THE MODIFIED NEWTONIAN DYNAMICS AS AN ALTERNATIVE TO HIDDEN MATTER

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ABSTRACT. The mass discrepancy, which has led to the notion of dark matter may, in fact, be due to a breakdown of the Newtonian laws which are used to determine the masses of galactic systems. We describe a nonrelativistic theory which departs from Newton's in the limit of small accelerations. When one uses the modified dynamics to deduce gravitational masses, the need to invoke large quantitites of dark matter disappears. We outline the theory and give criteria for deciding which systems are expected to exhibit marked departures from Newtonian behaviour. The main body of the talk is a succinct description of the major predictions of the theory regarding dynamics within galaxies.

1. INTRODUCTION

We were asked to discuss an approach which is somewhat outside the mainstream of dark matter studies in that it advocates that dark matter does not actually exist (at least not in quantities as large as are required to bridge the galactic mass discrepancy). Instead we take the view that masses of galaxies and systems

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of galaxies are grossly overestimated when they are deduced from Newton's laws (second law and law of gravity).

A modified set of dynamical laws (MOND) is used to describe the motions in galactic systems and, in particular, to obtain the masses of such systems. We find that the mass discrepancy disappears and a number of the observed traits of galaxies are unavoidable consequences of MOND.

The nature of MOND and its implications have been described in detail in a series of papers (Milgrom 1983a,b,c, Bekenstein and Milgrom 1984 and Milgrom 1984, 1985a,b, referred to hereafter as papers I-VII respectively). We thought it fit to concentrate, in the present talk, on the main predictions which MOND makes concerning dynamics within galaxies. We shall list these predictions and discuss each of them briefly. A more detailed discussion can be found in the references.

We would also like to bring to your attention the fact that there have been suggestions to explain away the mass discrepancy by adopting, unlike MOND, modified forms of the distance dependence of the gravitational force (e.g., Finzi 1963, Tohline 1983, Sanders 1984).

2. THE NONRELATIVISTIC FORMULATION

2.1 The Basic Postulates

Most of the major predictions of MOND follow from the following assumptions (Papers I, II).

(i) Newtonian dynamics breakdown when the accelerations involved are small.

(ii) The acceleration a of a test body at a distance r from a mass M is given by $a^2/a_0 \approx MGr^{-2}$ in the limit $MGr^{-2} \ll a_0$ (or $a \ll a_0$) .

Here a_0 is an acceleration constant which plays both the role of a transition acceleration from the Newtonian to the Non-Newtonian regime and the role of a proportionality constant in the modified equation of motion. The value of a_0 was determined (Paper II) to be about $2 \times 10^{-8}(H_0/50\,\mathrm{km\,s^{-1}\,Mpc^{-1}})^2\,\mathrm{cm\,s^{-2}}$. Interestingly, this value is very close to that of cH_0 .

2.2 The Theory We Now Use

Of the various interpretations of the basic assumptions and theories which may incorporate them, we have found the following the most appealing thus far (Paper IV). It is assumed that MOND signifies a breakdown of the Newtonian law of gravity (leaving the 2nd law intact). The gravitational acceleration field $\vec{g}$ is still taken to be derivable from a potential $\vec{g} = -\vec{\nabla}\varphi$. However, φ is now related to the density distribution ρ which induces it by the following field equation (derivable from a Lagrangian):

$$\vec{\nabla} \cdot [\mu(g/a_0)\vec{g}] = -4\pi G\rho, \tag{1}$$

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instead of the Poisson equation. Here $\mu(x) \approx 1$ for $x \gg 1$ so that Poisson's equation is restored in the limit $g \equiv |\vec{g}| \gg a_0$, but $\mu(x) \approx x$ for $x \ll 1$ so that the desired low acceleration behaviour is obtained. Equation (1) is supplemented by the boundary condition at infinity: $\vec{g} \to 0$ for an isolated system and $\vec{g} \to \vec{g}_\infty$ for a system in a constant external acceleration field $\vec{g}_\infty$. Other than these requirements (and the monotonicity of μ which we always require) μ has remained undetermined.

We do not yet have a satisfactory relativistic extension of MOND.

2.3 A Simplified (approximate) Formulation

The theory given by eq.(1) is nonlinear and practically impossible to solve exactly for all but the simplest configurations. By eliminating ρ between eq.(1) and the Poisson equation for the Newtonian acceleration field $\vec{g}_N$, ($\vec{\nabla} \cdot \vec{g}_N = -4\pi G\rho$) we get $\vec{\nabla} \cdot [\mu(g/a_0)\vec{g} - \vec{g}_N] = 0$. Equating the field in parentheses to zero (when in fact it is in general a non-zero curl field)

$$\mu(g/a_0)\vec{g} = \vec{g}_N, \tag{2}$$

seems to give a very good approximation for the field $\vec{g}$ when test particle motion is considered (see e.g. Paper VI). For example we find (paper VI) that a galaxy's rotation curve derived from eq.(2) differs by at most five percent from that which is derived from the exact eq.(1) for the many galaxy models which we have tried. Equation (2) is the formulation originally used for MOND in Papers I-III, and it is exact when the system has a plane cylindrical or spherical symmetry. The solution of eq.(2) is straightforward for an arbitrary mass distribution.

2.4 Some General Properties of the Field Equation

We have derived in paper IV the following results for systems which are governed by eq.(1):

(i) A system of gravitating masses with local accelerations given by the solution of eq.(1) conserves energy, momentum, and angular momentum.

(ii) A small, low mass, object in the field of a large massive body is accelerated like a test particle (irrespective of whether the accelerations within the object are large or small). Thus, for example, stars, binary stars, globular clusters etc. may, to a very good approximation, be regarded as test bodies when their motion in the field of a galaxy is considered.

(iii) The motions (relative to the c.o.m.) within a system s, which itself is in a field of a mother system S, are affected by the external field (Papers I, IV,VI) of S when the latter is not negligible compared with the internal acceleration. Thus let m and r be the mass and average radius of s and M and R those of S. Comparing the (Newtonian and thus not the actual) accelerations $g^N_{in} \equiv mG/r^2$ and $g^N_{ex} \equiv MG/R^2$ with each other and each of them with a_0 will help us decide

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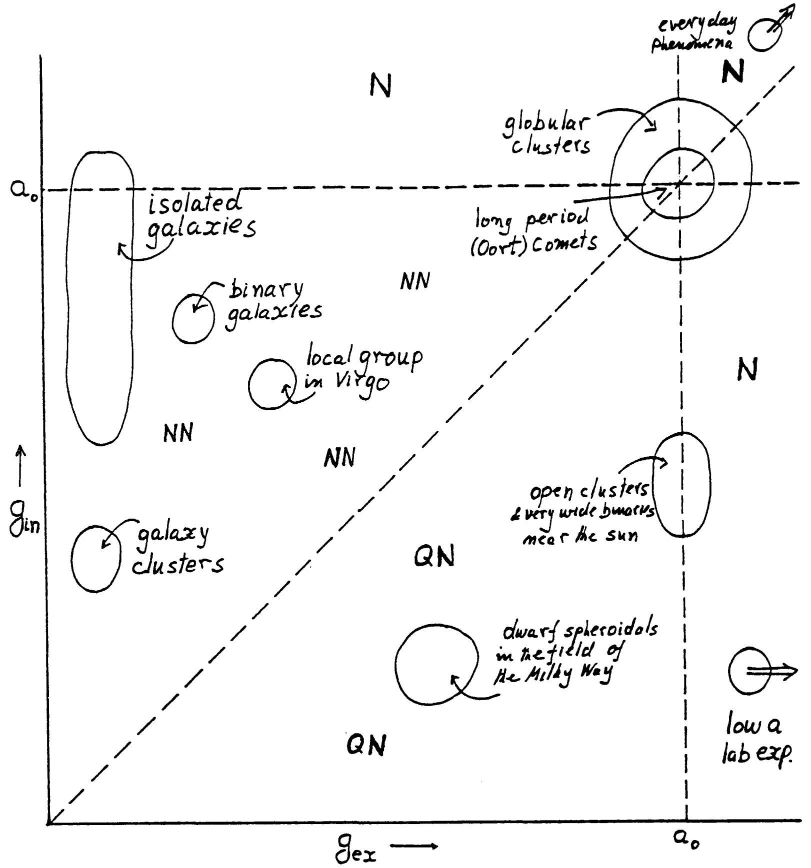

Fig. 1. A classification of various galactic systems according to their intrinsic and c.o.m. accelerations.

whether to expect strong departures from Newtonian laws in the internal dynamics of s . We should, strictly speaking, use the actual accelerations g_{in} and g_{ex} but these cannot be written a priori in simple terms of the sizes and masses. However, all strong inequalities between g_{in}, g_{ex} and a_0 are the same as those between g_{in}^N, g_{ex}^N and a_0. Figure 1 shows schematically where various systems of interest fall in the g_{in}, g_{ex} plane.

The general rules which apply when the inequalities between g_{in}, g_{ex} and a_0 are strong are as follows:

(i) When either $g_{in} \gg a_0$ or $g_{ex} \gg a_0$, the dynamics within s are Newtonian (region marked N in Figure 1).

(ii) When $g_{ex} \ll g_{in} \ll a_0$, the system is approximately isolated (in the MOND sense) and the small acceleration limit of MOND applies (region marked NN in Figure 1).

(iii) When $g_{in} \ll g_{ex} \ll a_0$, the dynamics are quasi-Newtonian but with a

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value of the effective gravitational constant being $G_{eff} = G/\mu(g_{ex}/a_0) \gg G$ (see detailed discussion in Paper VI). This case corresponds to the region marked QN in Figure 1.

3. PREDICTIONS WHICH ARE INCOMPATIBLE WITH DARK MATTER

One may ask whether all the predictions of MOND can be mimicked with hidden mass, maintaining Newtonian dynamics. This, as we shall now demonstrate, is not the case.

3.1 Negative "Dark Matter"

If a density $\rho(\vec{r})$ gives rise to an acceleration field $\vec{g}(\vec{r})$ according to MOND, the only way we could make the measured accelerations in this field consistent with Newtonian dynamics is by assuming that the actual density distribution is $\rho^* = -(4\pi G)^{-1}\vec{\nabla} \cdot \vec{g}$. However, it can be shown (Paper VII) that for various configurations $\rho^* < \rho$ or even $\rho^* < 0$. Insisting on describing such systems with Newtonian dynamics will imply that the system contains negative mass densities which is unacceptable. For example in any binary galaxy system there is a region (shown in Figure 2 for galaxies of equal mass) where one will find negative density if he insists on using Newtonian dynamics to explain the measured acceleration field.

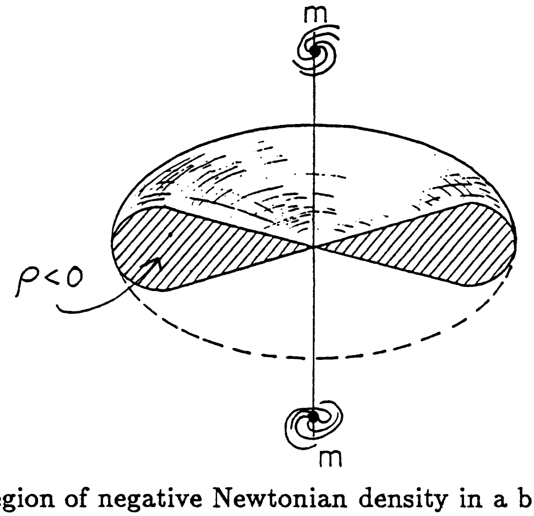

Fig. 2. The region of negative Newtonian density in a binary galaxy system.

3.2 Breakdown of the Strong Equivalence Principle (SEP)

MOND does not satisfy the strong equivalence principle (SEP) even in the nonrelativistic regime. Any observed manifestation of this fact will point to the

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breakdown of Newton laws. Examples are discussed in Papers IV and VI. For instance, a self-gravitating many-particle system (such as a gas cloud) with isotropic pressure (or velocity dispersion) in an external field $\vec{g}_{ex}$ and with $g_{in} < g_{ex} < a_0$ will not be spherical (as in the Newtonian case) even when $\vec{g}_{ex}$ is exactly constant. It will be an ellipsoid of revolution with its long (symmetry) axis along $\vec{g}_{ex}$, in conflict with the SEP.

3.3 Light Bending

As we do not yet have a relativistic generalization of MOND, we cannot predict the nature or strength of light bending in a gravitational field. There is however no reason we can think of why the same fictitious mass distribution will be required to explain the trajectories of light rays and those of massive particles in the field of an object such as a galaxy. In fact, in a toy relativistic model we studied, an attempt to explain light bending and say the rotation curve of a massive body (in the regime $g \ll a_0$), assuming the conventional dynamics, will fail.

4.OTHER PREDICTIONS CONCERNING GALAXIES

Many of the consequences of MOND which we list below involve galaxy properties that have already been observed. However, it has not been known how strong and general these observed characteristics are and, for that matter, what their exact nature is. These observations do not conflict with the dark matter hypothesis and, in fact, have been taken to reflect various properties of dark halos (for example, asymptotically flat rotation curves are interpreted as resulting from a r^{-2} behaviour of the dark halo's density law).On the other hand, those regularities in galaxy appearance are also not predictions of the dark matter hypothesis.In MOND they are exact general and unavoidable predictions.

4.1 Disc Galaxies

4.1.1. Rotation curves. The rotation curve of a galaxy deduced from the "observed" mass distribution using MOND should agree with the observed rotation curve. There are sub-predictions of this general one which do not require the full knowledge of the galaxy's rotation curve or mass distribution (see paper II).

(i) The velocity of a test body in a circular orbit around an isolated galaxy should become independent of the radius of the orbit at large radii.

(ii) The asymptotic circular velocity V_∞ depends only on the total mass M of the galaxy via $V_\infty^4 = a_0 GM$.

(iii) In high rotational velocity galaxies (such that $V_\infty^2/h > a_0$, where h is the galaxy's scale length), the local M/L value (as deduced from Newton's laws) should be constant at small radii and then start to increase around the radius where $V^2/r = a_0$.

(iv) Very low surface density (LSD) galaxies are particularly good test cases because of the following reasons:

a. When the average surface density is very small $\langle\Sigma\rangle \ll \Sigma_0 \equiv a_0 G^{-1}$, the accelerations are much smaller than a_0 and hence we predict large departures from Newtonian behaviour.

b. LSD'S tend to be bulgeless so there are fewer parameters involved (Carignan and Freeman 1985).

c. Since we are dealing with a system where $g \ll a_0$ everywhere we do not require the exact form of $\mu(x)$ and it is a good approximation to use $\mu(x) = x$.

d. All the uncertainties involved in comparing calculated and measured rotation curves (galaxy's distance, inclination, extinction, M/L, a_0 etc.) lump into one multiplicative factor and the test of MOND which such galaxies offer is much more clear-cut. [In Newtonian dynamics the dimensionless rotation curve $v(r) = V(r)/V_\infty$, depends only on the mass distribution in the galaxy but not on the total amount of mass say. In MOND $v(r)$ also depends on an additional parameter, say the average surface density. For instance, two pure exponential disc galaxies may have very different - looking rotation curves (see e.g. the model curves in paper II). The point we are making here is that when $\langle\Sigma\rangle \ll \Sigma_0$ there exist some similarity laws which eqs.(1)(2) obey and which make $v(r)$ depend on the mass distribution only.All this is discussed in detail in papers II,VI].

On the other hand, LSD disks tend to contain relatively large quantities of hydrogen which may contribute subtantially or even dominantly to the radial acceleration. Such cases may thus involve additional parameters and they provide less of a clear-cut test.

As a mere demonstration we give in Figure 3 the measured and calculated rotation curves for three galaxies. The data for NGC 3198 are taken from van Albada et al.(1985). This galaxy has one of the cleanest and the furthest reaching rotation curves to date (in optical radii).It is a relatively low-acceleration galaxy (maximum acceleration is $\approx .5a_0$).The data for NGC 247 and NGC 300 are taken from Carignan and Freeman(1985). These galaxies may be considered very low acceleration ones. In calculating the MOND curves we used the approximate eq.(2), assumed pure exponential discs, and used the values of the scale length s given in the above references. The choice of $\mu(x)$ does not affect the fit much. We have only one free parameter for each galaxy. This may be taken as the value of M/L assuming that all the others are as given by the observers (they all lump into one factor anyway when $g \ll a_0$). The value of M/L used is given beside each curve.

4.1.2. Surface densities. The constant a_0 defines a quantity with the dimension of mass surface density $\Sigma_0 \equiv a_0 G^{-1}$ which we predict to play an important role in galaxy dynamics. When a galaxy has an average surface density $\langle\Sigma\rangle \gg \Sigma_0$, its dynamics will be Newtonian out to large radii(compared say with the half-mass

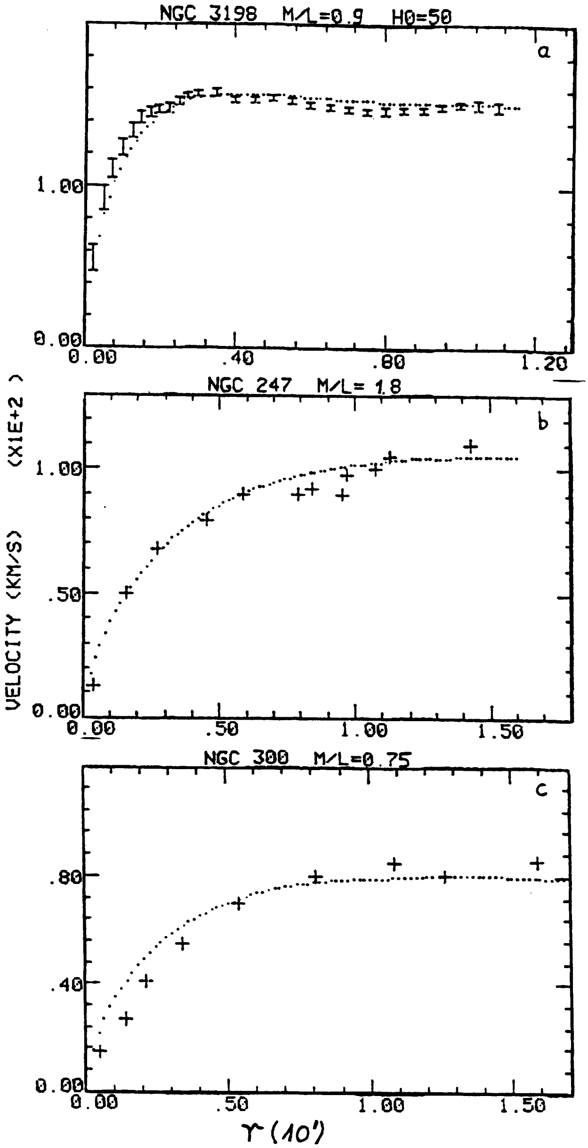

Fig. 3 MOND rotation curves compared with observations for three galaxies.

radius). In this case there will be a range of radii with a Keplerian decline of the rotational velocity before it reaches the asymptotic value V_∞. We found (Paper II) that for the velocity curve to remain approximately flat down to small radii we must have $\langle\Sigma\rangle \approx \Sigma_0$. When $\langle\Sigma\rangle \gg \Sigma_0$, the rotation curve should exhibit an appreciable hump. When $\langle\Sigma\rangle \ll \Sigma_0$, the velocity rises slowly, peaks at a few scale

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length s, and then decreases by a few percent to its asymptotic value. Thus *if galaxies are not observed to have considerbly humped rotation curves they should all have* $\langle\Sigma\rangle \lesssim \Sigma_o$.

4.1.3. The Oort discrepancy. Further predictions can be made which concern the dynamics of motions perpendicular to the plane of a thin planar galactic disc. Recent results of such analysis near the sun by Bahcall (1984) strengthen results from earlier reports starting with Oort(1960) showing that, near the sun, the dynamically determined mass is larger than that which is accounted for by known components. A detailed analysis of the expected fictitious "dark matter" distribution in very thin discs is given in Paper VII. Here we give only the predictions which are based on the further approximations that a. The accelerations perpendicular to the plane are small compared with the radial acceleration. b. The density in the disc is large compared with the density of the galaxy averaged within the galactic radius r ($\rho \gg M/4\pi r^3$). Both approximations are good near the sun up to a height of a few hundred parsecs above the galactic plane. Under these conditions we find that the dynamics are Newtonian but with an effective gravitational constant $G_{eff} = G/\mu(V_\odot^2/r_\odot a_0)$, where $V_\odot$ and $r_\odot$ are the galactic orbital velocity and radius at the sun's position. Thus we predict that when the above approximations are valid:

(i) The distribution of disc "dark matter" will be found to be the same as that of the visible mass.

(ii) The Oort discrepancy factor,which according to MOND is $1/\mu(V_\odot^2/r_\odot a_0)$, is the same as that for the total galactic mass discrepancy within the orbit of the sun.

(iii) The same factor appears (albeit multiplied by an additional parameter of order unity) in the dynamics of open cluster or very wide binaries in the solar neighbourhood (see Paper IV on the asymptotic field of a mass in a constant external field and also Paper VI on an N-body system in an external field).

4.2 Elliptical Galaxies

We work on the premise that galaxies contain no appreciable quantities of dark matter. We should thus be able to understand elliptical galaxies' light distribution and velocity dispersions selfconsistently. Unfortunately our analysis is beset by the same uncertainties which stand in the way of conventional analyses, i.e., those involved in deducing space mass distributions from surface brightnesses and velocity distributions from line-of-sight velocity dispersions. We can follow one of the two avenues which others before us have taken. We can make some assumptions about the stellar distribution function leaving certain parameters which specify it free. One then asks how such model systems look if they obey MOND instead of the Poisson equation, and to what extent they resemble the astronomical systems which they purport to represent.

It has been popular with model makers to describe ellipticals as some variety of isothermal spheres. We have studied in paper V self-gravitating, many-particle, spherical systems with radius independent radial and tangential velocity dispersions, assuming MOND. The following are the major traits of such models which are independent of the values of the parameters which determine their exact structure (velocity dispersion,ratio of radial to tangential dispersion, etc.).

(i) All such spheres have a finite mass (unlike their Newtonian cousins) and their density distribution tends to a power law asymptotically $\rho(r) \rightarrow r^{-\alpha}$, with $\alpha > 3$.

(ii) The surface density constant Σ_0 introduced earlier is an upper limit on the average surface density which such isothermal spheres can have.

(iii) The total mass M of a sphere is approximately proportional to the fourth power of the space velocity dispersion σ_s:

$$(a_0 G)^{-1} \leq M/\sigma_s^4 \leq 2(a_0 G)^{-1}. \quad (3)$$

Alternatively we may simply try to map the test-particle acceleration field of ellipticals and see if it agrees with that calculated from the observed light distribution, with reasonable M/L values, using MOND. In this conection we can make the following predictions:

(i) In ellipticals with test-particle gas discs the rotation curve of the disc will be that which MOND dictates for the observed light distribution.

(ii) Dwarf ellipticals or spheroidals with $\langle\Sigma\rangle \ll \Sigma_0$ will be found to contain large quantities of dark matter when treated Newtonically (see more details in paper II).

(iii)The observed temperature and density distributions in x-ray-emitting envelopes of ellipticals (such as described by Forman et al. 1985) will be those given by MOND (straightforwardly deduced from eq.(2), which is exact for spherical systems, or numerically from eq.(1) in the more general case).

The M/L values one obtains in this way, for individual galaxies from the existing data are rather uncertain. But it is interesting to see if there is a correlation of the Newtonian M/L values with the accelerations. We plot in Figure 4 Forman et al.'s values of M/L against the (Newtonian) accelerations g_N at R_{max}(value of radius where the M/L values are estimated, taken from Forman et al.) We use g_N rather than g because the first depends only on observed quantities and not e.g. on the form of $\mu(x)$ or the value of a_0. We take $g_N = L(M/L)_t G/R_{max}^2$. Here $(M/L)_t$ is an assumed stellar value of M/L.

The values of M/L given by Forman et al. are based on a single temperature of $T = 1$ KeV for all the galaxies. For some, an actual uncertainty range of T is given (in some cases not including 1 KeV).We also plot the range of M/L values which corresponds to the temperature uncertainties.

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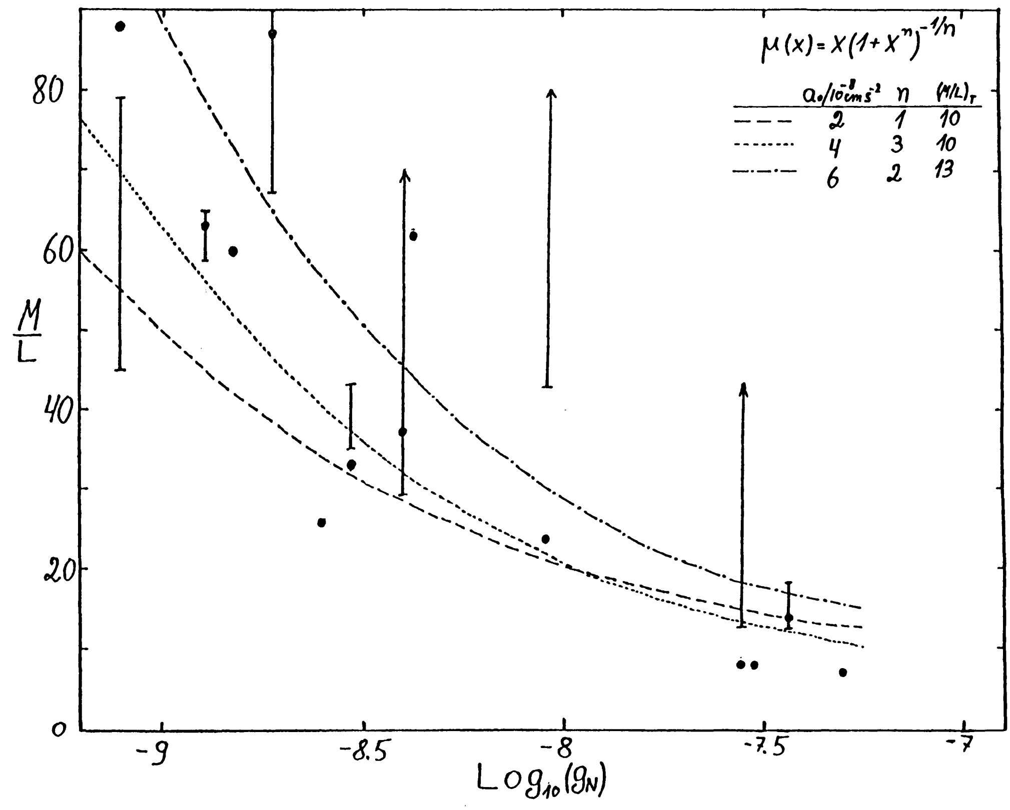

Fig. 4 Newtonian M/L values (as given by Forman et.al.1985) plotted against the Newtonian accelerations for early-type galaxies with x-ray envelopes (•). Vertical bars indicate the range of M/L corresonding to the envelope's temperature uncertainty range. The lines show the predictions of MOND for point-mass galaxies with stellar M/L value $(M/L)_t$ for different forms of $\mu(x)$ and values of a_0.

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DISCUSSION

SCHECHTER: Would you tell us something about wide binaries?

MILGROM: For wide binaries, the acceleration is < a_0; in fact, the acceleration around a 1 $M_\odot$ star becomes equal to a_0 at a fraction of a parsec, ~ 5000 AU radius. On the other hand, these binaries are in the solar neighborhood, so even if you make the separations very large, the size of the mass discrepancy is limited by the acceleration in the Galaxy. So you expect to find a mass discrepancy which is of the order of the Oort limit.

J. BAHCALL: Perhaps Scott Tremaine could say what is required observationally to test the modified dynamics using wide binaries.

TREMAINE: The Milgrom-Bekenstein theory predicts that there should appear to be "dark matter" in wide binary stars in the solar neighborhood. To measure the masses of wide binaries, you would need velocity measurements of ~ 10^2 binaries with a resolution of 0.1 km s^{-1} or better. The mass measurement is subject to most of the problems which plague mass measurements in binary galaxies; in addition, you have to worry about velocity perturbations due to dark companion stars.

OSTRIKER: What happens when we apply your formulation to the Local Group? It should apply, because the motions are non-relativistic and the Local Group is quite isolated. When I looked at this, I had trouble getting the orbits of Andromeda and the Galaxy to come out right in the time available.

MILGROM: So I'll answer you for the fifth time, Jerry, just for the record. What Jerry is saying is that, if we assume that M31 and the Galaxy are on radial orbits toward each other, and if we try to calculate, assuming modified dynamics, how long ago the Galaxy and Andromeda were close together, we find a few × 10^9 years, considerably less than the Hubble time. Jerry worried that this might be destructive to the two galaxies. My standard answer is that if the systems have a tangential velocity of only 50 - 60 km s^{-1}, they would never have gotten closer together than ~ 150 kpc.

OSTRIKER: I just wanted to give you the opportunity to say it to this large audience (laughter).

VAN DER KRUIT: As a specific case of what Vera Rubin illustrated earlier, I would like to mention the two edge-on disk galaxies NGC 7814 and NGC 891. These have comparable distances and angular sizes. Between 1' and 6' radius, both systems have flat rotation curves of 220 km s^{-1} amplitude. Despite this, the light distribution of NGC 7814 consists almost entirely of a centrally concentrated $r^{1/4}$-law spheroid and NGC 891 almost entirely of a much more distended exponential disk. I do not understand how any gravitational law can give identical rotation curves if these light distributions trace mass distributions.

MILGROM: Vera mentioned two apparently odd phenomena: galaxies with similar light distributions but different rotation curves and galaxies like yours with different light distributions but similar rotation curves. The latter is actually not odd at all. You can have a spherical mass and a thin disk with arbitrary spheroid-to-disk mass ratio and with exactly the same rotation curve. This is true in Newtonian mechanics as well as in MOND. The first phenomenon, however, is truly puzzling. If you stick to Newtonian dynamics, you have to say that the two galaxies must have very different halos resulting in different rotation curves. MOND predicts exactly such different rotation curves for galaxies with similar light distributions, as I have explained in connection with the surface density constant Σ_0. You can see this effect clearly in Figure 2 of my paper on galaxies (Paper II of the references).

FELTEN: I think it should be mentioned that a theory like this causes big problems for cosmology.

MILGROM: It is not true at all that a theory like ours has problems with cosmology. What is true is that we cannot derive cosmology using Newtonian arguments, so we have to await a relativistic generalization of MOND. I do not forsee any potential problems.

FABER: Without a theory of relativity, it is not fair to ask you to do cosmology, but to what distances do you think you can apply your formulation? In particular, can you apply it to the Local Supercluster?

MILGROM: The formulation can be applied when the mass contained in the region of overdensity you're considering is large compared with the expected average mass within the same volume. The Local Supercluster is such a case.

FABER: Why is that a good criterion?

MILGROM: Because I can then assume that the acceleration is primarily determined by the mass that is actually there in the region of overdensity. Of course, another criterion is that velocities should be non-relativistic, which is certainly the case in the Local Supercluster.

FABER: So the Virgo Cluster meets your requirements.

MILGROM: Yes. I haven't kept up with the new developments, but when I wrote my first paper I analyzed the then-existing information on Virgocentric infall, and found $M/L \sim 1$.

DAVIS: I don't think that the Virgo Cluster has a big mass overdensity.

MILGROM: Then the corresponding uncertainty in M/L is a factor of order 2. There are other factor-of-two uncertainties also, such as any non-Virgocentric acceleration components due to matter outside Virgo.

WHITE: The large hydrostatic atmospheres in galaxy clusters offer an interesting test of your theory. The x-ray data provide a good lower limit on the observed M(r), namely the observed mass in gas. The accelerations in clusters place them just inside your weak-acceleration regime, and your theory makes a prediction for the velocity dispersion of the galaxies and for the temperature of the gas. Have you made any models for the structure of these systems to check that you can get agreement with observations for the same value of a_0 that you need in galaxies?

MILGROM: No, we haven't looked at this question.

ISHIZAWA: An action principle has been applied to the gravitational model by Bekenstein and Milgrom (1984, Ap. J., **286**, 7). Starting from the action of classical general relativity with a negative cosmological constant Λ in the weak limit of gravitation, we have obtained a system of the Poisson equation and natural boundary conditions. They are completely equivalent to an incompressible, irrotational flow surrounded by a constant-pressure gas. As a natural consequence, our model leads to a two-phase Universe, the gravitational channels in which the gravitational lines of force are confined (filamentary matter fields) and the gravitational vacuum (voids).

VISHNIAC: I wonder if you could comment on the constraints imposed on your theory by laboratory experiments designed to measure the gravitational constant.

MILGROM: A system which is embedded in a strong external acceleration field such as that of the Earth ($g \sim 10^{11}\, a_0$) is very nearly Newtonian. The deviation can be expressed by a slightly larger effective gravitational constant, $G_{eff} \approx G/\mu(10^{11})$. We have no idea how fast $\mu(x)$ approaches unity when its argument becomes large, but there are limits which we can put from the perihelion shift of Mercury and other solar-system measurements (Paper I). These imply that detecting the effect in laboratory experiments is far beyond present capabilities.