DECODING OF THE LIGHT CHANGES IN WOLF-RAYET ECLIPSING BINARIES: AN APPLICATION TO HD 5980 IN THE SMALL MAGELLANIC CLOUD

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ABSTRACT. A method of light curve analysis is described which allows the study of an eccentric partially-eclipsing system containing one component possessing an extended atmosphere. The effects of transparency as well as limb-darkening are taken into account. Preliminary results obtained for the Wolf-Rayet eclipsing binary HD 5980 in the SMC are presented.

1. INTRODUCTION

HD 5980≡SMC/AB 5 (Azzopardi and Breysacher, 1979) is located in NGC 346, the largest H II region of the Small Magellanic Cloud. The eclipsing nature of the star was recognized by Hoffmann et al. (1978) but the correct orbital period, P = 19.266 ± 0.003 days, was found by Breysacher and Perrier (1980). The obtained light curve revealed a strongly eccentric orbit: e=0.47 for i=80°, however, the shape of this light curve was not defined well enough to allow a detailed quantitative analysis.

The relatively long period together with the large eccentricity making HD 5980 a potentially interesting object in which to study the structure of a W-R envelope, numerous new photometric observations in the Strömgren system were carried out in order to significantly improve the light curve. What has been achieved in this respect is presented in Figure 1.

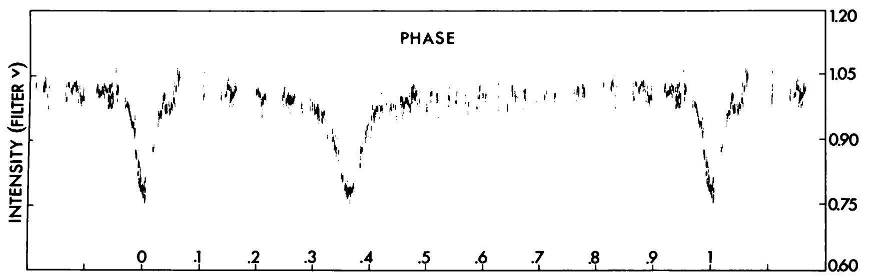

Figure 1. Light curve in the Strömgren v band of SMC/AB 5 (705 observations) normalized to I=1 around apastron, i.e. for 0.5 ≤ Φ ≤ 0.7 .

K. A. van der Hucht and B. Hidayat (eds.),
Wolf-Rayet Stars and Interrelations with Other Massive Stars in Galaxies, 229–235.

Concerning the interpretation of the data, as none of the existing "tools" turned out to be really suited to our purpose, i.e. the decoding of the light changes of a **partially-eclipsing system** characterized by a **strong eccentric orbit** and containing one component with an **extended atmosphere**, a non-classical approach aiming at the solution of light curves in the frequency-domain (cf. Kopal, 1979; Smith and Theokas, 1980) was attempted.

2. ANALYSIS OF THE LIGHT CHANGES IN THE FREQUENCY-DOMAIN

2.1. The basic equations

Reference being made to Kopal's fundamental work (cf. Kopal, 1979), let us first consider an eclipsing system which consists of two spherical stars revolving around the common centre of gravity in circular orbits, and appearing in projection on the sky as uniformly bright discs. When star 1 of luminosity L_1 and radius r_1 is partly eclipsed by star 2 of luminosity L_2 and radius r_2, then the brightness l of the system (maximum light between minima taken as unit) is given by

$$l(r_1,r_2,\delta,J) = 1 - \int_A J(r)\, d\sigma \tag{1}$$

where δ is the apparent separation of the centres of the two discs and J represents the distribution of brightness over the apparent disc of the star undergoing eclipse, of surface element $d\sigma$. The assumption that star 1 is uniformly bright gives

$$J(r) = \frac{L_1}{\pi r_1^2} \tag{2}$$

Combining equations (1) and (2) we obtain for the "loss of light" suffered by the system when an area $A(r_1,r_2,\delta)$ of star 1 is eclipsed

$$1 - l(r_1,r_2,\delta,J) = \frac{L_1}{\pi r_1^2} \int_A d\sigma \tag{3}$$

As proposed by Kopal, let us focus our attention on the area subtended by the light curve in the $l - \sin^{2m}\theta$ coordinates ($m=1,2,3,\ldots$), where θ denotes the phase-angle. The areas A_{2m} between the lines $l=1$ and the actual light curve l, from the eclipse minimum $\sin^{2m}\theta=0$ to the first contact $\sin^{2m}\theta_1$, are given by the integrals

$$A_{2m} = \int_0^{\theta_1} (1-l)\, d(\sin^{2m}\theta) \tag{4}$$

hereafter referred to as **moments of the eclipse**, of index m.

Kopal (1979) has shown that it is possible:

a) to invert this relationship to determine the elements of the system in terms of the moments A_{2m} that can be empirically obtained from the data.

b) to extend this treatment to the case of a partial eclipse of a limb-darkened star by a star surrounded by an extended atmosphere.

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A convenient mathematical solution of the problem of atmospheric eclipses is due to Smith and Theokas (1980). To take into account the transparency effects a function F(s) is introduced in equation (1), so that the total amount of light emitted to the observer is now

$$l(r_1,r_2,\delta,J,F) = 1 - \int_A J(r)\ F(s)\ d\sigma \tag{5}$$

Considering that when a star with an extended atmosphere eclipses an ordinary one it may be advantageous to weigh the data nearer mid-minima, Smith and Theokas (1980) also introduce the moments B_{2m} defined as

$$B_{2m} = - \int_0^{\theta_1} (1-l)\ d(\cos^{2m}\theta) \tag{6}$$

The expressions for the A_{2m} and B_{2m} moments have been derived by Smith and Theokas (1980) for m=1→4 and m=1→5, respectively. We only give here, as an example, the final forms obtained for A_2, A_4 and B_2, B_4

$$A_2 = I_1R_1r_2^2\csc^2 i + \cot^2 iQ\ , \tag{7}$$

$$A_4 = I_1R_2r_2^4\csc^4 i - 2I_1R_1r_2^2\csc^2 i\cot^2 i + $$
$$+ I_2R_1r_1^2r_2^2\csc^4 i - \cot^4 iQ\ , \tag{8}$$

$$B_2 = \lambda - \csc^2 i(P-I_1R_1r_2^2-\psi_1)\ , \tag{9}$$

$$B_4 = \lambda - \csc^4 i(P-2I_1R_1r_2^2+I_2R_1r_1^2r_2^2+I_1R_2r_2^4-\psi_2)\ , \tag{10}$$

where i is the orbit inclination. The general expressions for the coefficients $I_m(r_1,J)$, $R_m(r_2,F)$, $Q(r_1,r_2,J,F)$, $P(r_1,r_2,J,F)$, $\psi_m(r_1,r_2,i,J,F)$ and λ can be found in the paper by Smith and Theokas (1980).

2.2. The transparency and limb-darkening functions

While for the transparency function F(s) of the eclipsing W-R star, of radius r_0, Smith and Theokas (1980) simply adopt

$$F(s) = F_y(r_0,\upsilon) = y\left[1 - \upsilon\left(\frac{s}{r_0}\right)^2\right] \qquad \text{for} \quad s < r_0 \tag{11}$$

where υ is the coefficient of transparency, we have adopted a law of the form (see Figure 2)

$$F(s) = F_{1-y}(r_3,0) + F_y(r_2,\upsilon) \tag{12}$$

where the radius of the opaque core of the W-R star is r_3 and that of the extended eclipsing envelope, r_2.

For the brightness distribution J(r) over the W-R disc, a law very similar to that of the transparency function was taken

$$J(r) = J(0)\left[J_{1-y}(r_3',0) + J_y(r_2',u)\right] \tag{13}$$

where J(0) is the central surface brightness and u the coefficient of limb-darkening. J_y is defined as

$$J_y(r_0,u) = y\left[1 - u\left(\frac{r}{r_0}\right)^2\right]^2 \tag{14}$$

When the W-R star is eclipsed the radius of the core, assumed to be of uniform brightness, is now r_3' and that of the limb-darkened envelope r_2' .

Using these laws of transparency and limb-darkening, we have then derived the corresponding expressions for I_m, R_m, Q, P and ψ_m and hence the final equations for the moments A_{2m} and B_{2m}.

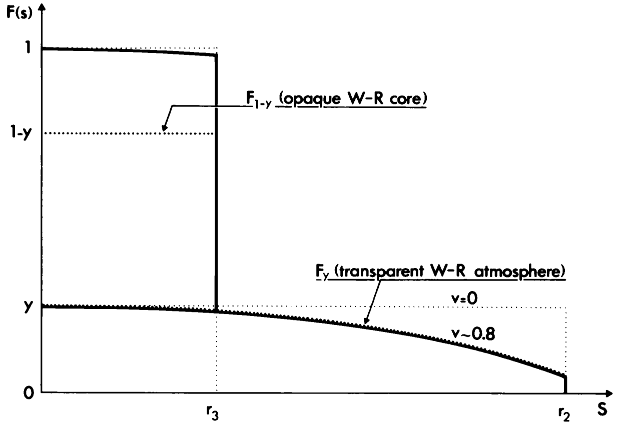

Figure 2. Opacity distribution across the disc of the W-R component.

2.3. The orbital eccentricity

In the case of an elliptical orbit the problem still consists of a determination of the elements of the eclipse from the moments A_{2m} (B_{2m}) derived from the light curve, but taking into account the eccentricity e and the longitude ω of periastron.

In the definition of the A_{2m}'s areas, the phase-angle θ is no longer identical with the mean anomaly M but rather a linear function of the true anomaly v

$$\theta = v + \omega - \frac{\pi}{2} \tag{15}$$

the element $d(\sin^{2m}\theta)$ of integration in the equation defining the moments A_{2m} becomes

$$d(\sin^{2m}\theta) = d\left[\cos^{2m}(v+\omega)\right] \tag{16}$$

As a consequence, the empirical values of A_{2m} cannot be ascertained from the observed data until a proper conversion into the true anomalies has

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been done. This can be accomplished by a resort to the expansion of elliptic motion (cf. Danjon, 1959)

$$v = \omega + M + (2e-\frac{1}{4}e^3)\sin M + (\frac{5}{4}e^2 - \frac{11}{24}e^4)\sin 2M + \frac{13}{2}e^3\sin 3M + \ldots(17)$$

The empirical "elliptic" moments of the light curve then furnish the elements of the eclipse exactly as in the "circular" case, care being taken only that the resulting values of the radii have to be reduced to the same unit of length.

For a given value of the inclination i, $\Delta\Phi$ being the phase displacement of the minima, e and ω can be derived by means of the following equations

$$\Delta\Phi = \pi + 2e\left[1 + \csc^2 i - \frac{e^2}{2}(\frac{8}{3}\cos^2\omega - 2)\right]\cos\omega \tag{18}$$

$$e\sin\omega = \frac{d_2 - d_1}{4\sin(\frac{d_1+d_2}{4})}\left\{1 - \frac{\cot^2 i}{\sin^2(\frac{d_1+d_2}{4})}\right\}^{-1} \tag{19}$$

where d_1 and d_2 are the respective durations of the primary and secondary stellar core eclipses, determined from the light curve.

It is important to note that, although each separate half-eclipse provides a self-reliant solution for the elements, due to the present composite model adopted for the W-R star (Figure 2), with in particular different radii introduced when this component is seen either as an eclipsing or an eclipsed disc, the complete determination of the elements necessarily requires a combination of the solutions furnished by the descending and ascending branches of both minima.

3. APPLICATION TO SMC/AB 5

Using this method we have analysed the light curve of SMC/AB 5 obtained with the Strömgren v filter (Figure 1). The elements for the primary eclipse (O star in front) and the secondary eclipse (W-R star in front) are derived from the moments A_{2m} and B_{2m}, respectively. The empirical values of the "elliptic" moments are determined from smooth curves resulting from a spline fit on the observed points. The solutions are obtained by inversion of Equations (7) to (10) with a Newton-Raphson method for non-linear equations. The ill-defined ascending branch of the primary minimum is, however, not included in the calculation.

No solution is found for $i < 86°$; the results obtained for $i = 86°$ are summarized in Table 1. All radii are reduced to the semi-major axis of the relative orbit. For the radius r_1 and the luminosity L_1 of the O star as well as for the radius r_3 of the eclipsing W-R core, mean values have been taken as this did not imply any "a priori" assumption concerning these particular elements. SMC/AB 5 being classified WN3+O7: (Breysacher et al., 1982), we have adopted u=0.3 for the limb-darkening coefficient of the O star (cf. Klinglesmith and Sobieski, 1970).

TABLE 1. Elements of the eclipse for i = 86° (e = 0.324, ω = 133°)

	PRIMARY (descending branch)	SECONDARY (descending branch)	SECONDARY (ascending branch)
O star	← r_1 = 0.163 ± 0.007 →		
		← L_1 = 0.410 ± 0.034 →	
WR star	r'_3 = 0.120 ± 0.009	← r_3 = 0.112 ± 0.007 →	
	r'_2 = 0.227 ± 0.012	r_2 = 0.245 ± 0.018	r_2 = 0.332 ± 0.016
	L_2 = 0.257 ± 0.011		
	y = 0.18 ± 0.06	y = 0.06 ± 0.04	y = 0.16 ± 0.03
	u = 0.5 ± 0.3	υ = 0.37 ± 0.15	υ = 0.28 ± 0.15

The results presented here, which relate to the v filter data only, are obviously too preliminary to allow a thorough discussion of the geometry of the system, nevertheless, some interesting conclusions can already be drawn regarding SMC/AB 5.

1. The size of the W-R core does not change significantly between the primary and secondary eclipses, i.e. when the star is seen as an eclipsed or as an eclipsing disc.
2. The W-R envelope appears highly asymmetrical when occulting the O star. Very different values for r_2 and y are indeed furnished by the descending and ascending branches of the secondary minimum.
3. The fact that L_1+L_2 is far from unity indicates that there exists very likely a third unresolved component in the line of sight, a conclusion which seems to be also supported observationally (Massey et al., 1989).

Absolute radii may be estimated by assuming that the mass-luminosity relation for W-R stars given by Maeder and Meynet (1987) applies to SMC/AB 5. With M_v = -7.3 (Breysacher, 1988) the sum of the masses derived for the O and W-R components, 76.4 $M_\odot$, leads to a semi-major axis of 0.597 AU for this binary. The resulting values for the radii are: 20.9 $R_\odot$ for the O star, about 15 $R_\odot$ for the W-R core and 30 to 40 $R_\odot$ for the W-R envelope.

Although of fundamental importance, an error analysis is beyond the scope of this short communication. The subject will be treated exhaustively in a forthcoming article.

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REFERENCES

Azzopardi, M., Breysacher, J.: 1979, Astron. Astrophys. **75**, 120.
Breysacher, J.: 1988, Thèse, Université de Paris VII.
Breysacher, J., Moffat, A.F.J., Niemelä, V.S.: 1982, Astrophys. J. **257**, 116.
Breysacher, J., Perrier, C.: 1980, Astron. Astrophys. **90**, 207.
Danjon, A.: 1959, Astronomie Générale, 2nd edition, Albert Blanchard, Paris, 1980.
Hoffmann, M., Stift, M.J., Moffat, A.F.J.: 1978, Publ. Astron. Soc. Pacific **90**, 101.
Klinglesmith, D.A., Sobieski, S.: 1970, Astron. J. **75**, 175.
Kopal, Z.: 1979, Language of the Stars, Reidel Publishing Company, Dordrecht.
Maeder, A., Meynet, G.: 1987, Astron. Astrophys. **182**, 243.
Massey, P., Parker, J.W., Garmany, C.D.: 1989, Astron. J. **98**, 1305.
Smith, S.A.H., Theokas, A.C.: 1980, Astrophys. Space Sci. **70**, 103.

DISCUSSION

Moffat: Maybe you have seen the poster outside by myself, Niemela and others that the polarization curve, one of the first with an elliptical orbit solution, gives an eccentricity a little bit different than yours, 0.22 ± 0.03-4. It is in the right direction, it is a smaller value.

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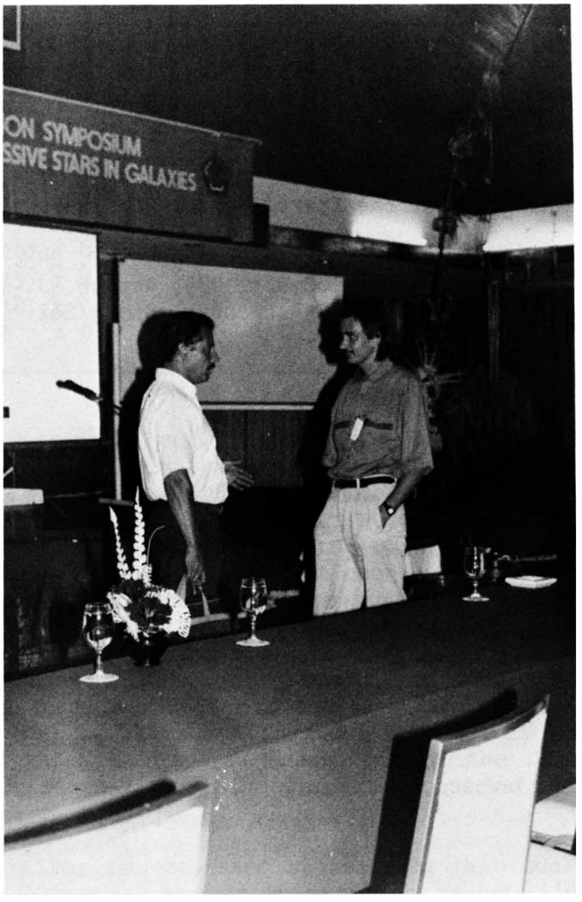

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