsars. But for its being in a binary system, 0820+02 could not have been differentiated from numerous single pulsars with similar characteristics (see Fig. 14).

2. The recent exciting discovery of Q.P.O.s by van der Klis et. al. (1985) shows an earlier phase in the evolution of such systems when the neutron star is still accreting matter from its less massive companion.

3. While there is no massive envelope to get rid of when the neutron star is formed this way, there is still the same amount of binding energy (10% of the rest mass) to be released. Although this is believed to be given off in neutrinos mostly, an identification - and explanation - of the associated light curve and radio or X-ray remnant remain to be carried out. Are these the subluminal SN seen in external galaxies? Which galactic SNRs that we know were formed this way?

4. Finally, to those who were looking for new channels of pulsar production to boost the birth rate statistics, I feel that there is not much hope here. Pulsars born this way seem doomed to be stuck with a companion even if it is as small as Jupiter. If so, they cannot be confused with single pulsars.

It is possible that I have given you the impression that all of these complicated histories are good only to account for a few obviously peculiar pulsars, most of them in binaries. In this last part of my talk, I shall try to convince you that they also account for the vast majority of observed pulsars most of which are single.

In an early and seminal study, Gunn and Ostriker (1970) identified the progenitors of pulsars as Population I objects. Pulsars were born in binaries in the galactic plane, but moved away due to the velocities acquired on the disruption of the systems. As I have pointed out already, in close binaries, mass transfer before the first supernova explosion leaves the system bound, and the neutron star will never appear as a single pulsar unless and until the system is disrupted. This can happen either by the explosion of the other star, or in some other way. In this picture therefore, it will have to be the disruption scenario that will characterise their velocities. So let us take a closer look at what is expected to happen.

If a binary system is formed in the plane of the Galaxy, it will remain there through the evolution of the primary star and mass transfer phase. It is only the sudden loss of mass associated with the first SN explosion that will give it a kick. But as the mass of the ejected envelope is only a small fraction of the total mass at this stage, the remaining binary will acquire only a small velocity, of the order of 30-50 km/sec. Between now and when something interesting happens again, the binary system will gradually drift away from the plane with the Z component of this velocity. Eventually, if the binary is broken up not by a second SN explosion, but by the total disruption of the companion, as in two of the cases I described, the pulsar left behind will have the same low velocity as the centre of mass of the parent system, of the order of 30-50 km/sec and a field commensurate with decay between the times of the SN and the disruption.

Note that if the neutron star was formed by accretion on to a white dwarf, there will be virtually no recoil, as very little mass is expelled, and the velocity of the binary system will be negligible.

On the other hand, if it is a second SN that breaks up the binary, the velocities will depend on the mass of the companion, and its separation from the neutron star, just before the explosion. The asymptotic velocity of the first born neutron star will be somewhat higher, and its field somewhat lower, than the corresponding quantities for the new born pulsar. To compare theory with observations, one could assume models for the binary system and ask what velocities and fields should be observed. Alternatively, if it were possible, one could attempt to *derive* the characteristics of the binary systems from the observations.

In the work of Gunn and Ostriker (1970), high velocities were attributed to pulsars simply on the basis of the distances from the galactic plane at which they were discovered. But since then, we have more and better estimates for the velocities of pulsars. Long baseline radio interferometry systems, while unable to produce radio pictures of pulsars because they are so small, have nevertheless made the very important contribution of measuring their proper motions. These combined with the distances lead to their transverse velocities. Such measurements of very high quality have been made by Lyne et al., (1982) on a sample of 26 pulsars. They found velocities ranging from about 10 to 400 km/sec, and that the distribution

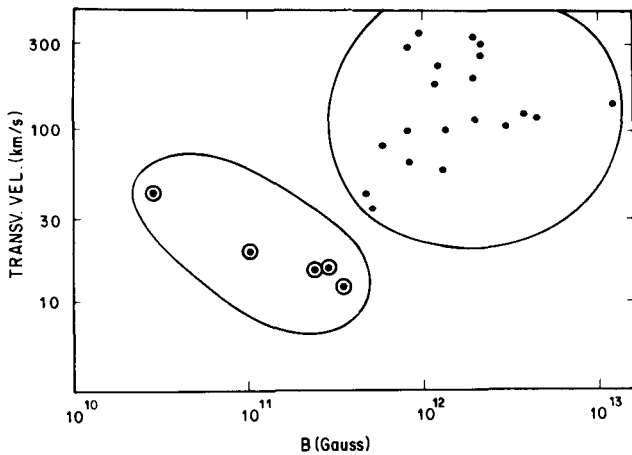


Fig. 15: Plot of transverse velocity versus magnetic field for 26 pulsars. Their distribution suggests two groups as indicated in the figure.

distribution did not seem to be Maxwellian, there being excesses of both high and low velocity objects. Interestingly, the high velocities seem to be associated with high field pulsars and low velocities with low field pulsars (Anderson and Lyne, 1983). Their observations are illustrated in Fig. 15 and one notices that the apparent correlation of field with velocity is really due to the 5 pulsars over in the lower left-hand corner, all of which, incidentally, fall below the

spin-up line on a field period plot. A very reasonable interpretation would therefore be that we are dealing here with two populations having different histories (Radhakrishnan, 1984). In particular, the low field, low velocity pulsars seem to have precisely the characteristics expected as the outcome

of Case II mentioned earlier and illustrated in Fig.16. All the experts agree that the first born neutron star will spiral completely into the core of its companion. But as to how long it will take eventually to completely disperse all of the companion's mass is not so clear. The problem is the

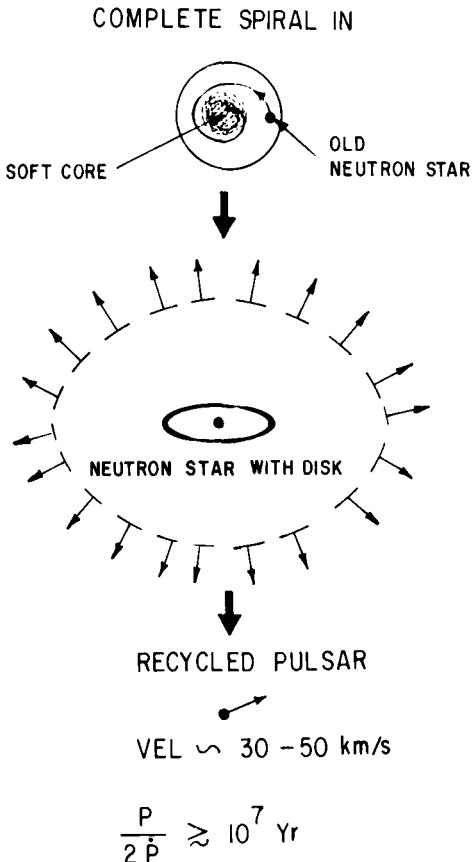


Fig. 16: Evolutionary scenario for low-velocity low-field pulsars.

energy required to send all this mass to infinity, and which may require some accretion. The neutron star may go right to the middle and create a so-called Thorne-Zytkow star which would appear like a red supergiant. But as there are too few such objects seen in the sky, either this phase is very short, or the star is disrupted into a disk to begin with. In any case, all the mass will finally be dispersed and this scenario can account for the origin of the minority group of single pulsars.

The vast majority of single pulsars are, however, high-field and also high-velocity as shown both by the interferometric measurements referred to earlier, and by scintillation studies on a larger sample (Cordes et. al., 1984). It must be clear from all I have said before, that the logical identification of their origin is with Case I, in which the characteristics of the binaries lead naturally to two SN explosions and disruption of the system. This scenario has been around for many years in the literature as a highly plausible one, but could not be tested in a quantitative way because of the lack of good pulsar velocity measurements. My colleague

Shukre and I have been worrying about pulsar velocities and what they mean for many years now, but meaningful progress had to await measurements such as those of Lyne et. al., (1982), that I have just mentioned. After removal of the disparate group of 5 very low velocity pulsars (Fig. 15), and correction to 3 dimensions from 2, one was left with a distribution of 20 velocities ranging from 100 to 450 km/sec and with a mean of 270 km/sec. If these velocities were due to the disruption of binaries, what were the parameters of those binary systems?

The first step was to assume circular orbits and to ask what combinations of masses and separations could lead to the velocities. The simple answer was that the allowed separations were far smaller than

the main sequence radii of the stars with corresponding masses. This was evidence from the velocities that a common envelope phase was involved, just as predicted by theory. This close encounter would circularise the orbit, justifying our assumption, and lead to shedding of the envelope prior to the second SN explosion. The sample of velocities must therefore represent some mixture of both first and second born neutron stars, and the formulae can be reapplied allowing for both possibilities (Radhakrishnan and Shukre, 1985).

The surprising but welcome result of this exercise is illustrated in Fig. 17. It shows extreme limits

of the binary parameters consistent with the observed velocity distribution. The ranges of the parameters are just those expected from the evolution of massive close binary systems, including the cut off at  $\sim 8 M_{\odot}$  for the Helium star. This corresponds to a main-sequence mass of around  $30 M_{\odot}$  above which, as I said earlier, Black Holes are expected to be formed and not neutron stars. My colleague and I consider this as strong supporting evidence for our implicit assumption in all of this analysis, that the velocities are not due to asymmetric kicks from the SN explosions, which even otherwise have no logical basis.

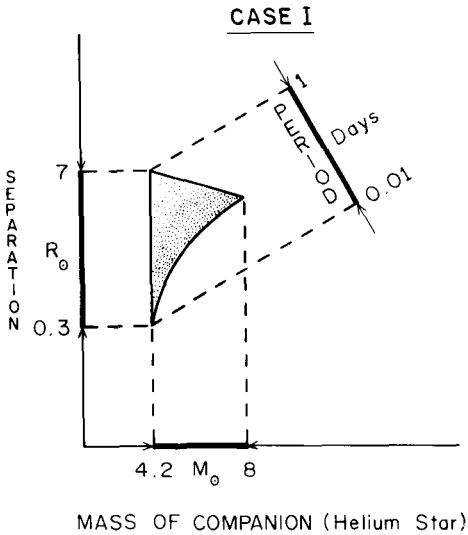


Fig. 17: Limits on parameters of binary systems prior to the 2nd SN explosion derived from pulsar velocities. Fig. adapted from Radhakrishnan and Shukre (1985).

which in any case may not substantially modify the simple picture I have tried to present. What would modify the picture, and drastically, is if the origin of pulsar velocities is not as I have supposed, but due to "asymmetric kicks" received at birth. The observed velocities require kick energies which are a minute fraction of the total energy in the explosion. Apparently the celestial footballer delivering these kicks moderates them carefully to mimic the velocity distribution expected naturally from the kick-free disruption of close-binary systems !

In any case, our conclusion from all of this is that the progenitors of all pulsars were originally in binary systems. The evolution of these stars, governed in various ways by the presence of their companions, results



in pulsars with wide ranging characteristics. We may still not understand exactly how pulsars pulse, but I think we are beginning to understand their genesis.

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#### REFERENCES

- Anderson, B. and Lyne, A.G. 1983, *Nature* 303, 597.
- Baade W. and Zwicky, F. 1934, *Proc. Nat. Acad. Sci. (U.S.A.)* 20, 254.
- Backer, D.C., Kulkarni, S.R., Heiles, C., Davis, M.M. and Goss, W.M. 1982, *Nature* 300, 615.
- Boriakoff, V., Buccheri, R. and Fauci, F. 1983, *Nature* 304, 417.
- Chevalier, R.A. and Emmering, R.T. 1985, (to appear in *Astrophys. J.*).
- Cordes, J.M. and Weisberg, J.M. 1984, in *Millisecond Pulsars*, eds. S.P. Reynolds & D.R. Stinebring (NRAO, Green Bank, W.Va., U.S.A.), p.138.
- Davidson, K. and Ostriker, J.P. 1973, *Astrophys.J.* 179, 583.
- Davis, M., Taylor, J., Weisberg, J. and Backer, D. 1984, in *Millisecond Pulsars*, eds. S.P. Reynolds & D.R. Stinebring (NRAO, Green Bank, W.Va., U.S.A.), p.12.
- Giacconi, R., Gursky, H., Paolini, F.R. and Rossi, B.R. 1962, *Phys. Rev. Lett.* 9, 439.
- Ginzburg, V.L. 1971, in *Highlights of Astronomy*, Vol. 2, ed. C. de Jager (Reidel, Dordrecht) p.20.
- Goldreich, P. 1972, in *The Physics of Pulsars*, ed. A.M. Lenchek (Gordon & Breach, New York), p.151.
- Green, D.A., and Gull, S.F. 1983, in *IAU Symp. 101: Supernova Remnants and Their X-ray Emission*, eds. J. Danziger & P. Gorenstein (Reidel, Dordrecht), p.329.
- Gunn, J.E. and Ostriker, J.P. 1970, *Astrophys.J.* 160, 979.
- Hewish, A., 1971, in *Highlights of Astronomy*, Vol. 2, ed. C. de Jager (Reidel, Dordrecht), p.18.
- Hewish, A., Bell, S.J., Pilkington, J.D.H., Scott, P.F. and Collins, R.A. 1968, *Nature* 217, 709.
- Hulse, R.A. and Taylor, J.H. 1974, *Astrophys. J.(Lett.)* 191, L59.
- Iben, I. and Tutukov, A.V. 1985, *Astrophys.J. Supp.* 58, 661.
- Large, M.I., Vaughan, A.E. and Mills, B.Y. 1968, *Nature* 220, 340.
- Lyne, A.G., Anderson, B. and Salter, M.J. 1982, *Mon. Not. R. Astr. Soc.* 201, 503.
- Narayan, R. and Vivekanand, M. 1983, *Astron. Astrophys.* 122, 45.
- Manchester, R.N. and Taylor, J.H. 1977, "Pulsars" (W.H. Freeman, San Francisco).
- Miller, J.S. and Wampler, E.J. 1969, *Nature* 221, 1037.
- Murray, S.S., Fabbiano, G., Fabian, A.C., Epstein, A. and Giacconi, R. 1979, *Astrophys.J.* 234, L69.

- Oppenheimer, J.R. and Volkoff, G.M. 1939, *Phys. Rev.* 55, 374.
- Ostriker, J.P. and Gunn, J.E. 1969, *Astrophys.J.* 157, 1395.
- Paczynski, B. 1971, *Ann. Rev. Astr. Astrophys.* 9, 183.
- Pringle, J.E. and Rees, M.J. 1972, *Astron. Astrophys.* 21, 1.
- Proszynski, M. and Przybycien, D. 1984, in *Millisecond Pulsars*, eds. S.P. Reynolds & D.R. Stinebring (NRAO, Green Bank, W.Va., U.S.A.), p.151.
- Radhakrishnan, V. 1984, in *Proceedings of Millisecond Pulsars*, eds. S.P. Reynolds & D.R. Stinebring (NRAO, Green Bank, W.Va., U.S.A.), p.130.
- Radhakrishnan, V., Cooke, D.J., Komisaroff, M.M. and Morris, D. 1969, *Nature* 221, 443.
- Radhakrishnan, V. and Cooke, D.J. 1969, *Astrophys.Lett.* 3, 225.
- Radhakrishnan, V. and Srinivasan, G. 1982, *Curr. Sci.* 51, 1096.
- Radhakrishnan, V. and Srinivasan, G. 1984, in the *Proceedings of the Second Asian-Pacific Regional Meeting on Astronomy*, eds. B. Hidayat & Feast, M.W. (Tira Pustaka, Jakarta, Indonesia), p.423.
- Radhakrishnan, V. and Shukre, C.S. 1985, in the *Proceedings of the workshop on Supernovae, their Progenitors and Remnants*, eds. G. Srinivasan & V. Radhakrishnan (Indian Acad. Sci., Bangalore, India), p.155.
- Richards, D.W. and Comella, J.M. 1969, *Nature* 222, 551.
- Ruderman, M. 1981, in *IAU Symp. 95: Pulsars*, eds. W. Sieber & R. Wielebinski, (Reidel, Dordrecht), p.87.
- Ruderman, M. 1986, in *Proceedings of the NATO Advanced Study Institute on "High Energy Phenomena Around Collapsed Stars"*, Cargese, Sept. 1985.
- Shklovsky, I.S. 1968, *Sov. Astron.* 11, 749.
- Smarr, L.L. and Blandford, R. 1976, *Astrophys.J.* 207, 574.
- Srinivasan, G. and van den Heuvel, E.P.J. 1982, *Astron. Astrophys.* 108, 143.
- Srinivasan, G., Bhattacharya, D. and Dwarakanath, K.S. 1984, *J. Astrophys. Astron.* 5, 403.
- Staelin, D.H. and Reifenstein, E.C. 1968, *Science* 162, 1481.
- Stokes, G.H., Taylor, J.H. and Dewey, R.J. 1985, *Astrophys.J.(Lett.)* 294, L.21.
- Stokes, G.H., Taylor, J.H., Weisberg, J.M. and Dewey, R.J. 1985, *Nature* 317, 787.
- Sturrock, P.A. 1971, *Astrophys.J.* 164, 529.
- Taylor, J.H. and Weisberg, J. 1982, *Astrophys.J.* 253, 908.
- Tutukov, A.V. 1981, in *IAU Symp. 93: Fundamental Problems in the Theory of Stellar Evolutions*, eds. D. Sugimoto, D.Q. Lamb & D.N. Schramm (Reidel, Dordrecht). p.137.
- van den Heuvel, E.P.J. 1981, in *IAU Symp. 93: Fundamental Problems in the Theory of Stellar Evolution*, Eds. D. Sugimoto, D.Q. Lamb & D.N. Schramm (Reidel, Dordrecht), p.155.
- van den Heuvel, E.P.J. 1983, in *Accretion Driven Stellar X-ray Sources*, in eds. W.H.G. Lewin & E.P.J. van den Heuvel (Cambridge Univ. Press.), p.303.
- van den Heuvel, E.P.J. and Bonsema, P.T.J. 1984, *Astron. Astrophys.* 139, L.16.
- van der Klis, M., Jansen, F., van Paradijs, J., Lewin, W.H.G., van den Heuvel, E.P.J., Trumper, J.E. and Sztajno, M. 1985, *Nature* 316, 225.

- Vivekanand, M. and Narayan, R. 1981, *J. Astrophys. Astron.* 2, 315.
- Vivekanand, M., Narayan, R. and Radhakrishnan, V. 1982, *J. Astrophys. Astron.* 3, 237.
- Vivekanand, M. and Narayan, R. 1982, *J. Astrophys. Astron.* 3, 399.
- Zeldovich, Ya.B. 1964, *Sov. Phys. Doklady* 9, 246.