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Imaging FP Scans: Comments on Interpolation, Sampling, Decomposition, Deconvolution, Binning, and Photometric Calibration

James Caplan

Observatoire de Marseille

Abstract. Several suggestions are made concerning Fabry-Perot data acquisition and reduction, in order to improve the quality of imaging spectroscopy.

1. Introduction

I describe several methods of data acquisition and reduction of Fabry-Perot scans. Although some of these have been used with old-fashioned photoelectric FP scanners, others were in fact developed with imaging FPs in mind. Some of the methods have been implemented whereas others are purely speculative.

2. The Airy Function and its Shortcomings

The transmittance profile of an ideal FP as a function of the order of interference, x, is given by the Airy function $A(x)$. It is convenient to express this function as a *replicated Lorentzian* (cf. Connes 1961, Steel 1983):

$$A(x) = \langle A \rangle \, L(x) * \mathrm{III}(x) \tag{1}$$

where $\langle A \rangle$ is the average of $A(x)$ over one order of interference, and

$$L(x) = \frac{2F/\pi}{1 + 4F^2x^2} \tag{2}$$

is the normalized Lorentzian profile, $F \equiv -\pi/\ln R$ being the finesse, where R is the intensity reflectance of each of the two semi-reflecting mirrors. Equation 2 states that the Airy function is equal to the sum of overlapping Lorentzians, each of FWHM exactly $1/F$, centered at every integral order of interference.

It is unfortunate that the peaks are not *Gaussians*. Compare the Lorentzian with the normalized Gaussian function of the same FWHM,

$$G(x) = 2F\sqrt{\ln 2/\pi}\, 16^{-F^2x^2}. \tag{3}$$

The wings of the Lorentzian fall off *much* more slowly than those of the Gaussian, and "pollute" the data cube.

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Now consider their Fourier transforms. The Lorentzian's transform is an exponential,

$$l(s) = R^{|s|} = e^{-\pi|s|/F}, \tag{4}$$

whereas that of the Gaussian $G(x)$ is another Gaussian,

$$g(s) = \exp\left(\frac{-\pi^2 s^2}{4F^2 \ln 2}\right). \tag{5}$$

So we see that the wings of $l(s)$ also fall off more slowly than $g(s)$; i.e. high frequency components are present, and closer sampling is required.

3. Interpolation

Sinc interpolation, i.e. convolution of a scan sampled N times per FSR (free spectral range—here equal to unity) with the function

$$\mathrm{sinc}(Nx) \equiv \frac{\sin(2\pi Nx)}{2\pi Nx}, \tag{6}$$

is frequently used in FP work. Since the scan is periodic, one gets the same result by convolving just *one* FP scan with the function $\mathrm{sinc}(Nx) * \mathrm{III}(x)$, if N is an integer. Sinc convolution of an Airy function will recover intermediate values with a maximum error of $\pm 1\%$ of the peak if $N \geq 3.4F$. This is more stringent than the generally assumed $N = 2F$.

4. Two reduction methods

4.1. Decomposition by least squares

This method assumes that the source spectrum is the sum of a continuum plus a certain number of Gaussian spectral lines. The scan is therefore represented by

$$\text{Signal} = \text{Continuum} + \text{Line}_1 + \text{Line}_2 + \ldots, \tag{7}$$

where each observed line is a Gaussian convolved with an Airy function. The continuum is either a constant (for filter FWHM $\gg$ FSR), or can be fitted by a Fourier series. It is convenient to express each such line as an infinite Fourier series (cf. Hernandez 1966, Caplan 1972, Caplan & Deharveng 1985):

$$\text{Line}_i = A_i\left[1 + 2\sum_{n=1}^{\infty} R^n \exp\left(\frac{-\pi^2 w_i^2}{4\ln 2} n^2\right) \cos\left(2\pi n \frac{t - t_i}{\Delta t}\right)\right]. \tag{8}$$

We solve for the unknown parameters by least squares. The reflectivity R and the FSR Δt (in "etalon code units") are characteristic of the FP and are best determined from calibration spectra, while the observed scans on the sky are used to find, for each line, the average line signal A_i, the FWHM of Gaussian broadening w_i, and the line center t_i. This method, although requiring assumptions about the form of the spectrum, gives us the quantitative information we want—line strengths, positions, and widths. Note that there is no "Nyquist criterion"; it is sufficient that the number of observed points be at least as great as the number of unknowns, although obviously the accuracy will be greater if the observed points are judiciously located.

4.2. Deconvolution

Knowing the instrumental profile, $\langle A \rangle L(x) * \mathrm{III}(x)$, we can remove its effect from the observed scan—to a certain extent. The effect of $\mathrm{III}(x)$ cannot be removed. We suggest *replacing* $L(x)$ *by* $G(x)$—i.e. changing the effective instrument profile from an Airy function to a replicated Gaussian.

In the transform domain, this means multiplying the transform of our scan by $q(s) = g(s)/l(s)$. Or we can work in the function domain and simply *convolve* our scan with $Q(x)$, the transform of $q(s)$. Equivalently, we convolve just *one free spectral range* of the scan (which must be sampled or re-sampled at an integral number of points N per FSR) with our "deconvolution function"

$$Q'(x) = Q(x) * \mathrm{III}(x) = 1 + 2\sum_{i=1}^{\infty} \exp\left(\frac{\pi i/F - \pi^2 i^2}{4F_g^2 \ln 2}\right) \cos(2\pi x i), \qquad (9)$$

where F is the Airy function finesse and F_g is the desired "Gaussian finesse" of the end result. With a replicated Gaussian as the effective profile, a maximum interpolation error of 1% only requires $N \geq 2.1F_g$.

5. Binning (With a Photon-Counting Camera)

5.1. Simple Binning

This method of data acquisition and reduction for imaging FP observations was used by Caplan et al. (1985). Thanks to the absence of readout noise with a photon-counting camera, very small FP steps ($\lesssim 1/2000$ FSR) are possible. The position in the field α, δ and the etalon code unit t are recorded for each photon. During reduction, the "phase correction" is applied to each photon to find its α, δ, x. But the data cube contains only N possible values of x; we choose the x_i nearest to x. Then the cell α, δ, x_i of the data cube is incremented by one. This method avoids interpolation, but is equivalent to convolution with a rectangle function.

5.2. Deconvolution Binning

A variant of this method consists of replacing the "increment by one" step by the following: add $Q'(x - x_i)$ to *each cell* $(\alpha, \delta, x_1) \ldots (\alpha, \delta, x_N)$ at the position α, δ in the data cube. The result, again, is to change to instrument profile's Lorentzian to a Gaussian.

6. Pre-Deconvolution of CCD Observations

If we wish to sample more sparsely, we must filter out the higher frequency components from our observed scan *before* sampling. Can a servo-controlled FP, capable of rapid tuning, be used to solve this problem? A simple example suggests this may be possible. Suppose that, in a given pixel, we wish to sample at x. Normally we would tune our FP to x and integrate. But if we want to measure what *would* have been measured at x with a "Gaussian-profile FP", we can, during the integration of a single frame, "jitter" the FP to $\sim$ 2000

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different values of $x + \Delta x$, where $-.5 \leq \Delta x < .5$, and stay at each Δx for a length of time proportional to $Q'(\Delta x)$ plus a constant (needed to avoid negative values). Subtracting the constant times the average signal, we have convolved with Q', and the Lorentzian has been replaced by a Gaussian, so fewer points are required for proper sampling. (We have not yet done the "phase correction", so for imaging observations interpolation will still be necessary.) The signal to noise ratio will be degraded. However, I suspect that a less brutal form of jittering, over a smaller range of Δx, can be concocted so as to remove just the high-frequency components of the Lorentzian, the rest being done numerically.

7. Photometric Calibration

I wholeheartedly agree with J. Bland that the FP is suitable for photometry. Here is a simple, photometrically rigorous way of analyzing FP scans (Caplan & Deharveng 1983, 1985). First consider an ordinary photometer *without* an FP. Observing a standard star of spectrum $S_\lambda(\lambda)$, we measure (ignoring extinction)

$$\text{Signal(star)} = A \int S_\lambda(\lambda) T_{\text{star}}(\lambda) d\lambda, \tag{10}$$

where $T_{\text{star}}(\lambda)$ is the filter transmittance. Now we solve this equation for A, which is in units of area, and in which all the parameters *except* the filter curve are mixed together: telescope collecting area, reflectivities, quantum efficiency of the detector, etc. Next we observe the signal from a nebular emission line,

$$\text{Signal(line)} = A F_{\text{line}} T_{\text{line}}(\lambda) \tag{11}$$

where $T_{\text{line}}(\lambda)$ refers to the (usually different) filter used for the nebula. Knowing A, we can solve for F_{line}, the line flux.

Let us add an FP and repeat this procedure. Now what we call "signal" is the *average* signal measured after scanning over one FSR; i.e. the FP's transmittance averaged over one FSR gets incorporated into A (which will be $\sim F$ times smaller). So of what use is the FP? It allows us to distinguish (e.g. by least squares) between the desired line and the unwanted spectral lines and continuum, so we can average only the *desired* signal to get Signal(line). The FP parallelism adjustment does not affect A. But filter calibration is critical ($T_{\text{line}}(\lambda)$ is *not* given by the white light cube, except approximately, under certain conditions).

Placing the filters near the pupil eliminates problems of non-uniformity. An angle between the filters and the FP avoids reflections.

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