DYNAMIC BEHAVIOR OF MULTI-LAYERED VISCOELASTIC NANOBEAM SYSTEM EMBEDDED IN A VISCOELASTIC MEDIUM WITH A MOVING NANOPARTICLE

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ABSTRACT

In this paper, dynamic behavior of multi-layered viscoelastic nanobeams resting on a viscoelastic medium with a moving nanoparticle is studied. Eringens nonlocal theory is used to model the small scale effects. Layers are coupled by Kelvin-Voigt viscoelastic medium model. Hamilton's principle, eigen-function technique and the Laplace transform method are employed to solve the governing differential equations. Analytical solutions for transverse displacements of double-layered is presented for both viscoelastic nanobeams embedded in a viscoelastic medium and without it while numerical solution is achieved for higher layered nanobeams. The influences of the nonlocal parameter, stiffness and damping parameter of medium, internal damping parameter and number of layers are studied while the nanoparticle passes through. Presented results can be useful in analysing and designing nanocars, nanotruck moving on surfaces, racing nanocars etc.

Keywords: Nanostructure, Multi-layered nanobeam, Viscoelastic nanobeam, Dynamic response.

1. INTRODUCTION

Nanobeams are one of the most important nanostructures used in nano-devices such as oscillators, clocks and sensor devices. The behavior of single layered and multi-layered nanobeams and nanoplates, have attracted a great deal of attentions in scientific community in different static [1-4] and dynamic [5-10] manners. One of the most important fields in dynamic researches is evaluating the behavior of nanobeams under a moving load, nanoparticle or nanocars. The first nanocar was built in 2005 at Rice University [11] in Houston. Later, researchers focused on understanding and improving the movement of naovehicles [12-17]. With the new discussion of racing between nanocars in 2016 [18] on different surfaces, it's important to understand the deflection made on the surfaces which the nanovehicle is passing through. To achieve this behavior, researchers modeled different type of nanocars with a concentrated mass, a nanoparticle or a moving loads. In this case, Kiani [19] reported the Longitudinal and transverse vibration of a single-walled carbon nanotube subjected to a moving nanoparticle. The governing equation of motions were achieved and solved by Galerkin method. Şimşek [20] studied the forced vibration of a simply supported single-walled carbon nanotube (SWCNT) subjected to a moving harmonic load by using nonlocal Euler–Bernoulli beam theory. The time-domain responses were obtained by using both modal analysis method and direct integration method. Şimşek [21] also investigated the vibration of an elastically connected double carbon nanotube system (DCNTS) carrying a moving nanoparticle based on the nonlocal elasticity theory. The two nanotubes were assumed identical and connected with each other continuously by elastic springs. Main equations were solved numerically by using the Galerkin method and the time integration method. Kiani and Wang [22] developed the vibration of a single-walled carbon nanotube (SWCNT) subjected to a moving nanoparticle in the framework of the nonlocal continuum theory. The SWCNT was modeled by an equivalent continuum structure (ECS). The ECS was simulated based on the Rayleigh, Timoshenko, and higher-order beam theories in the context of the nonlocal continuum theory. Ghorbanpour Arani *et al*. [23] reported the vibration of single-walled Boron Nitride nanotube embedded in bundle of CNTs using nonlocal piezoelasticity theory under a moving nanoparticle. Chang [24] studied the statistical dynamic behaviors of nonlinear vibration of the fluid-conveying double-walled carbon nanotubes (DWCNTs) under a moving load by considering the effects of the geometric nonlinearity and the nonlinearity of van der Waals (vdW) force. Ghorbanpour Arani and Roudbari [25] investigated the nonlocal longitudinal and transverse vibrations

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DOI : 10.1017/jmech.2016.91

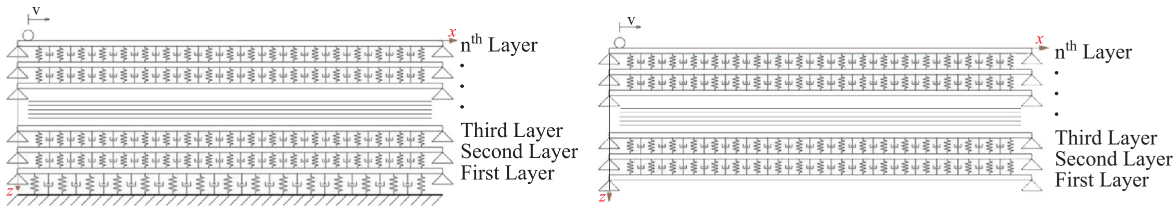

Fig. 1 Schematic representation of multi-layered viscoelastic nanobeam with a moving nanoparticle on top layer.

of coupled boron nitride nanotube (BNNT) system under a moving nanoparticle using piezoelastic theory and surface stress based on Euler–Bernoulli beam. Lü *et al.* [26] presented the transverse vibration of double carbon-nano-tubes (DCNTs) which were coupled through elastic medium. Both tubes were conveying moving nano-particles and their ends were simply supported. The system equations were discretized by applying Galerkin expansion method, and numerical solutions were obtained.

By the knowledge of the authors, there is no research done in order to understand the behavior of multi-layered viscoelastic nanobeams under a moving nanoparticle or a moving load with or without a viscoelastic medium. This study could be useful in different subjects of science. With improvements done in nanotechnology and the usage of smaller devices in designing different machines, multi layered nanobeam systems as a part of nanostructures have achieved more usages. Designing new nanocars with motors which leaded to self-driving nanocars, nanotricycles and nanotrucks, and having a nanorace between them on the surface of nanobeams and nanoplates, its necessary to understand the behavior of the lower surface while the nanocar passes through. Also, with the usage of nanobeam systems as sensors and actuators being forced bended with moving concentrated forces, it's necessary to understand the reaction of them to the applied moving load in order to design a more efficient nanosensors and nanoactuators. In Fig. 1, a schematic view of multi-layered viscoelastic nanobeam with a moving nanoparticle on top layer is presented for both type of having viscoelastic foundation and without it.

2. PROBLEM FORMULATION

To add the small scale effects in nanoscale beams, Eringen's nonlocal theory is employed. Basic equations for a linear homogenous nonlocal elastic body are given as

$$\begin{aligned} &\sigma_{ij}^{nl} = \int_V \alpha(|x-x'|,\tau)\,\sigma_{ij}^{l}(x')\,dV(x'), \qquad \forall x \in V \\ &\sigma_{ij}^{l}(x) = C_{ijkl}\,\varepsilon_{kl}^{l} \\ &\varepsilon_{ij}^{l} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \end{aligned} \tag{1}$$

Where σ_{ij}^{l} and ε_{ij}^{l} are the local stress and strain tensors, σ_{ij}^{nl} is the nonlocal stress tensor, C_{ijkl} is the fourth-order elasticity tensor, $|x-x'|$ is the distance in Euclidean form and $\alpha(|x-x'|, \tau)$ is the nonlocal modulus or attenuation function incorporating into constitutive equations the nonlocal effects at the reference point x produced by local strain at the source x'. α is the material constant which is defined as (e_0a/l) where a is an internal characteristic size depending on the internal lengths (e.g. lattice parameter, granular distance, distance between $C-C$ bonds), l is the external characteristic size depending on the external lengths (e.g. crack length, wavelength) and e_0 is a constant number depending on the material. Due to the difficulty of solving the integral constitutive Eq. (1), it can be simplified [27] to equation of differential form as

$$\left(1-\alpha^2 l^2 \nabla^2\right)\sigma^{nl} = \sigma^{l} \tag{2}$$

For one dimensional homogeneous material, Eq. (2) can be simplified as

$$\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\sigma_{xx}^{nl}(x,t) = E\left(1+\tau_d\frac{\partial}{\partial t}\right)\varepsilon_{xx}^{l}(x,t) \tag{3}$$

Where τ_d is the internal damping parameter and E is the Young's modulus of the nanobeam. Multiplying Eq. (3) by zdA and integrating the result over the area A leads to

$$M-(e_0a)^2\frac{d^2M}{dx^2} = E\left(1+\tau_d\frac{\partial}{\partial t}\right)I\frac{d^2w}{dx^2} \tag{4}$$

The equation of motion for the transverse vibration of nanobeam can be obtained from the second Newton law as

$$\frac{\partial^2 M_i}{\partial x^2} - F_i = \rho A\frac{\partial^2 w_i}{\partial t^2} \tag{5}$$

By substituting Eq. (5) into equation (4), the nonlocal bending momentum will be achieved as

$$M_i^{nl} = E\left(1+\tau_d\frac{\partial}{\partial t}\right)I\frac{\partial^2 w_i}{\partial x^2} + (e_0a)^2\left[\rho A\frac{\partial^2 w_i}{\partial t^2}+F_i\right] \tag{6}$$

With respect to Fig. 2, external forces applied to each layer due to the moving mass for multi-layered nanobeam resting in a viscoelastic medium are

https://doi.org/10.1017/jmech.2016.91 Published online by Cambridge University Press

$$F_i=\begin{cases} mg\delta(x-x_m)-K_{n-1}(w_n-w_{n-1})-C_{n-1}\left(\frac{\partial w_n}{\partial t}-\frac{\partial w_{n-1}}{\partial t}\right), & i=n \\ K_i(w_{i+1}-w_i)+C_i\left(\frac{\partial w_{i+1}}{\partial t}-\frac{\partial w_i}{\partial t}\right)-K_{i-1}(w_i-w_{i-1})-C_{i-1}\left(\frac{\partial w_i}{\partial t}-\frac{\partial w_{i-1}}{\partial t}\right), & 2\le i\le n-1 \\ K_1(w_2-w_1)+C_1\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right)-K_0w_1-C_0\frac{\partial w_1}{\partial t}, & i=1 \end{cases} \tag{7}$$

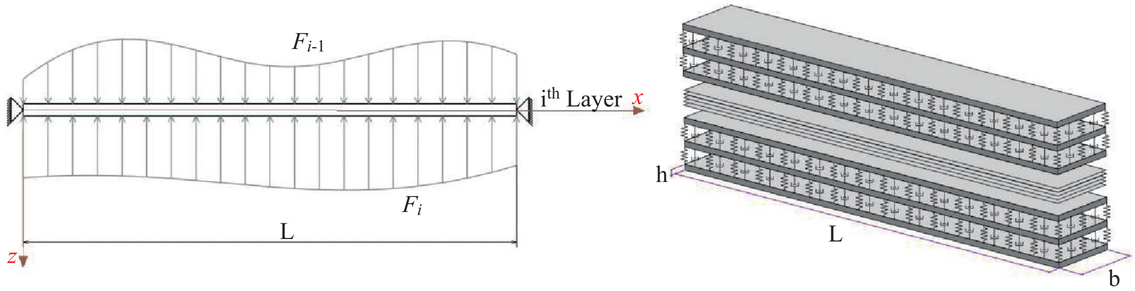

Fig. 2 Free diagram of the i^{th} layer of multi-layered viscoelastic nanobeam with external forces F_i and $F_{i\text{-}1}$.

Where K_i and C_i are the stiffness and damping parameters of Kelvin-Voigt viscoelastic theory which is applied between i^{th} and $(i + 1)^{th}$ layers. The coupling made with the viscoelastic medium is presented by using 0 index. By using Eq. (4) to Eq. (7), the governing equation of motions of multi-layered nanobeam system resting on a viscoelastic medium with a moving mass on top side of upper layer can be expressed in terms of transverse displacement for nonlocal constitutive relations as

$$\begin{aligned} &\rho A\frac{\partial^2 w_n}{\partial t^2}+mg\delta(x-x_m)-K_{n-1}(w_n-w_{n-1})-C_{n-1}\left(\frac{\partial w_n}{\partial t}-\frac{\partial w_{n-1}}{\partial t}\right)+E_iI\left(1+\tau_d\frac{\partial}{\partial t}\right)\frac{\partial^4 w_n}{\partial x^4} \\ &=\alpha^2\frac{\partial^2}{\partial x^2}\left[\rho A\frac{\partial^2 w_n}{\partial t^2}+mg\delta(x-x_m)-K_{n-1}(w_n-w_{n-1})-C_{n-1}\left(\frac{\partial w_n}{\partial t}-\frac{\partial w_{n-1}}{\partial t}\right)\right], \qquad i=n \\ &\rho A\frac{\partial^2 w_i}{\partial t^2}+K_i(w_{i+1}-w_i)+C_i\left(\frac{\partial w_{i+1}}{\partial t}-\frac{\partial w_i}{\partial t}\right)-K_{i-1}(w_i-w_{i-1})-C_{i-1}\left(\frac{\partial w_i}{\partial t}-\frac{\partial w_{i-1}}{\partial t}\right)+E_i\left(1+\tau_d\frac{\partial}{\partial t}\right)I\frac{\partial^4 w_i}{\partial x^4} \\ &=\alpha^2\frac{\partial^2}{\partial x^2}\left[\rho A\frac{\partial^2 w_i}{\partial t^2}+K_i(w_{i+1}-w_i)+C_i\left(\frac{\partial w_{i+1}}{\partial t}-\frac{\partial w_i}{\partial t}\right)-K_{i-1}(w_i-w_{i-1})-C_{i-1}\left(\frac{\partial w_i}{\partial t}-\frac{\partial w_{i-1}}{\partial t}\right)\right] \qquad 2\le i\le n-1 \\ &\rho A\frac{\partial^2 w_1}{\partial t^2}+K_1(w_2-w_1)+C_1\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right)+E_i\left(1+\tau_d\frac{\partial}{\partial t}\right)I\frac{\partial^4 w_1}{\partial x^4} \\ &=\alpha^2\frac{\partial^2}{\partial x^2}\left[\rho A\frac{\partial^2 w_1}{\partial t^2}+K_1(w_2-w_1)+C_1\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right)-K_0w_1-C_0\frac{\partial w_1}{\partial t}\right], \qquad i=1 \end{aligned} \tag{8}$$

It can be easily shown that for the time when there is no viscoelastic medium under the multi-layered nanobeams system, external forces applied and the equation of motion of each layered is achieved by neglecting the viscoelastic terms by having $K_0 = C_0 = 0$ in Eq. (7) and Eq. (8).

3. SOLUTION PROCEDURE

The total transverse dynamic deflection $w_i(x, t)$ in modal form is written as

$$w_i(x,t)=\sum_{j=1}^{\infty}\eta_i(x)W_i(t) \tag{9}$$

Where $W_i(t)$ are the unknown time-dependent generalized coordinates and $\eta_i(x)$ are the eigen-modes of simply supported beam which are expressed as

$$\eta_i(x)=\sin\left(\frac{i\pi x}{L}\right) \tag{10}$$

Moreover, the external force made by the moving particle could be expressed as

$$F_0(x,t) = m_p g\, \delta\left(x - x_p\right) \tag{11}$$

Where m_p is the mass of nanoparticle and $\delta(x - x_p)$ is the Dirac delta function which could be expressed in terms of sinusoidal as follow

$$\delta\left(x - x_p\right) = \frac{2}{L}\sum_{i=1}^{\infty} \sin\left(\frac{i\pi}{L}x_p\right)\sin\left(\frac{i\pi}{L}x\right) \tag{12}$$

Where x_p denotes the coordinate of the moving particle from the left end of the upper layer.

3.1 Solution Procedure for Multi-Layered Viscoelastic Nanobeam System

By substituting Eq. (9) to Eq. (12) into Eq. (8) and neglecting the viscoelastic medium terms with using new parameters defined as

$$X = \frac{x}{L},\quad W = \frac{w}{L},\quad \alpha = \frac{e_0 a}{L},\quad \gamma^2 = \frac{\rho A L^4}{EI},\quad P^2 = \frac{mgL^3}{EI},\quad \kappa = \frac{KL^4}{EI},\quad \varsigma = \frac{CL^4}{EI} \tag{13}$$

The set of equations of motions could be represented in matrix form as

$$\gamma^2 \frac{\partial^2}{\partial t^2}\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix} + \begin{bmatrix} \chi_1\tau_d - \varsigma_1 & \varsigma_1 & & & & & & \\ \varsigma_1 & \chi_2\tau_d - \varsigma_1 - \varsigma_2 & \varsigma_2 & & Zero & & & \\ & & \ddots & & & & & \\ & & \varsigma_{i-2} & \chi_{i-1}\tau_d - \varsigma_{i-2} - \varsigma_{i-1} & \varsigma_{i-1} & & & \\ & & & \varsigma_{i-1} & \chi_i\tau_d - \varsigma_{i-1} - \varsigma_i & \varsigma_i & & \\ & & & & \varsigma_i & \chi_{i+1}\tau_d - \varsigma_i - \varsigma_{i+1} & \varsigma_{i+1} & \\ & & & & & \ddots & & \\ & Zero & & & & \varsigma_{n-2} & \chi_{n-1}\tau_d - \varsigma_{n-2} - \varsigma_{n-1} & \varsigma_{n-1} \\ & & & & & & \varsigma_{n-1} & \chi_n\tau_d - \varsigma_{n-1} \end{bmatrix} \frac{\partial}{\partial t}\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix}$$
$$+ \begin{bmatrix} \chi_1 + \kappa_1 & -\kappa_1 & & & & & & \\ -\kappa_1 & \chi_2 + \kappa_1 + \kappa_2 & -\kappa_2 & & Zero & & & \\ & & \ddots & & & & & \\ & & -\kappa_{i-2} & \chi_{i-1} + \kappa_{i-2} + \kappa_{i-1} & -\kappa_{i-1} & & & \\ & & & -\kappa_{i-1} & \chi_i + \kappa_{i-1} + \kappa_i & -\kappa_i & & \\ & & & & -\kappa_i & \chi_{i+1} + \kappa_i + \kappa_{i+1} & -\kappa_{i+1} & \\ & & & & & \ddots & & \\ & Zero & & & & -\kappa_{n-2} & \chi_{n-1} + \kappa_{n-2} + \kappa_{n-1} & -\kappa_{n-1} \\ & & & & & & -\kappa_{n-1} & \chi_n + \kappa_{n-1} \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \Gamma \end{Bmatrix} \tag{14}$$

Where χ_i and Γ is defined as

$$\chi_i = \frac{(i\pi)^4}{1 + (i\pi)^2 \dfrac{\alpha^2}{L^2}}$$
$$\Gamma = -2\frac{mgL^3}{EI}\sin(i\pi X_p) \tag{15}$$

With assuming that the moving particle starts moving from the left end of the upper beam at $t = 0$ by having a constant velocity through the path, the dimensionless location of the moving nanoparticle would be

$$X_p = \frac{V}{L}t \tag{16}$$

Where V is the velocity of the moving nanoparticle. At $t = L/V$ moving mass reaches the other end of the beam. By having the same material of viscoelastic modulus, mass density, uniform cross section and same continuous linear Kelvin-Voigt viscoelactic medium the set of equations of motions could be represented as

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$$\gamma^2\frac{\partial^2}{\partial t^2}\begin{Bmatrix} w_1\\ w_2\\ \vdots\\ w_{i-1}\\ w_i\\ w_{i+1}\\ \vdots\\ w_{n-1}\\ w_n\end{Bmatrix}+\begin{bmatrix} \chi\tau_d-\varsigma & \varsigma & & & & & \\ \varsigma & \chi\tau_d-2\varsigma & \varsigma & & & & \\ & \ddots & & & Zero & & \\ & \varsigma & \chi\tau_d-2\varsigma & \varsigma & & & \\ & & \varsigma & \chi\tau_d-2\varsigma & \varsigma & & \\ & & & \varsigma & \chi\tau_d-2\varsigma & \varsigma & \\ & Zero & & & \ddots & & \\ & & & & \varsigma & \chi\tau_d-2\varsigma & \varsigma \\ & & & & & \varsigma & \chi\tau_d-\varsigma \end{bmatrix}\begin{Bmatrix} w_1\\ w_2\\ \vdots\\ w_{i-1}\\ w_i\\ w_{i+1}\\ \vdots\\ w_{n-1}\\ w_n\end{Bmatrix}$$

$$+\begin{bmatrix} \chi+\kappa & -\kappa & & & & & \\ -\kappa & \chi+2\kappa & -\kappa & & & & \\ & \ddots & & & Zero & & \\ & -\kappa & \chi+2\kappa & -\kappa & & & \\ & & -\kappa & \chi+2\kappa & -\kappa & & \\ & & & -\kappa & \chi+2\kappa & -\kappa & \\ & Zero & & & \ddots & & \\ & & & & -\kappa & \chi+2\kappa & -\kappa \\ & & & & & -\kappa & \chi+\kappa \end{bmatrix}\begin{Bmatrix} w_1\\ w_2\\ \vdots\\ w_{i-1}\\ w_i\\ w_{i+1}\\ \vdots\\ w_{n-1}\\ w_n\end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ \vdots\\ 0\\ 0\\ 0\\ \vdots\\ 0\\ \Gamma\end{Bmatrix} \tag{17}$$

For solving Eq. (17) in time domain, Laplace transform method is employed so the following set of equations are obtained:

$$\begin{bmatrix} \eta_1 & \eta_2 & 0 & 0 & 0 & \dots & 0\\ \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & 0 & \dots & 0\\ 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & \dots & 0\\ \vdots & \vdots & & & & \vdots & \vdots\\ 0 & 0 & \dots & 0 & \eta_2 & \eta_1-\eta_2 & \eta_2\\ 0 & 0 & \dots & 0 & 0 & \eta_2 & \eta_1 \end{bmatrix}\begin{Bmatrix} L\{W_1\}\\ L\{W_2\}\\ L\{W_3\}\\ .\\ .\\ .\\ L\{W_{n-1}\}\\ L\{W_n\}\end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ 0\\ .\\ .\\ .\\ 0\\ -2P^2\dfrac{i\pi V_p}{(i\pi V_p)^2+S^2}\end{Bmatrix} \tag{18}$$

Where η_1 and η_2 are defined as

$$\begin{cases}\eta_1=\gamma^2S^2+(\chi\tau_d-\varsigma)S+\chi+\kappa\\ \eta_2=\varsigma S-\kappa\end{cases} \tag{19}$$

By inversing the coefficient matrix and evaluating the inverse Laplace transform of Eq. (18), the results of transverse displacement for each layer with respect to time domain could be achieved. Further solution depends on the number of layers used for nanobeam system where the process is presented for double layered with moving nanoparticle and for Higher layered nanobeam system in both cases of having a viscoelastic medium or without it.

3.1.1 Double Layered Viscoelastic Nanobeam with Moving Nanoparticle

For two layered nanobeam system, Eq. (18) and Eq. (19) could be rewritten as

$$\begin{bmatrix}\gamma^2S^2+(\chi\tau_d-\varsigma)S+(\chi+\kappa) & \varsigma S-\kappa\\ \varsigma S-\kappa & \gamma^2S^2+(\chi\tau_d-\varsigma)S+(\chi+\kappa)\end{bmatrix}\begin{Bmatrix}L\{W_1\}\\ L\{W_2\}\end{Bmatrix}=\begin{Bmatrix}0\\ -2P^2\dfrac{i\pi V_p}{(i\pi V_p)^2+S^2}\end{Bmatrix} \tag{20}$$

With evaluating the inverse Laplace transforms by decomposing partial fraction and doing some calculations, the values of $W_1(t)$ and $W_2(t)$ are obtained as

$$W_1(t)=P^2\left[L_1\cosh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+L_3\cosh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t}\right.$$
$$+L_2\sinh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+\mathrm{L}_4\sinh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t} \tag{21}$$
$$\left.+L_5\cos\left(n\pi Vt\right)+L_6\sin\left(n\pi Vt\right)\right]$$

$$W_2(t)=P^2\left[-L_1\cosh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+L_3\cosh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t}\right.$$
$$-L_2\sinh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+\mathrm{L}_4\sinh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t} \tag{22}$$
$$\left.+L_7\cos\left(n\pi Vt\right)+L_8\sin\left(n\pi Vt\right)\right]$$

Where L_1 to L_8 are defined in appendix.

Substituting Eq. (21) and Eq. (22) into Eq. (9) the result for transverse displacement of double layered nanobeams with moving nanoparticle with respect to time is achieved as

$$w_1(X,t)=\sum_{i=1}^{\infty}P^2[L_1\cosh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+L_3\cosh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t}$$
$$+L_2\sinh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+\mathrm{L}_4\sinh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t} \tag{23}$$
$$+L_5\cos\left(n\pi Vt\right)+L_6\sin\left(n\pi Vt\right)]\sin(i\pi X)$$

$$w_2(X,t)=\sum_{i=1}^{\infty}P^2[-L_1\cosh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+L_3\cosh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t}$$
$$-L_2\sinh\left(\frac{\sqrt{\chi^2\tau_d^2-4\varsigma^2\chi\tau_d-8\kappa\gamma^2-4\chi\gamma^2+4\varsigma^2}}{2\gamma^2}t\right)e^{\frac{-\chi\tau_d+2\varsigma}{2\gamma^2}t}+\mathrm{L}_4\sinh\left(\frac{\sqrt{(\chi\tau_d^2-4\gamma^2)\chi}}{2\gamma^2}t\right)e^{-\frac{\chi\tau_d}{2\gamma^2}t} \tag{24}$$
$$+L_7\cos\left(n\pi Vt\right)+L_8\sin\left(n\pi Vt\right)]\sin(i\pi X)$$

3.1.2 Higher Layered Viscoelastic Nanobeam with Moving Nanoparticle

For higher layered viscoelastic nanobeams with moving nanoparticle, the same calculation procedure is done which causes long complex equations due to the inverse of matrix coefficient. For having a more accurate results and forbidding the errors, numerical solution is used to obtain the deflection of each layer in higher number of layers by solving Eq. (25).

$$\begin{Bmatrix} {}^n w_1 \\ {}^n w_2 \\ {}^n w_3 \\ . \\ . \\ . \\ {}^n w_{n-1} \\ {}^n w_n \end{Bmatrix}=\sum_{i=1}^{\infty}\left(L^{-1}\left\{inv\begin{bmatrix} \eta_1 & \eta_2 & 0 & 0 & 0 & \dots & 0 \\ \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & 0 & \dots & 0 \\ 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & \dots & 0 \\ \vdots & \vdots & & & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 \\ 0 & 0 & \dots & 0 & 0 & \eta_2 & \eta_1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2P^2\dfrac{i\pi V_p}{\left(i\pi V_p\right)^2+S^2} \end{Bmatrix}\right\}\right)\sin(i\pi X) \tag{25}$$

3.2 Solution Procedure for Multi-Layered Viscoelastic Nanobeam System Embedded in a Viscoelastic Medium

In the same way, for multi-layered viscoelastic nanobeams embedded in a viscoelastic medium, Substituting Eq. (9) to Eq. (12) into Eq. (8) the matrix form of equations of motions could be represented as

$$\gamma^2 \frac{\partial^2}{\partial t^2}\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix} + \begin{bmatrix} \chi\tau_d-\varsigma-\varsigma_0 & \varsigma & & & & & \\ \varsigma & \chi\tau_d-2\varsigma & \varsigma & & & Zero & \\ & \ddots & & & & & \\ & & \varsigma & \chi\tau_d-2\varsigma & \varsigma & & \\ & & & \varsigma & \chi\tau_d-2\varsigma & \varsigma & \\ & & & & \varsigma & \chi\tau_d-2\varsigma & \varsigma \\ & Zero & & & & \ddots & \\ & & & & \varsigma & \chi\tau_d-2\varsigma & \varsigma \\ & & & & & \varsigma & \chi\tau_d-\varsigma \end{bmatrix} \frac{\partial}{\partial t}\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix}$$

$$+\begin{bmatrix} \chi+\kappa+\kappa_0 & -\kappa & & & & & \\ -\kappa & \chi+2\kappa & -\kappa & & & Zero & \\ & \ddots & & & & & \\ & & -\kappa & \chi+2\kappa & -\kappa & & \\ & & & -\kappa & \chi+2\kappa & -\kappa & \\ & & & & -\kappa & \chi+2\kappa & -\kappa \\ & Zero & & & & \ddots & \\ & & & & -\kappa & \chi+2\kappa & -\kappa \\ & & & & & -\kappa & \chi+\kappa \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{i-1} \\ w_i \\ w_{i+1} \\ \vdots \\ w_{n-1} \\ w_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \Gamma \end{Bmatrix} \tag{26}$$

And by having the Laplace transform we have

$$\begin{bmatrix} \eta_1-\eta_0 & \eta_2 & 0 & 0 & 0 & \dots & 0 \\ \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & 0 & \dots & 0 \\ 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & \dots & 0 \\ \vdots & \vdots & & & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 \\ 0 & 0 & \dots & 0 & 0 & \eta_2 & \eta_1 \end{bmatrix}\begin{Bmatrix} L\{W_1\} \\ L\{W_2\} \\ L\{W_3\} \\ . \\ . \\ . \\ L\{W_{n-1}\} \\ L\{W_n\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ -2P^2 \dfrac{i\pi V_p}{(i\pi V_p)^2+S^2} \end{Bmatrix} \tag{27}$$

Where η_0 is defined as

$$\eta_0 = \varsigma_0 S - \kappa_0 \tag{28}$$

3.2.1 Double Layered Viscoelastic Nanobeam Embedded in a Viscoelastic Medium with Moving Nanoparticle

For two layered nanobeam system resting on a viscoelastic medium, Eq. (27) and Eq. (28) could be rewritten as

$$\begin{bmatrix} \gamma^2 S^2+(\chi\tau_d-\varsigma-\varsigma_0)S+\chi+\kappa+\kappa_0 & \varsigma S-\kappa \\ \varsigma S-\kappa & \gamma^2 S^2+(\chi\tau_d-\varsigma)S+(\chi+\kappa) \end{bmatrix}\begin{Bmatrix} L\{W_1\} \\ L\{W_2\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -2P^2 \dfrac{i\pi V_p}{(i\pi V_p)^2+S^2} \end{Bmatrix} \tag{29}$$

By inversing the coefficient matrix and assuming that the nondimensional stiffness and damping parameter for each layer and the viscoelastic medium is the same by having $\varsigma_0 = \varsigma$ and $\kappa_0 = \kappa$, evaluation results for transverse displacements are

$$W_1(t) = \frac{2P^2}{\Delta_1}\left[L_9 \sin(n\pi Vt) + L_{10}\cos(n\pi Vt) - \sum_{\xi_i} L_{11}/L_{12}\right] \tag{30}$$

$$W_2(t) = \frac{2P^2}{\Delta_1}\left[L_{13} \sin(n\pi Vt) + L_{14}\cos(n\pi Vt) - \sum_{\xi_i} L_{15}/L_{12}\right] \tag{31}$$

Where L_9 to L_{15} are defined in appendix and Δ_1 and ξ_i are defined as

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$$\begin{aligned}\Delta_1 = & V^8\gamma^8n^8\pi^8 - 2V^6\chi\gamma^4n^6\pi^6\tau_d + 6\kappa V^6\gamma^6n^6\pi^6 + 4V^6\chi\gamma^6n^6\pi^6 - 7\varsigma^2V^6\gamma^4n^6\pi^6 - V^4\chi^4n^4\pi^4\tau_d^{\ 4} + 6\varsigma V^4\chi^3n^4\pi^4\tau_d^{\ 3} \\ & + 6\kappa V^4\chi^2\gamma^2n^4\pi^4\tau_d^{\ 2} + 4V^4\chi^3\gamma^2n^4\pi^4\tau_d^{\ 2} - 11\varsigma^2V^4\chi^2n^4\pi^4\tau_d^{\ 2} - 8\varsigma\kappa V^4\chi\gamma^2n^4\pi^4\tau_d - 12\varsigma V^4\chi^2\gamma^2n^4\pi^4\tau_d \\ & - 11\kappa^2V^4\gamma^4n^4\pi^4 - 18\kappa V^4\chi\gamma^4n^4\pi^4 - 6V^4\chi^2\gamma^4n^4\pi^4 + 6\varsigma^3V^4\chi n^4\pi^4\tau_d + 6\varsigma^2\kappa V^4\gamma^2n^4\pi^4 + 14\varsigma^2V^4\chi\gamma^2n^4\pi^4 \\ & - \varsigma^4V^4n^4\pi^4 - 7\kappa^2V^2\chi^2n^2\pi^2\tau_d^{\ 2} - 6\kappa V^2\chi^3n^2\pi^2\tau_d^{\ 2} - 2V^2\chi^4n^2\pi^2\tau_d^{\ 2} + 6\varsigma\kappa^2V^2\chi n^2\pi^2\tau_d + 8\varsigma\kappa V^2\chi^2n^2\pi^2\tau_d \\ & + 6\varsigma V^2\chi^3n^2\pi^2\tau_d + 6\kappa^3V^2\gamma^2n^2\pi^2 + 22\kappa^2V^2\chi\gamma^2n^2\pi^2 + 18\kappa V^2\chi^2\gamma^2n^2\pi^2 + 4V^2\chi^3\gamma^2n^2\pi^2 - 2\varsigma^2\kappa^2V^2n^2\pi^2 \\ & - 6\varsigma^2\kappa V^2\chi n^2\pi^2 - 7\varsigma^2V^2\chi^2n^2\pi^2 - \kappa^4 - 6\kappa^3\chi - 11\kappa^2\chi^2 - 6\kappa\chi^3 - \chi^4 \end{aligned} \tag{32}$$

$$\begin{aligned}\xi_i = & \mathit{Roots\ of\ equation}\,(\gamma^4\xi^4 + (2\chi\gamma^2\tau_d - 3\varsigma\gamma^2)\xi^3 + (\chi^2\tau_d^2 - 3\varsigma\chi\tau_d + 3\kappa\gamma^2 + 2\chi\gamma^2\varsigma^2)\xi^2 \\ & + (3\kappa\chi\tau_d + 2\chi^2\tau_d - 2\varsigma\kappa - 3\varsigma\chi)\xi + \kappa^2 + 3\kappa\chi + \chi^2) \end{aligned} \tag{33}$$

Substituting Eq. (30) and Eq. (31) into Eq. (9) the result for double layered nanobeams resting on viscoelastic medium with moving nanoparticle with respect to time is achieved as

$$w_1(X,t) = \sum_{i=1}^{\infty} \frac{2P^2}{\Delta_1}\left[L_9\sin(n\pi Vt) + L_{10}\cos(n\pi Vt) - \sum_{\xi_i} L_{11}/L_{12}\right]\sin(i\pi X) \tag{34}$$

$$w_2(X,t) = \sum_{i=1}^{\infty} \frac{2P^2}{\Delta_1}\left[L_{13}\sin(n\pi Vt) + L_{14}\cos(n\pi Vt) - \sum_{\xi_i} L_{15}/L_{12}\right]\sin(i\pi X) \tag{35}$$

3.2.2 Higher layered viscoelastic nanobeam embedded in a viscoelastic medium with moving nanoparticle

In the same way for higher layered viscoelastic nanobeams resting on viscoelastic medium, results are achieved by numerically solving Eq. (36)

$$\begin{Bmatrix} {}^n w_1 \\ {}^n w_2 \\ {}^n w_3 \\ \cdot \\ \cdot \\ \cdot \\ {}^n w_{n-1} \\ {}^n w_n \end{Bmatrix} = \sum_{i=1}^{\infty}\left(L^{-1}\left\{ inv\begin{bmatrix} \eta_1-\eta_0 & \eta_2 & 0 & 0 & 0 & \ldots & 0 \\ \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & 0 & \ldots & 0 \\ 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 & 0 & \ldots & 0 \\ \vdots & \vdots & & & & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & \eta_2 & \eta_1-\eta_2 & \eta_2 \\ 0 & 0 & \ldots & 0 & 0 & \eta_2 & \eta_1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2P^2\dfrac{i\pi V_p}{(i\pi V_p)^2 + S^2} \end{Bmatrix}\right\}\right)\sin(i\pi X) \tag{36}$$

4. RESULTS AND DISCUSSIONS

The presented analysis, describes the dynamical behavior of simply supported Euler-Bernoulli multi-layered viscoelastic nanobeam resting on an viscoelastic medium with a moving nanoparticle on top. For different number of layers, stiffness, damping and nonlocal parameter, transverse deflection of multi-layered nanobeams is studied. The geometrical and mechanical properties of the multi-layered nanoribbon is considered as [28]: E = 1.0 TPa, ρ = 2.25 g/cm^3, t = 0.34 nm, L = 10 t. In order to achieve a nondimensional dynamical deflection parameter, static deflection [26] is assumed as $w_{st} = m_p g L^3/48EI$.

To verify the validation of present solution procedure the number of layers is assumed to be two (double layered) and analysis is done for carbon nanotubes to compare the present solution with the forced vibration of an elastically connected double-carbon nanotube system under a moving nanoparticle presented by Şimşek [21]. In Table 1 and Table 2 the maximum non-dimensional deflection of the first and second layer of double carbon nanotube for various values of stiffness is presented and compared to those achieved by Şimşek [21]. In Table 1 the nonlocal parameter is assumed to be 0.1 while in Table 2 the nondimensional maximum deflection are presented for α = 0.5 which shows a great equality in the results. In order to verify the numerical solving procedure, numerical results are calculated for double layered nanobeam systems which were in a good agreement with those achieved by analytically solving the problem using Eq. (23), Eq. (24), Eq. (30) and Eq. (31).

In Fig. 3 and Fig. 4 the dynamic deflection for each layer of double layer viscoelastic nanobeam with moving nanoparticle is presented for different number of nonlocal parameters. Results are shown for the full length of the nanobeam with respect to the nondimensional time parameter which in time T = 0 nanoparticle enters the system and at T = 1 leaves it. It could be seen that with increasing the nonlocal parameter from 0.1 to 0.3, the maximum nondimensional deflection in all the time domain increases for both first and second layer of the system. Same analysis has been done for double layered

Table 1 Nondimensional maximum deflection of double layered carbon nanotubes with $\alpha = 0.1$

ς	κ	First Layer			Second Layer		
		Analytical Solution	Numerical Solution	Şimşek [21]	Analytical Solution	Numerical Solution	Şimşek [21]
0	1	0.023090	0.023053	0.02302	1.203969	1.203736	1.20434
0	10	0.176841	0.176838	0.17726	1.080981	1.080946	1.08202
0	100	0.429840	0.429816	0.42985	0.793449	0.793294	0.79397
0	1000	0.576821	0.576757	0.57646	0.640477	0.640354	0.64050

Table 2 Nondimensional maximum deflection of double layered carbon nanotubes with $\alpha = 0.5$

ς	κ	First Layer			Second Layer		
		Analytical Solution	Numerical Solution	Şimşek [21]	Analytical Solution	Numerical Solution	Şimşek [21]
0	1	0.033901	0.033896	0.03393	1.845947	1.845439	1.84672
0	10	0.278719	0.278652	0.27898	1.679870	1.679471	1.68053
0	100	0.833204	0.833168	0.83337	1.062783	1.062477	1.06288
0	1000	0.918833	0.918716	0.91893	0.952466	0.952278	0.95263

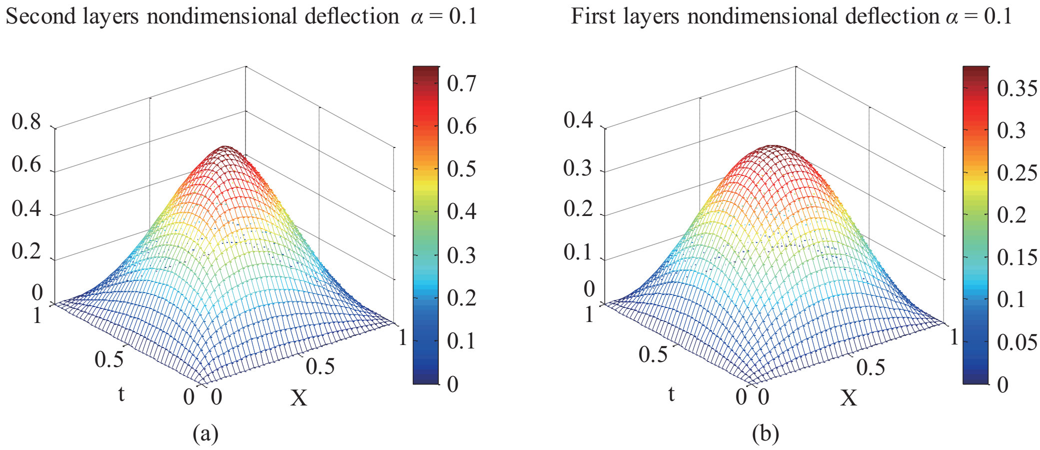

Fig. 3 Dynamic respond of double layered viscoelastic nanobeam with nondimensional nonlocal parameter = 0.1 (a) second layers nondimensional transverse displacement (b) first layers nondimensional transverse displacement.

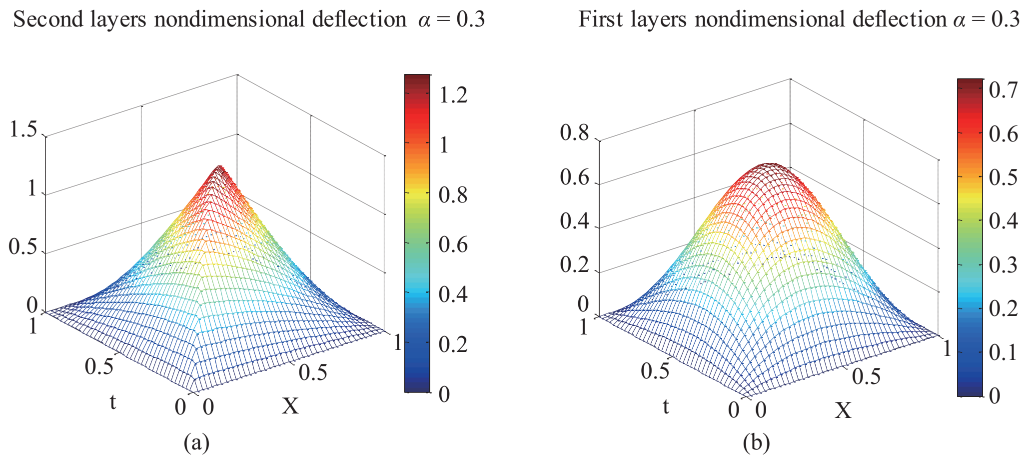

Fig. 4 Dynamic respond of double layered viscoelastic nanobeam with nondimensional nonlocal parameter = 0.3 (a) second layers nondimensional transverse displacement (b) first layers nondimensional transverse displacement.

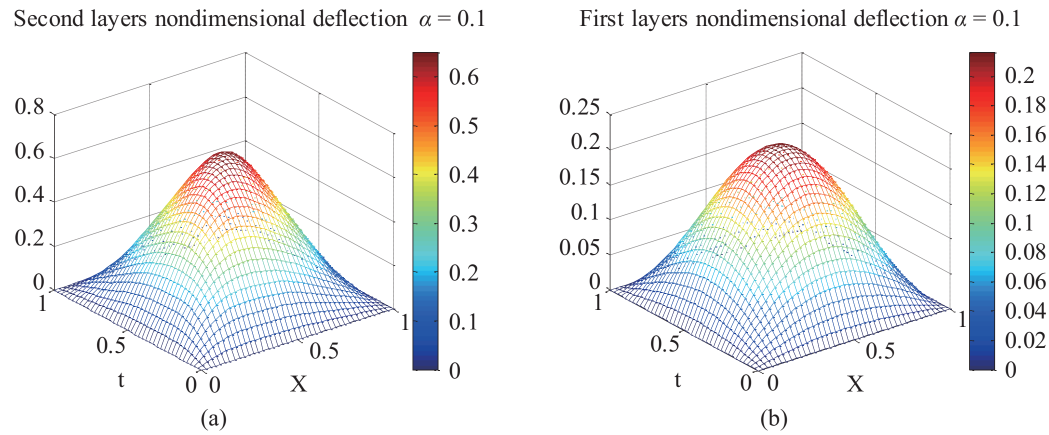

Fig. 5 Dynamic respond of double layered viscoelastic nanobeam embedded in a viscoelastic medium with nondimensional nonlocal parameter = 0.1 (a) second layers nondimensional transverse displacement (b) first layers nondimensional transverse displacement.

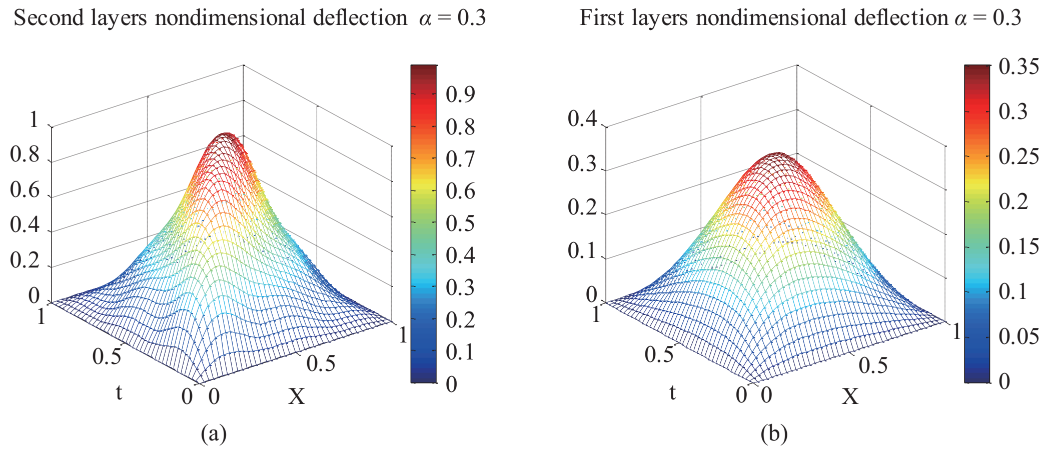

Fig. 6 Dynamic respond of double layered viscoelastic nanobeam embedded in a viscoelastic medium with nondimensional nonlocal parameter = 0.3 (a) second layers nondimensional transverse displacement (b) first layers nondimensional transverse displacement.

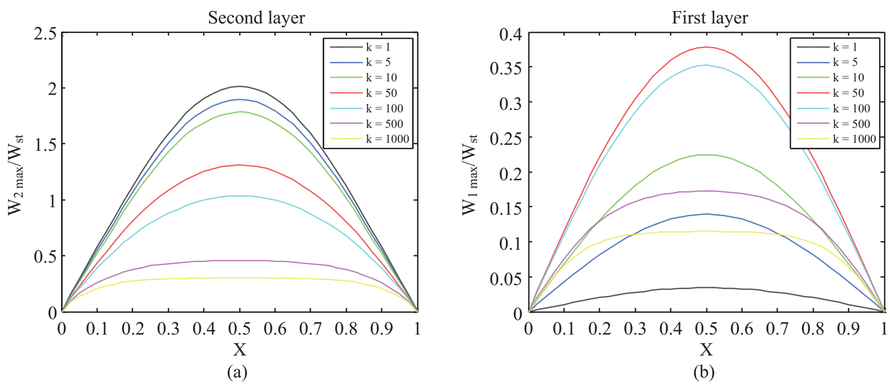

Fig. 7 Nondimensional maximum deflection of double layered viscoelastic nanobeam embedded in a viscoelastic medium with respect to nondimensional length parameter for different stiffness parameters while $\varsigma = -1$ and $\alpha = 0.3$ (a) second layer (b) first Layer.

nanobeam with viscoelastic medium and the results for each layer for different nonlocal parameters are presented at Fig. 5 and Fig. 6. In the same way, for the time that viscoelastic medium is also taken into account, increasing the nondimensional parameter leads to a higher nondimensional deflection in all the process of passing nanoparticle through the system. It is also shown that adding a viscoelastic medium to the nanobeam system, decreases the nondimensional deflection on both layers but it has more effect on decreasing the deflection in the lower layer of nanobeam system.

The effects of stiffness and damping parameters of Kelvin-Voigt viscoelastic coupling between layers with each other and the medium on the maximum deflection is presented in Fig. 7 and Fig. 8 for double layered and in Fig. 9 and Fig. 10 for three layered nanobeam system resting on a medium for all length of the beam. In Fig. 7 and Fig. 9, the stiffness parameter $\kappa = \kappa_0$ is assumed to vary from 1 to 1000 while the damping parameter is $\varsigma = \varsigma_0 = -1$ and the nonlocal parameter is $\alpha = 0.3$. While the nanoparticle passes through it is shown that for lower stiffness parameters, coupling between the layers are weak and the forces caused by the moving nanoparticle is carried by the top layer by having the most deflection

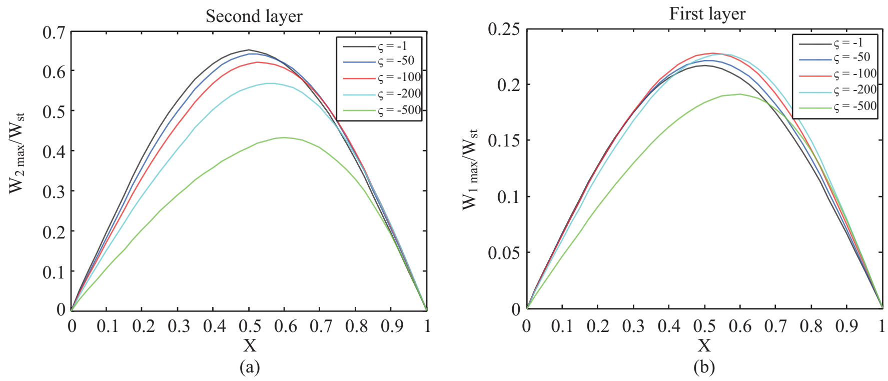

Fig. 8 Nondimensional maximum deflection of double layered viscoelastic nanobeam embedded in a viscoelastic medium with respect to nondimensional length parameter for different damping parameters while $\kappa = 100$ and $\alpha = 0.1$ (a) second layer (b) first Layer.

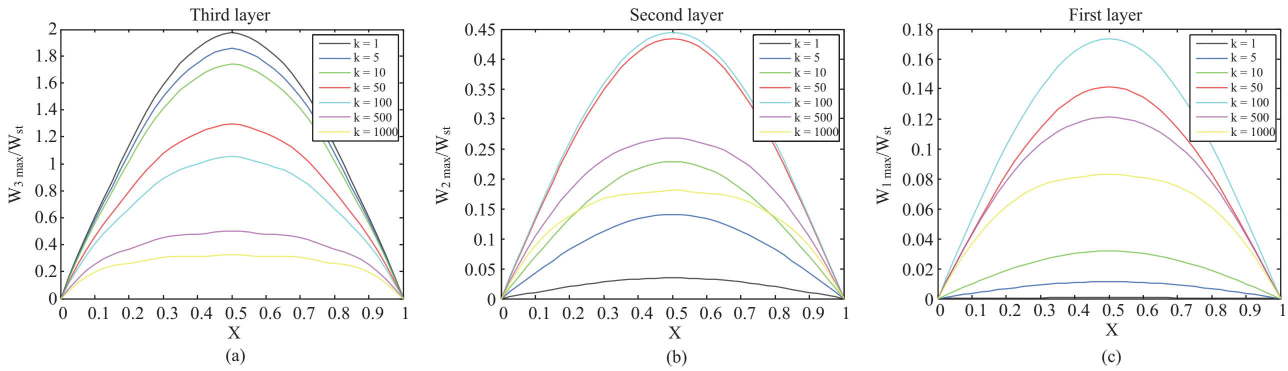

Fig. 9 Nondimensional maximum deflection of Three layered viscoelastic nanobeam embedded in a viscoelastic medium with respect to nondimensional length parameter for different stiffness parameters while $\varsigma = -1$ and$\alpha = 0.3$ (a) third layer (b) second layer (c) first Layer.

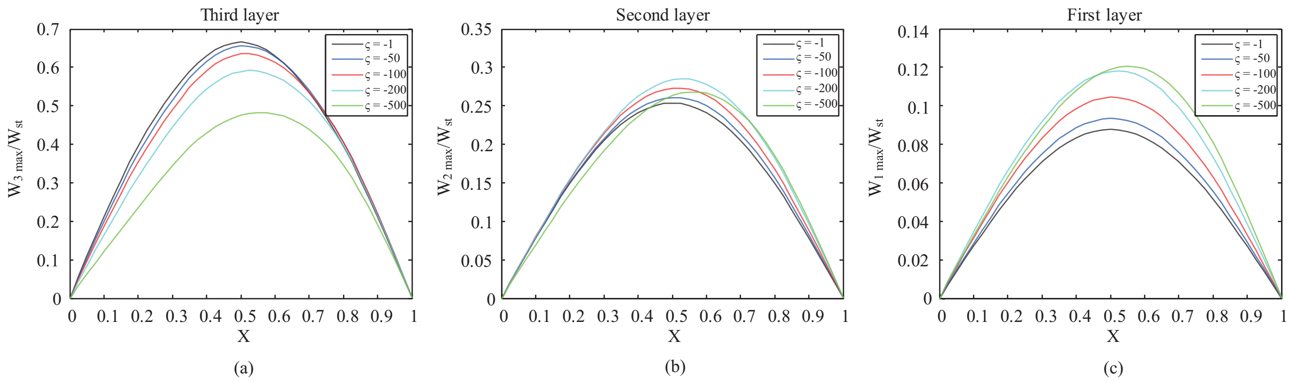

Fig. 10 Nondimensional maximum deflection of Three layered viscoelastic nanobeam embedded in a viscoelastic medium with respect to nondimensional length parameter for different damping parameters while $\kappa = 100$ and $\alpha = 0.1$ (a) third layer (b) second layer (c) first Layer.

while the other layers are almost not participated in system. Both Fig. 7 and Fig. 9 declare that the sensitivity to the changes in stiffness parameter is more in the lower amount of stiffness parameter and the results for each layer converge to a specific number by increasing the stiffness parameter.

In Fig. 7, it's shown that increasing the stiffness parameter from 1 to 50 leads to a higher deflection in the first layer of nanobeam system and a lower deflection in second layer but after that by reaching the stiffness parameter to κ = 100, 500 and 1000 in both layers the maximum nondimensional deflection decreases in all the time domain. In the same way in Fig. 9, the third layers deflection decreases continuously by increasing the stiffness parameter. The first and second layers maximum deflections increases by increasing the coupling stiffness parameter for κ = 1, 5, 10, 50 and 100 but for κ = 500 and 1000, the maximum deflection decreased. This shows that the amount of the stiffness parameter from which the behavior of lower layers of nanobeam systems starts to change depends on the number of layers and it raises by increasing the number of layers. In the same way, damping effects are studied by assuming that the stiffness parameter is $\kappa = \kappa_0 = 100$ and the nonlocal parameter is $\alpha = 0.1$ while the damping parameter varies from 1 to 500. Increasing the damping parameter makes two different changes, it may help the coupling between layers so the deflection will be distributed more between layers and meanwhile it could absorb more energy and decrease the total energy and deflections. As shown if Fig. 8, increasing the damping parameter from 1 to 100 caused more coupling between layers by having higher deflection in the first and lower deflection in the second layer. Afterward, by increasing the damping parameter for higher numbers than 100, the second behavior is obtained in which the absorption of the energy overcame the coupling behavior which leaded to lower deflections in both layers of nanobeam system. Also for three layered nanobeams resting on viscoelastic medium, dynamic behavior with respect to damping changes is presened in Fig. 10. Likewise, the top layers displacement decreases with increasing the damping parameter due to the coupling between layers and absorbing energy and the lower layer participate more in system by increasing the damping parameter. It also can be seen that increasing the damping parameter in all multi-layered nanobeam systems changes the position of the maximum deflection of each layer and moves it to the right side (by assuming that the nanoparticle joined the system from the left side).

The effects of nonlocal parameter is also shown in Fig. 11 for double layered, three layered and four layered viscoelastic nanobeam while the nondimensional damping and stiffness parameters are $\varsigma = \varsigma_0 = -1$ and $\kappa = \kappa_0 = 100$ and the nonlocal parameter is assumed to be $\alpha = 0.1$, 0.3 and 0.5. Results show that by increasing the nonlocal parameter, the maximum nondimensional deflection increases for all layers in all kind of multi-layered nanobeam systems. It's also shown that increasing the layers of multi-layered nanobeam systems, decreases the sensibility of the maximum deflection to the changes of nonlocal parameter. Also for double and three layered nanobeam system resting on a viscoelastic medium, by changing the stiffness parameter, maximum deflection parameter is presented in Fig. 12 and Fig. 14 for different number of nonlocal parameter. It is shown that increasing the nonlocal parameter leads to a bigger deflection in all layers of nonlocal viscoelastic nanobeams. In both Fig. 12 and Fig. 14 it's obtained that increasing the stiffness parameter decreases the sensibility of deflection to nonlocal parameter. Also, it can be seen that in all multi-layered nanobeam systems, for a specific number of stiffness and nonlocal parameter, the behavior of lower layers of nanobeam system changes to variation of stiffness parameter. In order to study the effect of varying damping parameter on the maximum deflections for different number of small scale parameter, results have been achieved in Fig. 13 and Fig. 15 for each layer of double and three layered systems. It can be seen that in multilayered viscoelastic nanobeams resting on viscoelastic medium, increasing the nonlocal parameter leads to higher maximum deflection independent from the damping parameter. Also, for higher damping parameters, the sensitivity of maximum deflection to changes in nonlocal parameter decreases and the effects of coupling layers and absorbing the energy is shown for lower layers of double layered and three layered systems.

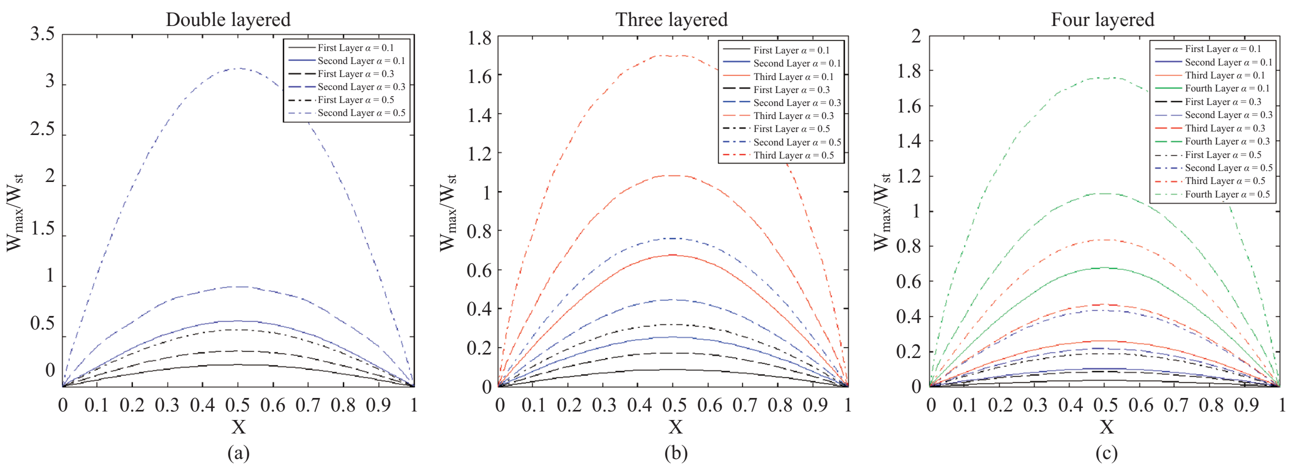

Fig. 11 Nondimensional maximum deflection of multi-layered nanobeams embedded in a viscoelastic medium with different nondimensional nonlocal parameters with ς = -1 and κ = 100 (a) double layered (b) three layered (c) four layered.

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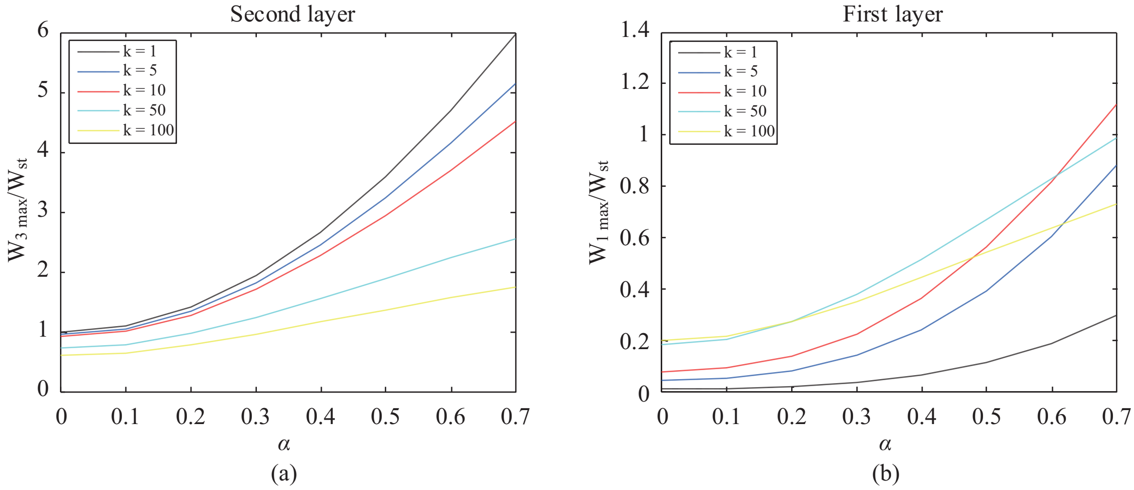

Fig. 12 Nondimensional maximum deflection of double layered nanobeam embedded in a viscoelastic medium for different stiffness parameters with respect to nonlocal parameter and $\varsigma = -1$ (a) second layer (b) first layer.

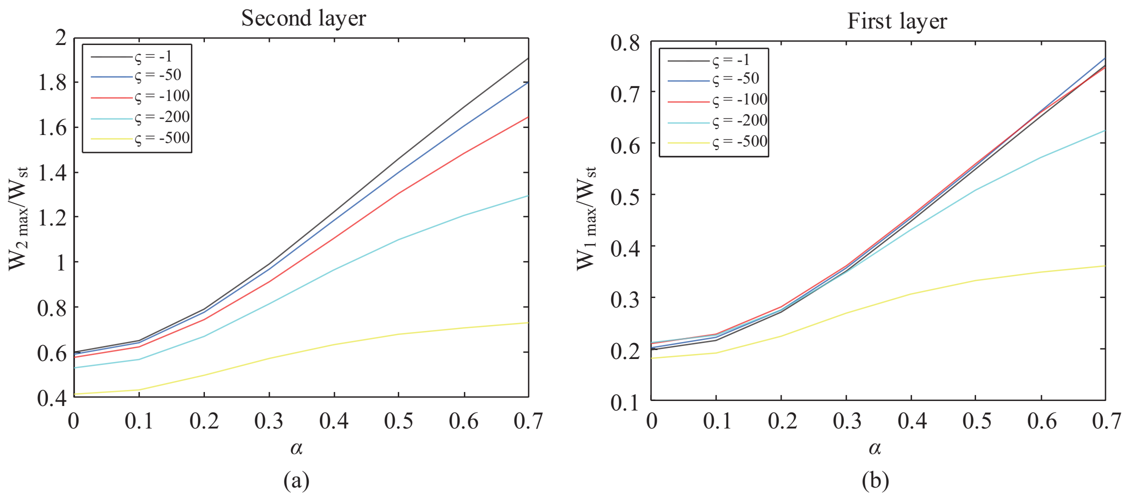

Fig. 13 Nondimensional maximum deflection of double layered nanobeam embedded in a viscoelastic medium for different damping parameters with respect to nonlocal parameter and $\kappa = 100$ (a) second layer (b) first layer.

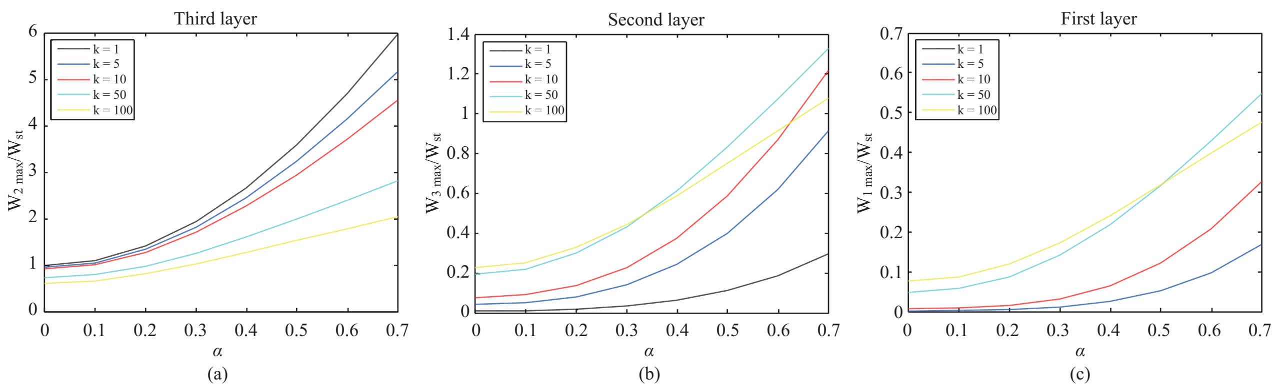

Fig. 14 Nondimensional maximum deflection of three layered nanobeam embedded in a viscoelastic medium for different stiffness parameters with respect to nonlocal parameter and $\varsigma = -1$ (a) third layer (b) second layer (c) first layer.

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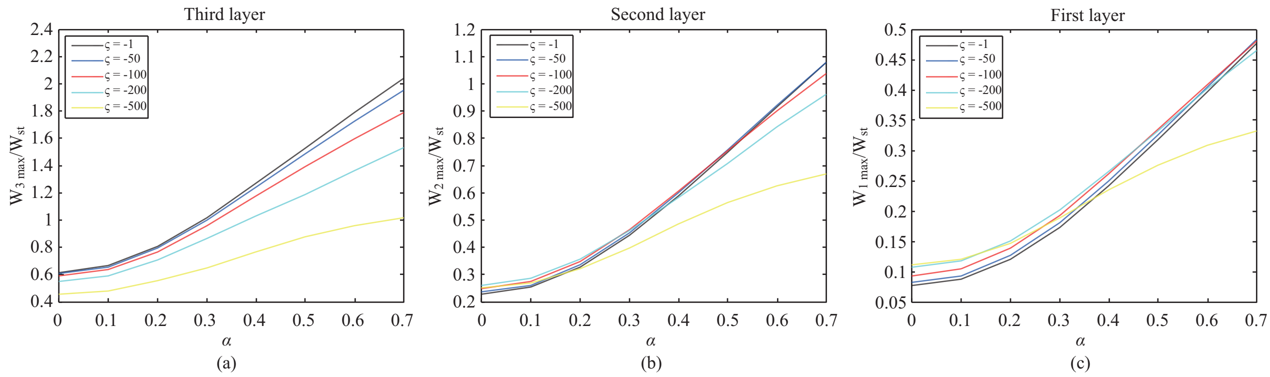

Fig. 15 Nondimensional maximum deflection of three layered nanobeam embedded in a viscoelastic medium for different damping parameters with respect to nonlocal parameter and κ = 100 (a) third layer (b) second layer (c) first layer.

5. CONCLUSIONS

In the present paper, dynamic behavior of multi-layered viscoelastic nanobeam embedded on a viscoelastic medium is studied while a nanoparticle passes through with a constant velocity. Small scale effects are modeled after Eringen's nonlocal theory. The moving nanoparticle is modeled by a concentrated mass at the top surface of upper layer of nanobeam. Coupling between the layers are modeled after Kelvin-Voigt viscoelactic medium model. Main equations are achieved and solved by using Hamilton's principle, eigen-function technique and the Laplace transform method. Analytical solution is presented for double layered viscoelastic nanobeam carrying a moving nanoparticle for both situation resting on a viscoelastic medium and without it. The effects of various material parameters, coupling parameters and viscoelastic medium usage on nondimensional deflection of each layer is discussed for different number of nanobeams. It is shown that the small scale effect has a significant influence on the enhancement of the nondimensional deflection. Parametric study is done for different number of stiffness, damping and nonlocal parameters and the results, effects and influences are fully discussed. Also, it is noted that increasing the stiffness parameters of the Kelvin-Voigt viscoelactic medium and viscoelastic coupling between layers causes a reduction in the influence of the small scale effect on the nondimensional transverse deflection. The verification of the current results is justified by comparing the results to those published before in literatures.

APPENDIX

Parameters L_1 to L_{15} are defined as

$$L_1 = (-V^4\gamma^4n^4\pi^4 - V^2\chi^2n^2\pi^2\tau_d^2 + 2V^2\chi\gamma^2n^2\pi^2 - \chi^2)(-\chi\tau_d + 2\varsigma)n\pi V/[(V^4\gamma^4n^4\pi^4 + V^2\chi^2n^2\pi^2\tau_d^2 - 4\varsigma V^2\chi n^2\pi^2\tau_d \\ -4\kappa V^2\gamma^2n^2\pi^2 - 2V^2\chi\gamma^2n^2\pi^2 + 4\varsigma^2V^2n^2\pi^2 + 4\kappa^2 + 4\kappa\chi + \chi^2)(V^4\gamma^4n^4\pi^4 + V^2\chi^2n^2\pi^2\tau_d^2 - 2V^2\chi\gamma^2n^2\pi^2 + \chi^2)]$$

$$L_2 = [n\pi V(2V^2n^2\pi^2\gamma^4 + \chi^2\tau_d^2 - 4\varsigma\chi\tau_d + 4\kappa\gamma^2 - 2\chi\gamma^2 + 4\varsigma^2)]/\Big[\sqrt{\chi^2\tau_d^2 - 4\varsigma\chi\tau_d - 8\kappa\gamma^2 - 4\chi\gamma^2 + 4\varsigma^2}\,(V^4\gamma^4n^4\pi^4 + \\ V^2\chi^2n^2\pi^2\tau_d^2 - 4\varsigma V^2\chi n^2\pi^2\tau_d - 4\kappa V^2\gamma^2n^2\pi^2 - 2V^2\chi\gamma^2n^2\pi^2 + 4\varsigma^2V^2n^2\pi^2 + 4\kappa^2 + 4\kappa\chi + \chi^2)\Big]$$

$$L_3 = (-V^4\gamma^4n^4\pi^4 - V^2\chi^2n^2\pi^2\tau_d^2 + 4\varsigma V^2\chi n^2\pi^2\tau_d + 4\kappa V^2\gamma^2n^2\pi^2 + 2V^2\chi\gamma^2n^2\pi^2 - 4\varsigma^2V^2n^2\pi^2 - 4\kappa^2 - 4\kappa\chi - \chi^2)\chi\tau_d$$

$$L_4 = [(2V^2\gamma^4n^2\pi^2 + \chi^2\tau_d^2 - 2\chi\gamma^2)n\pi V]/[(-V^4\gamma^4n^4\pi^4 - V^2\chi^2n^2\pi^2\tau_d^2 + 2V^2\chi n^2\pi^2\gamma^2 - \chi^2)\sqrt{(\chi\tau_d^2 - 4\gamma^2)\chi}]$$

$$L_5 = 2(\varsigma V^4\gamma^4n^4\pi^4 - \varsigma V^2\chi^2n^2\pi^2\tau_d^2 - 2\kappa V^2\chi\gamma^2n^2\pi^2\tau_d + 2\varsigma^2V^2\chi n^2\pi^2\tau_d - 2\varsigma V^2\chi\gamma^2n^2\pi^2 + 2\kappa^2\chi\tau_d + 2\kappa\chi^2\tau_d + \varsigma\chi^2)$$

$$L_6 = 2(-2\varsigma V^4\chi\gamma^2n^4\pi^4\tau_d - \kappa V^4\gamma^4n^4\pi^4 + 2\varsigma^2V^4\gamma^2n^4\pi^4 + \kappa V^2\chi^2n^2\pi^2\tau_d^2 + 2\varsigma V^2\chi^2n^2\pi^2\tau_d + 2\kappa^2V^2\gamma^2n^2\pi^2 \\ +2\kappa V^2\chi\gamma^2n^2\pi^2 - 2\varsigma^2V^2\chi n^2\pi^2 - 2\kappa^2\chi - \kappa\chi^2)$$

$$L_7 = 2(V^4\chi\gamma^4n^4\pi^4\tau_d - \varsigma V^4\gamma^4n^4\pi^4 + V^2\chi^3n^2\pi^2\tau_d^3 - 3\varsigma V^2\chi^2n^2\pi^2\tau_d^2 - 2\kappa V^2\chi\gamma^2n^2\pi^2\tau_d - 2V^2\chi^2\gamma^2n^2\pi^2\tau_d \\ +2\varsigma^2V^2\chi n^2\pi^2\tau_d + 2\varsigma V^2\chi\gamma^2n^2\pi^2 + 2\kappa^2\chi\tau_d + 2\kappa\chi^2\tau_d + \chi^3\tau_d - \varsigma\chi^2)$$

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$$L_8 = 2(V^6\gamma^6 n^6\pi^6 + V^4\chi^2\gamma^2 n^4\pi^4\tau_d^2 - 2\varsigma V^4\chi\gamma^2 n^4\pi^4\tau_d - 3\kappa V^4\gamma^4 n^4\pi^4 - 3V^4\chi\gamma^4 n^4\pi^4 + 2\varsigma^2 V^4\gamma^2 n^4\pi^4 - \kappa V^2\chi^2 n^2\pi^2\tau_d^2 - V^2\chi^3 n^2\pi^2\tau_d^2 + 2\varsigma V^2\chi^2 n^2\pi^2\tau_d + 2\kappa^2 V^2\gamma^2 n^2\pi^2 + 6\kappa V^2\chi\gamma^2 n^2\pi^2 + 3V^2\chi^2\gamma^2 n^2\pi^2 - 2\varsigma^2 V^2\chi n^2\pi^2 - 2\kappa^2\chi - 3\kappa\chi^2 - \chi^2)$$

$$L_9 = 2\varsigma V^4\chi\gamma^2 n^4\pi^4\tau_d + \kappa V^4\gamma^4 n^4\pi^4 - 3\varsigma^2 V^4\gamma^2 n^4\pi^4 - \kappa V^2\chi^2 n^2\pi^2\tau_d^2 - 2\varsigma V^2\chi^2 n^2\pi^2\tau_d - 3\kappa^2 V^2\gamma^2 n^2\pi^2 - 2\kappa V^2\chi\gamma^2 n^2\pi^2 + \varsigma^2\kappa V^2 n^2\pi^2 + 3\varsigma^2 V^2\chi n^2\pi^2 + \kappa^3 + 3\kappa^2\chi + \kappa\chi^2$$

$$L_{10} = n\pi V(-\varsigma V^4\gamma^4 n^4\pi^4 + \varsigma V^2\chi^2 n^2\pi^2\tau_d^2 + 2\kappa V^2\chi\gamma^2 n^2\pi^2\tau_d - 3\varsigma^2 V^2\chi n^2\pi^2\tau_d + 2\varsigma V^2\chi\gamma^2 n^2\pi^2 + \varsigma^3 V^2 n^2\pi^2 - 3\kappa^2\chi\tau_d + \varsigma\kappa^2 - \varsigma\chi^2)$$

$$\begin{aligned}
L_{11} = n\pi V e^{\xi_i t}[&3\kappa^4\gamma^2 + V^2 n^2\pi^2(\varsigma\chi^2\xi_i^3\gamma^4\tau_d + 2\kappa\chi\xi_i^3\gamma^6\tau_d - 3\varsigma^2\chi\xi_i^3\gamma^4\tau_d + 2\varsigma\chi^3\xi_i^2\gamma^2\tau_d^3 + 2\varsigma\chi\xi_i^3\gamma^6 + 3\kappa\chi^2\xi_i^2\gamma^4\tau_d^2 \\
&+ \varsigma^3\xi_i^3\gamma^4 - 9\varsigma^2\chi^2\xi_i^2\gamma^2\tau_d^2 - 6\varsigma\kappa\chi\xi_i^2\gamma^4\tau_d + \varsigma\chi^4\xi_i\tau_d^4 + 2\varsigma\chi^2\xi_i^2\gamma^4\tau_d - 3\kappa^2\xi_i^2\gamma^6 - 2\kappa\chi\xi_i^2\gamma^6 + 11\varsigma^3\chi\xi_i^2\gamma^2\tau_d \\
&+ \varsigma^2\kappa\xi_i^2\gamma^4 - 6\varsigma^2\chi^3\xi_i\tau_d^3 - 3\varsigma^2\chi\xi_i^2\gamma^4 + 3\kappa^2\chi\xi_i^2\gamma^4\tau_d - \kappa\chi^4\tau_d^4 + 2\kappa\chi^2\xi_i\gamma^4\tau_d - 3\varsigma^4\xi_i^2\gamma^2 + 11\varsigma^3\chi^2\xi_i\tau_d^2 \\
&- 5\varsigma^2\kappa\chi\xi_i\gamma^2\tau_d + 8\varsigma\kappa^2\chi\xi_i\gamma^4 + 6\varsigma\kappa\chi^3\tau_d^3 + 12\varsigma\kappa\chi\xi_i\gamma^4 + 5\varsigma\chi^2\xi_i\gamma^4 + 6\kappa^2\chi^2\gamma^2\tau_d^2 + 4\kappa\chi^3\gamma^2\tau_d^2 - 6\varsigma^4\chi\xi_i\tau_d \\
&- 5\varsigma^3\chi\xi_i\gamma^2 - 11\varsigma^2\kappa\chi^2\tau_d^2 - 6\varsigma\kappa^2\chi\gamma^2\tau_d - 6\varsigma\kappa\chi^2\gamma^2\tau_d + 2\varsigma\chi^3\gamma^2\tau_d - 10\kappa^3\gamma^4 - 15\kappa^2\chi\gamma^4 - 5\kappa\chi^2\gamma^4 \\
&+ \varsigma^5\xi_i + 6\varsigma^3\kappa\chi\tau_d + 3\varsigma^2\kappa^2\gamma^2 + 5\varsigma^2\kappa\chi\gamma^2 - 3\varsigma^2\chi^2\gamma^2 - \varsigma^4\kappa) - \chi^4\tau_d(2\kappa\xi_i\tau_d^2 + \varsigma\xi_i\tau_d + 3\kappa\tau_d + 2\varsigma) \\
&+ \gamma^2(-6\kappa^2\chi^2\tau_d^2 - 4\kappa\chi^3\tau_d^2 + 11\varsigma\kappa^2\chi\tau_d + 6\varsigma\kappa\chi^2\tau_d - 2\varsigma\chi^3\tau_d + \kappa^3\gamma^2 + 3\kappa^2\chi\gamma^2 + \kappa\chi^2\gamma^2 - 3\varsigma^2\kappa^2 \\
&+ 3\varsigma^2\chi^2)\xi_i^2 + \xi_i^3\gamma^4(-3\kappa^2\chi\tau_d - 2\kappa\chi^2\tau_d + \varsigma\kappa^2 - \varsigma\chi^2) + (\varsigma\xi_i - \kappa)\gamma^4 n^4\pi^4 V^4(\gamma^4 n^2\pi^2 V^2 - \xi_i^2\gamma^4 \\
&+ 2\chi^2\tau_d^2 - 6\varsigma\chi\tau_d - 6\kappa\gamma^2 - 4\chi\gamma^2 + 7\varsigma^2) + (10\varsigma\kappa\xi_i\tau_d^2 - 6\kappa\xi_i\gamma^2\tau_d - 2\varsigma^2\xi_i\tau_d - 6\varsigma\xi_i\gamma^2 - 8\kappa^2\tau_d^2 \\
&+ 6\varsigma\kappa\tau_d + 9\kappa\gamma^2 + 3\varsigma^2\chi)\kappa\chi^2 + \chi^3(-3\kappa^2\xi_i\tau_d^3 + 6\varsigma\kappa\xi_i\tau_d^2 - 2\kappa\xi_i\gamma^2\tau_d + 3\varsigma^2\xi_i\tau_d - 2\varsigma\xi_i\gamma^2 - 9\kappa^2\tau_d^2 \\
&+ 2\kappa\gamma^2 + 3\varsigma^2) + (-7\chi\xi_i\gamma^2\tau_d + 6\varsigma\chi\tau_d + 11\chi\gamma^2 - \varsigma^2)\kappa^3 + \varsigma\xi_i(-6\varsigma\kappa^2\chi\tau_d - 7\kappa^2\chi\gamma^2 + \varsigma^2\kappa^2 - \varsigma^2\chi^2)]
\end{aligned}$$

$$L_{12} = 4\xi_i^3\gamma^4 + 6\chi\xi_i^2\gamma^2\tau_d - 9\varsigma\xi_i^2\gamma^2 + 2\chi^2\xi_i\tau_d^2 - 6\varsigma\chi\xi_i\tau_d + 6\kappa\xi_i\gamma^2 + 4\chi\xi_i\gamma^2 + 2\varsigma^2\xi_i + 3\kappa\chi\tau_d + 2\chi^2\tau_d - 2\varsigma\kappa - 3\varsigma\chi$$

$$L_{13} = -V^6\gamma^6 n^6\pi^6 - V^4\chi^2\gamma^2 n^4\pi^4\tau_d^2 + 4\varsigma V^4\chi\gamma^2 n^4\pi^4\tau_d + 5\kappa V^4\gamma^4 n^4\pi^4 + 3V^4\chi\gamma^4 n^4\pi^4 - 5\varsigma^2 V^4\gamma^2 n^4\pi^4 + \kappa V^2\chi^2 n^2\pi^2\tau_d^2 + V^2\chi^3 n^2\pi^2\tau_d^2 - 2\varsigma\kappa V^2\chi n^2\pi^2\tau_d - 4\varsigma V^2\chi^2 n^2\pi^2\tau_d - 7\kappa^2 V^2\gamma^2 n^2\pi^2 - 10\kappa V^2\chi\gamma^2 n^2\pi^2 - 3V^2\chi^2\gamma^2 n^2\pi^2 + 2\varsigma^2\kappa V^2 n^2\pi^2 + 5\varsigma^2 V^2\chi n^2\pi^2 + 2\kappa^3 + 7\kappa^2\chi + 5\kappa\chi^2 + \chi^3$$

$$L_{14} = n\pi V(V^4\chi\gamma^4 n^4\pi^4\tau_d + \varsigma V^4\gamma^4 n^4\pi^4 - V^2\chi^3 n^2\pi^2\tau_d^3 + 5\varsigma V^2\chi^2 n^2\pi^2\tau_d^2 + 4\kappa V^2\chi\gamma^2 n^2\pi^2\tau_d + 2V^2\chi^2\gamma^2 n^2\pi^2\tau_d - 7\varsigma^2 V^2\chi n^2\pi^2\tau_d - 2\varsigma\kappa V^2\gamma^2 n^2\pi^2 - 2\varsigma V^2\chi\gamma^2 n^2\pi^2 + 2\varsigma^3 V^2 n^2\pi^2 - 5\kappa^2\chi\tau_d - 4\kappa\chi^2\tau_d - \chi^3\tau_d + 2\varsigma\kappa^2 + 2\varsigma\kappa\chi + \varsigma\chi^2)$$

$$\begin{aligned}
L_{15} = n\pi V e^{\xi_i t}[&-\chi^5\tau_d^2 + 5\kappa^4\gamma^2 + V^2 n^2\pi^2(-\chi^3\xi_i^3\gamma^4\tau_d^3 + 5\varsigma\chi^2\xi_i^3\gamma^4\tau_d^2 + 4\kappa\chi\xi_i^3\gamma^6\tau_d - 2\chi^4\xi_i^2\gamma^2\tau_d^4 + 2\chi^2\xi_i^3\gamma^6\tau_d \\
&- 7\varsigma^2\chi\xi_i^3\gamma^4\tau_d - 2\varsigma\kappa\chi\xi_i^3\gamma^6 + 13\varsigma\chi^3\xi_i^2\gamma^2\tau_d^3 - 2\varsigma\chi\xi_i^3\gamma^6 + 9\kappa\chi^2\xi_i^2\gamma^4\tau_d^2 - \chi^5\xi_i\tau_d^5 + 5\chi^3\xi_i^2\gamma^4\tau_d^2 + 2\varsigma^3\xi_i^3\gamma^4 \\
&- 29\varsigma^2\chi^2\xi_i^2\gamma^2\tau_d^2 - 18\varsigma\kappa\chi\xi_i^2\gamma^4\tau_d + 8\varsigma\chi^4\xi_i\tau_d^4 - 14\varsigma\chi^2\xi_i^2\gamma^4\tau_d - 7\kappa^2\xi_i^2\gamma^6 + 3\kappa\chi^3\xi_i\gamma^2\tau_d^3 - 10\kappa\chi\xi_i^2\gamma^6 \\
&+ 2\chi^4\xi_i\gamma^2\tau_d^3 - 3\chi^2\xi_i^2\gamma^6 + 25\varsigma^3\chi\xi_i^2\gamma^2\tau_d + 8\varsigma^2\kappa\xi_i^2\gamma^4 - 23\varsigma^2\chi^3\xi_i\tau_d^3 + 11\varsigma^2\chi\xi_i^2\gamma^4 - 6\varsigma\kappa\chi^2\xi_i\gamma^2\tau_d^2 - 9\varsigma\chi^3\xi_i\gamma^2\tau_d^2 \\
&+ 3\kappa^2\chi\xi_i^2\gamma^4\tau_d - 2\kappa\chi^4\tau_d^4 - 2\kappa\chi^2\xi_i\gamma^4\tau_d - \chi^5\tau_d^4 - \chi^3\xi_i\gamma^4\tau_d - 6\varsigma^4\xi_i^2\gamma^2 + 28\varsigma^3\chi^2\xi_i\tau_d^2 - \varsigma^2\kappa\chi\xi_i\gamma^2\tau_d + 16\varsigma^2\chi^2\xi_i\gamma^2\tau_d \\
&+ 13\varsigma\kappa^2\xi_i\gamma^4 + 12\varsigma\kappa\chi^3{\tau_d}^3 + 18\varsigma\kappa\chi\xi_i\gamma^4 + 6\varsigma\chi^4\tau_d^3 + 4\varsigma\chi^2\xi_i\gamma^4 + 11\kappa^2\chi^2\gamma^2\tau_d^2 + 11\kappa\chi^3\gamma^2\tau_d^2 + 3\chi^4\gamma^2\tau_d^2 - 13\varsigma^4\chi\xi_i\tau_d \\
&- 2\varsigma^3\kappa\xi_i\gamma^2 - 13\varsigma^3\chi\xi_i\gamma^2 - 22\varsigma^2\kappa\chi^2{\tau_d}^2 - 11\varsigma^2\chi^3{\tau_d}^2 - 12\varsigma\kappa^2\chi\gamma^2\tau_d - 20\varsigma\kappa\chi^2\gamma^2\tau_d + 8\varsigma\chi^3\gamma^2\tau_d - 17\kappa^3\gamma^4 - 29\kappa^2\chi\gamma^4 \\
&- 16\kappa\chi^2\gamma^4 - 3\chi^3\gamma^4 + 2\varsigma^5\xi_i + 12\varsigma^3\kappa\chi\tau_d + 6\varsigma^3\chi^2\tau_d + 7\varsigma^2\kappa^2\gamma^2 + 19\varsigma^2\kappa\chi\gamma^2 + 9\varsigma^2\chi^2\gamma^2 - 2\varsigma^4\kappa - \varsigma^4\chi) + V^4 n^4\pi^4\gamma^4 \\
&(-\chi\xi_i^3\gamma^4\tau_d + \varsigma\xi_i^3\gamma^4 - 3\chi^2\xi_i^2\gamma^2\tau_d^2 + 9\varsigma\chi\xi_i^2\gamma^2\tau_d + 5\kappa\xi_i^2\gamma^4 - 2\chi^3\xi_i\tau_d^3 + 3\chi\xi_i^2\gamma^4 - 8\varsigma^5\xi_i^2\gamma^2 + 10\varsigma\chi^2\xi_i\tau_d^2 \\
&+ 3\kappa\chi\xi_i\gamma^2\tau_d + 2\chi^2\xi_i\gamma^2\tau_d - 19\varsigma^2\chi\xi_i\tau_d - 10\varsigma\kappa\xi_i\gamma^2 - 5\varsigma\kappa\chi\gamma^2 - 4\kappa\chi^2\tau_d^2 - 2\chi^3\tau_d^2 + 14\varsigma^3\xi_i + 12\varsigma\kappa\chi\tau_d + 6\varsigma\chi^2\tau_d \\
&+ 11\kappa^2\gamma^2 + 11\kappa\chi\gamma^2 + 3\chi^2\gamma^2 - 14\varsigma^2\kappa - 7\varsigma^2\chi) + V^6 n^6\pi^6\gamma^8(-\xi_i^2\gamma^2 - \chi\xi_i\tau_d + 2\varsigma\xi_i - 2\kappa - \chi) + \varsigma^3\xi_i(2\kappa^2 + 2\kappa\chi + \chi^2) \\
&+ \gamma^2\xi_i^2(-10\kappa^2\chi^2\tau_d^2 - 8\kappa\chi^3\tau_d^2 - 2\chi^4\tau_d^2 + 19\varsigma\kappa^2\chi\tau_d + 16\varsigma\kappa\chi^2\tau_d + 5\varsigma\chi^3\tau_d + 2\kappa^3\gamma^2 + 7\kappa^2\chi\gamma^2 + 5\kappa\chi^2\gamma^2 + \chi^3\gamma^2 \\
&- 6\varsigma^2\kappa^2 - 6\varsigma^2\kappa\chi - 3\varsigma^2\chi^2) + \xi_i^3\gamma^4(-5\kappa^2\chi\tau_d - 4\kappa\chi^2\tau_d - \chi^3\tau_d + 2\varsigma\kappa^2 + 2\varsigma\kappa\chi + \varsigma\chi^2) + \chi^2\kappa(17\varsigma\kappa\xi_i\tau_d^2 - 8\kappa\xi_i\gamma^2\tau_d \\
&- 10\varsigma^2\xi_i\tau_d - 8\varsigma\xi_i\gamma^2 - 13\kappa^2\tau_d^2 + 12\varsigma\kappa\tau_d + 18\kappa\gamma^2 - 3\varsigma^2) + \chi^3(14\varsigma\kappa\xi_i\tau_d^2 - \kappa\xi_i\gamma^2\tau_d - \xi_i\tau_d^3(5\kappa^2 + 4\kappa\chi + \chi^2) \\
&- 4\varsigma^2\xi_i\tau_d - \varsigma\xi_i\gamma^2 - 15\kappa^2\tau_d^2 + 6\varsigma\kappa\tau_d + 7\kappa\gamma^2 - 2\varsigma^2) + \chi^4(4\varsigma\xi_i\tau_d^2 - 6\kappa\tau_d^2 + 2\varsigma\tau_d + \gamma^2) - \kappa^2\varsigma(11\chi\xi_i\gamma^2 + \varsigma(11\chi\xi_i\tau_d \\
&+ 2\kappa + 3\chi) + \kappa^3\chi(-11\xi_i\gamma^2\tau_d + 10\varsigma\tau_d + 19\gamma^2)]
\end{aligned}$$

https://doi.org/10.1017/jmech.2016.91 Published online by Cambridge University Press

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(Manuscript received March 25, 2016,
accepted for publication August 21, 2016.)