(SECOND PAPER)

Introducing the next speaker, the CHAIRMAN said Professor OWEN is a past Principal Scientific Officer at the Royal Aircraft Establishment and he was later Chief Scientist at the Naval Construction Research Establishment, Rosyth Prof Owen has held the John William Hughes Chair of Civil Engineering at Liverpool University since mid-1950, and he is well known in the rotating wing world for his work, which dates back to his term of office at Farnborough over the period 1936-1948

General Principles of the Structural Design of Helicopter Blades*

By J B B OWEN, D SC

Civil Engineering Department, University of Liverpool

Summary

In designing rotor blades their strength on the ground, their torsional stiffness, their aerodynamic shape and smoothness, etc , have all to be considered, but the primary purpose of rotor blades is to sustain helicopters in controlled flight in the air, and one of the most difficult problems arising is that of assessing the bending stresses to which rotor blades are then subjected The present treatment is, therefore, largely concerned with the estimation of the bending stresses to which rotor blades are subject in flight

The loads tending to bend blades are first taken as being identical with those to which an idealised inflexible blade is subjected These loads are analysed along and perpendicular to the blade length, and equations for the bending equilibrium of flexible blades in the lift and drag planes are developed Difficulties associated with the solution of these equations are overcome by the construction of special solutions, called " Type Solutions," which satisfy these equations and the end support conditions of the blade These type solutions give a physical picture of the problem and help in the calculation of blade stresses

The differences between the loads on an idealised inflexible blade and flexible blades are discussed In the lift plane it is demonstrated that they are, in general, small, but the possibility of dynamic amplification of higher harmonics in the loading system is brought out, and it is suggested that there may be a need for further work on this aspect of the problem, particularly for high speed helicopters

GENERAL CONSIDERATIONS

Structures in general are designed to withstand peak loading conditions and are not expected to fail until the predicted factored peak loads are exceeded Structures must also have a reasonable endurance so that fatigue and corrosion failures do not occur in their working life Again they must be stiff enough to ensure that the structure may be used satisfactorily Considerations of this kind are fundamental also in the structural design of helicopter blades

Needs other than those of a purely structural character, such as the necessity to provide a smooth aerodynamic profile and to mass balance the blade, may make it difficult to use the material in the blade to the best structural advantage

In endeavouring to use structural material economically it is desirable to design so that the peak loads and working load fluctuations are kept as low as practicable This objective is often unnecessarily hampered by some design requirements which specify arbitrary factors which are applicable alike to both good and bad designs Thus, the arbitrary specification of a factor of $1 5 \times 2 67$ to cover blades striking the flapping stops when the helicopter is taxying over rough ground, does not encourage the designer to incorporate means of preventing loads of such a magnitude arising Some assumptions based on judgement and experience seem inevitable but it would be better to choose these so that the designer is free to reduce the loads imposed on

* Permission to reproduce Figs 3, 4, 5 8, 9, 10 has been given by the Controller of H M Stationery Office and Fig 11 by "Aircraft Engineering "

the structure when he finds this necessary Thus, in the example quoted, it would be better to specify the gust to be considered and a contour for the ground over which the helicopter might taxi with the blades stationary The designer may then choose to absorb impact loads on the blade and to restrict the maximum loads occurring to some value of his own choosing

The evaluation of the loads to which a rotor blade is subjected requires, in general, considerably more detailed information than, for example, that which is necessary to assess the overall performance of a helicopter The algebra involved can be quite cumbersome, but this may be reduced by careful consideration of the relative magnitudes and importance of the quantities involved A lead on the aerodynamic side of the problem was given by Lock in R & M 1127 (1927)

The predominant force present in flight on a rotor blade is the centrifugal force of intensity $m\Omega^2rdr$ at radius r, where mdr is the blade mass (in slugs) at this point and Ω is the angular velocity of the rotor The components of the centrifugal force along and perpendicular to the blade length, in both the plane of the blade chord and normal to this direction, are of primary importance in considering the bending of a blade The normal and chord-wise components of the centrifugal force are comparable in magnitude with the other force components in these directions

The coning angle β and the drag angle γ are usually small and so too is the blade pitch θ, but small changes in pitch can seriously change the lift loading on a blade

General considerations of the peak and fluctuating loads to which a blade may be subjected would involve a review of design requirements Indeed, it is most important that a reasonably accurate assessment should be made of the maximum rotor speed, of the worse distribution of rotor torque between blades, of the loads due to hitting stops and setting dampers, of the stiffness needed to ensure adequate control and freedom from flutter and vibration, etc Some aspects of these problems have already been considered in the journal , but, since helicopter blades are primarily needed to provide sustentation in flight, it is perhaps most appropriate to deal in the present paper with the bending of blades in flight

Blade Bending in Flight

It is usual to base helicopter performance calculations on the assumption that the rotor blades are inflexible They are generally nowadays assumed to remain straight Account is taken of twisting actions on the blade and it is then possible to estimate the distribution of both aerodynamic and inertia loads on the blade These loads, it will be observed, are evaluated on the assumption that the blades are flexurally rigid, that is, it has been assumed that the actual blades may be replaced by some idealised blades which are rigid in flexure The significance and validity of this assumption will be discussed later but for the present it will be assumed that at any instant these equivalent blades, which will not bend, lie in a position defined by an azimuth angle φ, a coning angle β and a drag angle γ as shown in Figs 1 and 2

Dynamical problems may be reduced to statical problems by utilizing d'Alembert's principle, that is by introducing balancing inertia forces This can be done in the case of the helicopter blade and at any one instant the whole of the loading on the idealised blade, which will not bend, may be analysed into a longitudinal component w_x, an upward transverse component w_z, Fig 1, and an anti-drag transverse component w_y, Fig 2 Of all these components w_x is in general by far the larger and approximately equal to $m\Omega^2X$, since β and γ are small

But the blades of an actual helicopter are not rigid in flexure and they will deflect transversely to their length small amounts y and z and extend to radii $(r+\Delta r)$ Will these deflections, which in general will be small, change the loads to which the blades are subjected? The rate of deflection y and z and their derivatives y and z, will give rise to damping and inertia forces respectively, which will generally be small and comparable with the inaccuracies in the evaluation of w_y and w_z, the transverse load components on the flexurally rigid blade At present it is proposed to omit them But the direction of the large centrifugal force in Fig 2 will be changed by the deflexion y, and although its longitudinal component will be sensibly the same on the rigid as well as on the flexible blade, an anti-drag component of approximately $m\Omega^2rdrxy/r$ will appear when flexibility is introduced and the anti-drag loading on a flexible blade will be $(w_y + m\Omega^2y)\,dX$ Small blade bending deflections will not appreciably alter the lift loading w_z

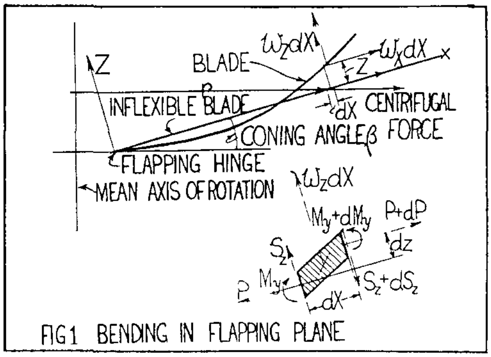

FIG 1 BENDING IN FLAPPING PLANE

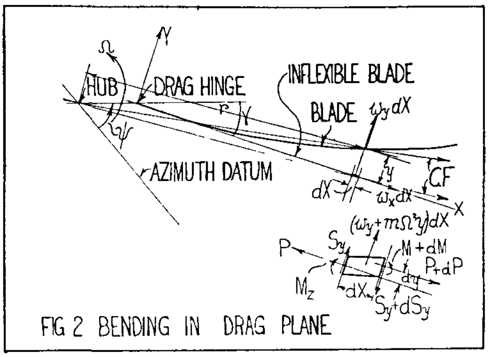

FIG 2 BENDING IN DRAG PLANE

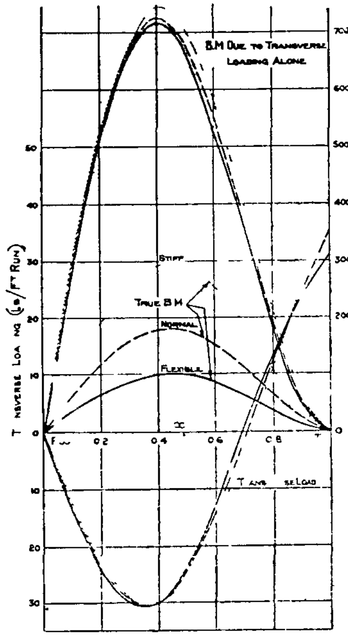

FIG 3.—Effect of blade bending stiffness (Type B)

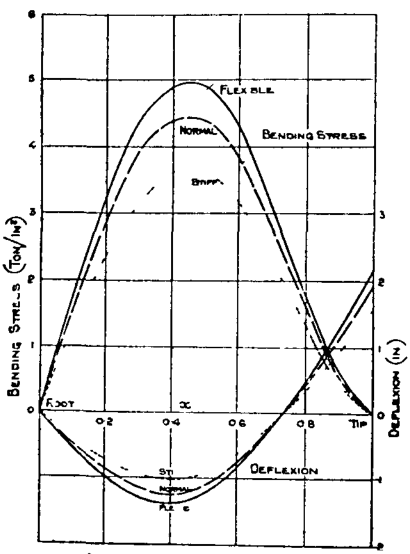

FIG 4 —Effect of blade bending stiffness (Type B)

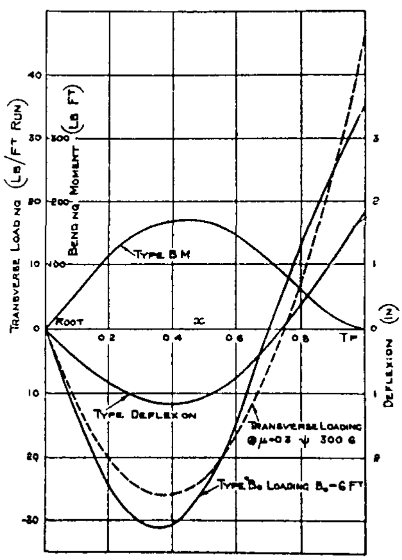

FIG 5 —Type B fitted to the given loading curve

Equations of Equilibrium

Considering now the equilibrium of a small length of blade shown inset in Fig 1 and 2, where M_y and M_z are the bending moments about axes parallel to the y and z axes respectively, and S_y and S_z are the shear forces in these directions, for equilibrium of moments

$$\left.\begin{aligned} dM_y &= S_z dX + P dz \\ \text{and} \quad dM_z &= S_y dX + P dy \end{aligned}\right\} \qquad (1),$$

and for equilibrium of the transverse forces

$$\left.\begin{aligned} dS_z &= w_z dX \\ dS_y &= (w_y + m\Omega^2 y) dX \end{aligned}\right\} \qquad (2)$$

Since the shear vanishes at the tip of the blade, integrating (2) gives

$$S_z = -\int_X^L w_z dX \quad = \text{Shear due to the transverse load } w_z \text{ acting alone,} \quad = d\mu_y/dX$$

and

$$S_y = -\int_X^L w_y dX - \Omega^2 \int_X^L m y dX \quad = \text{Shear due to the transverse load } w_y \text{ acting alone minus } \Omega^2 \text{ times the moment of the blade outboard of X about the X axis}$$

$$= d\mu_z/dX - \Omega^2 \int_X^L m y dX$$

Dividing (1) by dX and substituting from the above gives

$$dM_y/dX = d\mu_y/dX + P dz/dX \qquad (3),$$

and

$$dM_z/dX = d\mu_z/dX + P dy/dX - \Omega^2 \int_X^L m y dX \qquad (4)$$

Integrating these equations gives, since the bending moment vanishes at the tip of the blade,

$$M_y = \mu_y - \int_X^L P(dz/dX) dX \qquad (5),$$

and

$$M_z = \mu_z - \int_X^L P(dy/dX) dX + \Omega^2 \int_X^L \int_X^L m y dX dX \qquad (6)$$

In these equations the bending moments μ_y and μ_z, as well as their differentials $d\mu_y/dX$ and $d\mu_z/dX$ are directly calculable from the transverse loads w_z and w_y The bending moments μ_y and μ_z ate those due to the transverse loads, w_z and w_y, acting alone, and are the bending moments to which an idealised straight and inflexible blade is subjected P is the total centrifugal pull at a distance X from the root and is approximately

$$\int_X^L m\Omega^2 X dX$$

The bending moments M_y and M_z are the bending moments to which the blade is actually subjected From equations (5) and (6) it will be observed that while the integral containing the extensional force P remains positive, the effect of the extensional centrifugal pull is to reduce the bending moment due to the transverse loads acting alone In equation (6) the action is further modified because the deflection of the blade results in the centrifugal pull having a component transverse to the blade length

Solutions of the Equations of Equilibrium

In general the equations of equilibrium (5) and (6) are non-linear Thus, for example, when the blades are of uniform mass, m, and stiffness, EI, along their length L, $P = \frac{1}{2}m\Omega^2(L^2 - X^2)$ and equation (3) becomes on putting $M_y = EI d^2z/dX^2$ which assumes that the bending is about a principal axis of the cross section,

$$d^3z/dX^3 - (m\Omega^2/2EI)(L^2 - X^2) dz/dX = d\mu_y/dX \qquad (7)$$

Orthodox series solutions of this equation converge only slowly but there is now available a variety of ways in which to deal with it Probably one of the quickest methods of dealing with one particular given loading condition is to divide the blade up into a number of lengths over which the extensional pull P may be considered as roughly constant, that is over which a mean value of $(L^2 - X^2)$ is taken in (7) Then only a set of linear equations has to be dealt with

But , a better physical grasp of how bending actions are modified, as well as a more rapid method of assessing the effect of a large number of various loading conditions, is obtained by constructing special solutions which satisfy the blade end conditions and the equations of equilibrium Such solutions have been called " type solutions " in R & M 1875 (1939) For a helicopter blade they are such that at the tip of the blade the bending moment and the shear vanishes At the root of the blade the type of design will decide what conditions are appropriate If a frictionless flapping hinge is provided then the bending moment about this hinge will vanish at the root If a damper is provided on the drag hinge, appropriate moments will need to be reproduced by the " type solutions " In practice the transverse loading actions towards the blade root are small and it may be advantageous to choose type solutions so that the transverse loading towards the root is small, say zero

In constructing " type solutions " instead of proceeding from a given transverse loading and finding its effect, one chooses a deflected form for the blade and then it is quite easy to find the transverse loading which gives rise to this deflected form Proceeding from the deflected form to the transverse loading associated therewith, is a comparatively easy process, but the reverse process, of proceeding from a given transverse loading to find the deflected form it causes, is more difficult

If a few appropriate forms for the deflection of the blade are chosen then in practice it is often found that the combination of the transverse loads needed to produce these forms can be made to approximate to a given transverse loading It may be convenient sometimes to work with the bending moment due to the transverse loading acting alone, *i e* , the bending moment on an inflexible blade, rather than the transverse load itself

Analytically what is being done is to choose, instead of an ordinary power series, selected groups of terms of a power series The selection of the groups of terms is such that the end conditions of the blade are satisfied Thus one group of terms, *i e* , one " Type Solution," is a solution which is correct at the ends of the blade and if we so choose may be made correct at any other one point A combination of two groups of terms, that is two " Type Solutions," gives a solution which is correct at the ends of the blade and at two points if we so choose A combination of three groups of terms can ensure that conditions are correct at the ends and at three points and so on Usually such combinations are correct at more than the chosen points and a few appropriate " Type Solutions " combined may then give a sufficiently close approximation to an estimated transverse loading action

Construction of Particular " Type Solutions "

In the case of a uniform blade, bent about principal axes, the construction of particular solutions, which satisfy end conditions, is facilitated if equations (3) and (4) are differentiated and by writing $x = X/L$ and $k = m\Omega^2 L^4/2EI$, a constant for a particular condition since Ω, the angular velocity of the rotor is reasonably constant Then

$$w_z = (EI/L^4)\left[D^4 - k\left\{(1 - x^2)D^2 - 2xD\right\}\right]z \qquad (8),$$

and

$$w_y = (EI/L^4)\left[D^4 - k\left\{(1 - x^2)D^2 - 2xD + 2\right\}\right]y \qquad (9),$$

where D stands for the operator d/dx
In these equations w_y and w_z are the transverse loads on an inflexible blade, which if applied to a blade of flexibility k will give rise to deflections y and z

It will be observed that if we choose particular values, say (Y_1 and Z_1) of y and z, that these will give rise to transverse loads $(w_z)_1$ and $(w_y)_1$ which are directly obtainable from (8) and (9), provided we can obtain the derivatives of Y_1 and Z_1 Similarly, deflections Y_2 and Z_2 will give rise to transverse loads $(w_z)_2$ and $(w_y)_2$ It follows from equation (8) and (9) that the deflection $(Y_1 + Y_2)$ and $(Z_1 + Z_2)$ will be caused by transverse loads $(w_z)_1 + (w_z)_2$ and $(w_z)_1 + (w_y)_2$ respectively, *i e* , the Principle of Superposition is applicable

Returning now to equations (8) and (9) if the transverse loading is integrated the result is the shear force due to the transverse loading acting alone The integrals give respectively the shear in the Z direction as

$$(EI/L^3)\left[\left\{D^3 - k(1 - x^2)D\right\}z + c\right] \qquad (10),$$

and the shear in the Y direction on an inflexible blade, as

$$(EI/L^3)\left[\left\{D^3 - k(1 - x^2)D\right\}y - 2k\int^x ydx + d\right] \qquad (11),$$

In these expressions c and d are constants of integration and no further constant is introduced in evaluating the integral shown

Integrating again gives the bending moments on an inflexible blade, in the " lift " and " drag " planes respectively as

$$\mu_y = (EI/L^2)\left[(D^2 - k)z + k\int^x x^2Dzdx + cx + e\right] \qquad (12),$$

and

$$\mu_z = (EI/L^2)\left[(D^2 - k)y + k\int^x x^2Dydx - 2k\int^x\int^x y(dx)^2 + xd + f\right] \qquad (13)$$

In these equations e and f are again constants of integration These equations have been developed in order to derive the conditions which y and z must satisfy if they are to conform to the end conditions to which the blade is subject

Thus, considering first flapping motion about a free root hinge, the chosen values of z should be such that they will give

(*a*) zero transverse loading w_z at the root, *i e*, from (8) at x = 0

$$\left[D^4 - kD^2\right]z = 0 \qquad (14),$$

(*b*) zero shear due to transverse loading at the tip which from (10) gives at x = 1

$$D^3z + c = 0 \qquad (15),$$

(*c*) zero bending moment at the root and the tip of the blade for both the idealised inflexible and the flexible blade, *i e*, at x = 0 and 1 for the flexible blade since the bending moment is identical with

$$(EI/L^2)D^2z, \quad D^2z = 0 \qquad (16),$$

and for the inflexible blade from (12) using (16) at x = 0

$$k\int^0 x^2Dzdx + e = 0$$

and at x = 1

$$-kz + k\int^1 x^2Dzdx + c + e = 0 \qquad (17),$$

Using (16), (14) also reduces to

$$D^4z = 0 \text{ at } x = 0 \qquad (18),$$

Similarly considering the drag hinge as providing a moment M, the deflection y must be chosen so that there is

(*a*) zero transverse loading w_y at the root, which gives from (9)

$$\left[D^4 - kD^2\right]y = 0 \qquad (19),$$

(*b*) zero shear at the tip, *i e*, from (11)

$$D^3y - 2k\int^1 ydx + d = 0 \qquad (20),$$

(*c*) zero bending moment in both the flexible and the inflexible forms at the blade tip, *i e*, for the flexible blade at x = 1

$$D^2y = 0 \qquad (21),$$

and from (13) for the inflexible blade using (21) at x = 1

$$-ky + k\int^1 x^2Dydx - 2k\int^1\int^x y(dx)^2 + d + f = 0 \qquad (22),$$

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d) a moment M at the blade root in the flexible and inflexible forms, *i e* at $x = 0$ $D^2y = ML^2/EI$ (23), and from (13) using (23)

$$k\int^{0} x^2 Dy dx - 2k\int^{0}\int^{x} y(dx)^2 + f = 0 \qquad (24),$$

(*e*) the same rotor torque for both flexible and inflexible blades

Since (i) the extensional pull is for both flexible and inflexible blades taken as equal in magnitude and along the direction of the X axis, (ii) the root moment is also the same at the value M, then (iii) the only remaining way for transmitting torque is by root shear which must be the same for both flexible and inflexible blades The shear is the integral of the transverse loads Deflexion alters the transverse loads an amount $m\Omega^2 y$ Then since the tip shear is zero for there to be no change due to flexibility in the root shear

$$\int_0^1 m\Omega\ y dx$$

or since m and Ω are constant,

$$\int_0^1 y dx = 0 \qquad (25)$$

In creating a " type solution " for bending in the flapping direction it is then necessary to satisfy conditions (14) to (18) and in creating a " type solution " for bending in the drag direction to satisfy conditions (19) to (25)

Construction of Type Solutions for Bending in the Flapping Plane

In R & M 1875 (1939) several solutions are developed which satisfy the equations of elastic equilibrium for bending in the flapping plane and also the end conditions Here it is proposed to consider only one of them, namely

$$D^2 z = B_0(x - 2x^3 + x^5) = B_0\, \upsilon_0(x) \qquad (26)$$

which is the first term of the series

$$D^2 z = (x - 2x^3 + x^5)(B_0 + B_2x^2 + B_3x^3 + \quad + B_n x^n + \quad) \qquad (27)$$

Bending of the amount given by (26) is caused by a transverse loading which on an inflexible blade will give rise to a bending moment

$$B_0(EI/L^2)\Big[\{x - 2x^3 + x^5\} + k\Big\{-\frac{x^3}{6} + \frac{x^5}{5} - \frac{2}{3}x^7 - \frac{1}{54}x^9 + \frac{41}{1890}(3x - x^3)\Big\}\Big]$$
$$= B_0(EI/L^2)\,[\upsilon_0(x) + k\Upsilon_0(x)] \qquad (28)$$

The values of $\upsilon_0(x)$ and $\Upsilon_0(x)$, which are the polynomials in (26) and (28), are tabulated in Table 4 of the R & M 1875 and from this table Fig 3 has been plotted The upper curves for bending moment in this figure show values of bending moment on a rigid blade which are approximately equal and which, when account is taken of blade flexibility, are reduced by centrifugal force to the lower set of bending moment curves These curves correspond with flexibility coefficient $k = m\Omega^2L^4/2EI$, values of 27 5 (stiff), 55 (normal), 110 (flexible) From this figure it will be observed that the centrifugal force has a marked effect in reducing the bending moment which one would estimate if the presence of its longitudinal component were ignored The most flexible blade is the one which is most changed and in which the bending moment is most reduced It does not, however, follow that the most flexible blade is the blade which is subjected to the least bending stress, other conditions being unaltered Fig 4 shows the bending stresses estimated from Fig 3 on the assumption that the modulus of section of the blade is proportional to the blade's stiffness From this figure it will be observed that the least stressed blade is the stiffest blade These stresses have, however, been evaluated on the assumption that the blade weight and the rotor aerodynamic characteristics are the same for all three blades In practice a reduction in blade stiffness might well be accompanied by a reduction in blade weight and a change in the aerodynamic loading on the blade The illustrations for Figs 3 and 4 must not therefore be used beyond their range of applicability The Type

Solution (26) and (28) may, however, be used to obtain the first approximation to the true bending moment in a blade and hence the stresses to which it is subjected when the magnitude of the transverse load component has been estimated. Thus if the transverse loading shown by the broken curve of Fig 5 is given, then taking an amount $B_0 = 6$ ft of the type solution (26) gives the transverse loading curve shown by a full line in this figure. The bending moment corresponding with this amount of the type solution is also shown in the figure and this is a first approximation to the bending moment caused by the actual loading

Construction of Type Solutions for Bending in the Drag Plane

In the same manner as in R & M 1875 where type solutions were constructed for bending in the flapping plane, it is possible to construct solutions which satisfy the equation of equilibrium for bending in the drag direction and also the end conditions (19) to (25)

If a polynomial of the form

$$y = a_1x + a_2x^2 + a_3x^3 + \qquad (29)$$

is chosen for the deflected shape of a blade, and it is also assumed arbitrarily that $D^3y = 0$ at the blade tip the end conditions reduce to

$$\left.\begin{aligned} &[(D^4 - kD^2)y]_0 = (D^3y)_1 = (D^2y)_1 = f = \int^1 ydx = d = 0 \\ &(D^2y)_0 = (L^2/EI)M \\ &-y_1 + \int^1 x^2Dydx - 2\int^1\int^x y(dx)^2 = 0 \end{aligned}\right\} \qquad (30)$$

where the suffix indicates the appropriate value of x

It is now not difficult to show that the expression

$$D^2y = (L^2/2EI)M\left[(2 - 3x + x^3) + \frac{k}{2}(- x + 2x - x^3)\right] \qquad (31)$$

for the bent form of the blade can satisfy the above conditions (30) and the equation of equilibrium (13). These are satisfied if the bending moment on the rigid blade from which (31) is derived is given by

$$\mu_z = \frac{M}{2}\left[(2 - 3x + x^3) + \frac{k}{30}(- 2x + 10x^4 - 9x^5 + x^7) + \frac{k^2}{60}(- x + 5x^3 - 5x^4 - x^5 + 3x^6 - x^7)\right] \qquad (32),$$

where M is the moment at the blade root. In Fig 6 the bending moment distribution corresponding with these expressions is shown when the flexibility coefficient k has values 30, the stiffer, and 60

Although the damper may be present on the drag hinge the rotor hinge moment can still vanish because the blade may bave moved to such a position that the inertia loads have a moment about the drag hinge which exactly equals the moments due to the aerodynamic loads. In this case the end conditions reduce to

$$\left.\begin{aligned} &(D^4y)_0 = (D^3y)_1 = (D^2y)_0 = (D^2y)_1 = \int^1 ydx = f = d = 0 \\ &-y_1 + \int^1 x^2Dydx - 2\int^1\int^x y(dx)^2 = 0 \end{aligned}\right\} \qquad (33),$$

A first Type Solution which satisfies these equations is

$$D^2y = (x - 2x^3 + x^5) \qquad (34)$$

and gives a possible curvature of a flexible blade. The corresponding bending moment on an inflexible blade is

$$\mu_z = \frac{EI}{L^2}\left[(x - 2x^3 + x^5) + \frac{k}{840}(47x - 140x^3 + 154x^5 - 76x^7 + 15x^9)\right] \qquad (35)$$

These two expressions with $k = 30$ and 60 are plotted in Fig 7. The effect of centrifugal force is to pull out each of the μ_z curves shown to the same bending moment curve. The reduction in bending moment is greatest for the most flexible blade

Since any given drag bending action may be analysed into (*a*) one action producing a given hinge moment and (*b*) actions which produce zero hinge moments, it follows that the solution (26) together with a suitable number of solutions of which (34) is but one may be combined together to give the solution corresponding to any given drag loading. This device should facilitate the routine use of type solutions of the above form.

THE NATURE OF THE TRANSVERSE LOADING SYSTEMS ON BLADES

In choosing the value of the transverse loads such that they vanished at the root, some knowledge of transverse load distribution was presupposed. At this stage, it might be desirable to review their nature. Figs 8 and 9 which have been taken from R & M 1875 show the distribution of transverse load obtained for the C 30 A autogiro. The component shown is in the flapping plane, *i e*, in the notation of the present paper w_z.

Considering a point about 0 4 of the blade length from the root it will be observed that as the blade rotates from the downwind position where $\psi = 0$ the trough in the transverse loading decreases until the loading reaches a positive maximum just after the blade has passed the position where it is square on to the wind. Thereafter the load falls and reaches its maximum negative value at about $\psi = 300$ 6 degrees. At the blade tip the loading is of opposite sign to that discussed above, with this difference the variation is similar in kind to that previously described and the change in sign is consistent with the condition that the load systems have a moment about the blade root which is hinged for flapping motion. From Fig 7 and the nature of the Type Solutions which have been constructed it will be apparent that the greatest convexity of blade bending viewed from below will occur when $\psi = 300$ 6, *i e*, the blade is about 60 degrees short of the downwind position. The greatest concavity, which will be very slight compared with the greatest convexity, will occur when $\psi = 120$ degrees.

Load systems in the region of $\psi = 300$ and $\psi = 120$ will be those causing the most severe bending in forward flight and for other values of the tip speed ratio only such extreme curves are shown in Fig 9. In plotting these curves the blade position giving the maximum load at 0 4 of the span from the root has been taken as representing the value of the worst load distribution. From Fig 9 it will be observed that the transverse loading actions become more severe as the tip speed ratio is increased, *i e*, at the greatest forward speeds.

Perhaps of greater physical significance is the continual change which is taking place in the transverse loading systems. This indicates that the problem of the bending of a blade in forward flight is one of the response of an elastic beam to cyclically varying loads. The response of the blade will depend on the natural frequency of the blade and the damping present.

This problem has been dealt with in part by some writers by analysing the blade loads into their harmonic components, *i e*, by writing the transverse load w_y or w_z in the form

$$\begin{aligned} w_z = w_o &+ w_1 \sin \Omega t + w_2 \sin 2\Omega t + \\ &+ \bar{w}_1 \cos \Omega t + \bar{w}_2 \cos 2\Omega t + \end{aligned} \qquad (36),$$

where w_o, w_1, $\bar{w}_1$, etc, are constants for any one particular value of x and Ω is the constant angular velocity of the rotor. Then since these loads are repeated cyclically the deflection they cause will be of a similar character and may be written as

$$\begin{aligned} z &= z_1 + z_1 \sin \Omega t + z_2 \sin 2\Omega t + \\ &+ \bar{z}_1 \cos \Omega t + \bar{z}_2 \cos 2\Omega t + \end{aligned} \qquad (37),$$

where again the z_1, $\bar{z}_1$, etc, terms are constants for any one particular point on a blade. This approach is equivalent to modifying the rigid blade transverse loads by amounts $-m\partial^2 z/\partial t^2$ to

$$\begin{aligned} w_z &= w_o + (w_1 + m\Omega^2 z_1) \sin \Omega t + (w_2 + m\Omega^2 z_2) \sin 2\Omega t + \\ &+ (\bar{w}_1 + m\Omega^2 z_1) \cos \Omega t + \end{aligned} \qquad (38)$$

Substituting from (37) and (38) in equations (8) and equating constant and the corresponding harmonic terms gives, when terms up to the second harmonic only are considered, five equations of the type (8), with an additional Z term on the right hand side. The result of this approach is evidently a considerable increase in the work involved and, in view of the inaccuracy usually present in the estimation of transverse loads, it does not appear to the writer to be justifiable. In estimating the curves shown in Fig 8 and 9 no account was taken of tip losses or stalling and the induced

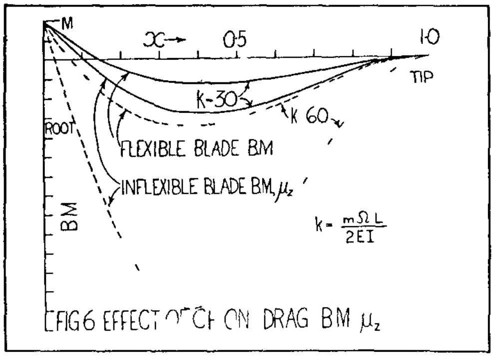

FIG 6 EFFECT OF CF ON DRAG BM μ_z

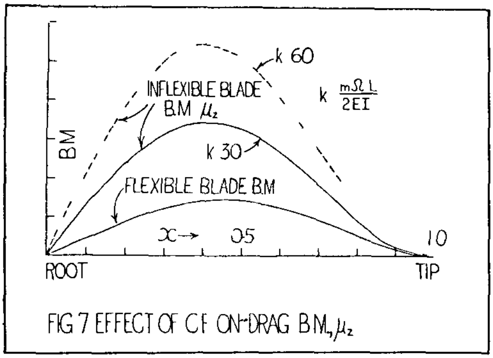

FIG 7 EFFECT OF CF ON DRAG BM, μ_z

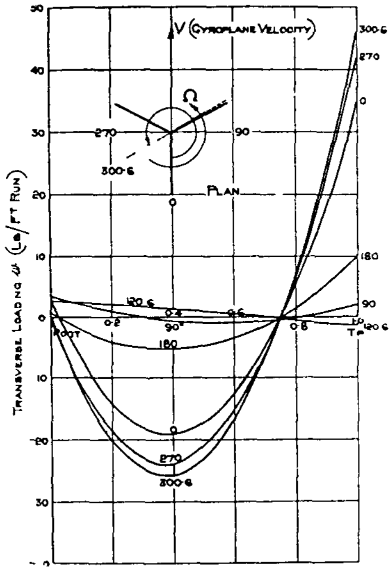

FIG 8 —Transverse load distribution along a C 30A blade for various angular positions from downwind at a tip speed ratio $\mu = 0\ 3$

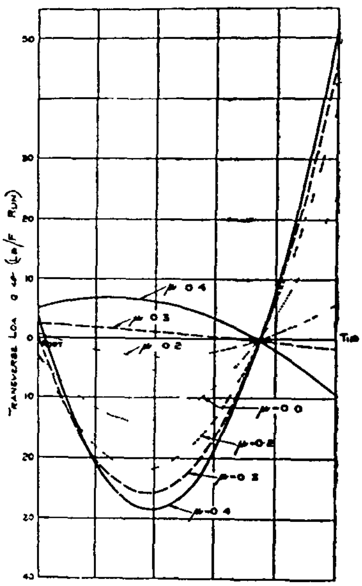

FIG 9 —Most severe transverse loading in steady flight μ = The tip speed ratio

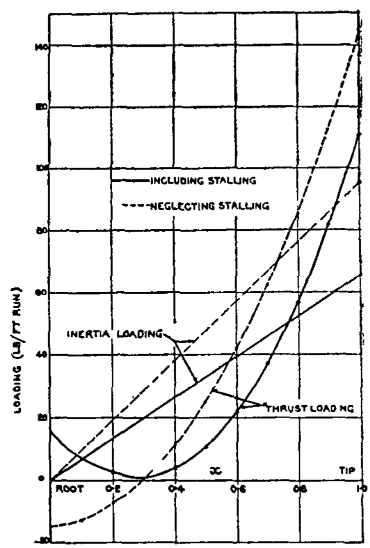

FIG 10 —Gyroplane blade thrust and inertia loading just after a 25 f p s up gust at $\psi = 270$ $\mu = 0\ 3$

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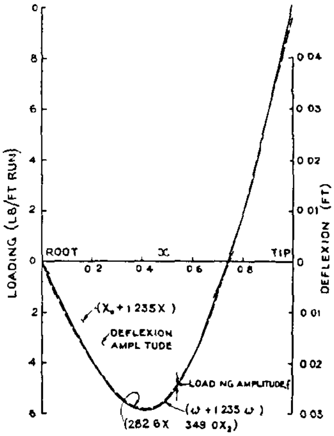

Fig 11 —Fundamental frequency 647 vibrations per minute

velocity was assumed constant over the rotor disc Again first harmonics only in the flapping angles were included and although these give rise to second and third harmonics in the transverse loading system the conditions for zero moment about the flapping hinge of the second and third harmonics were not satisfied, and only first harmonic terms in the loading system are included in Figs 8 and 9 In further illustration of the approximate nature of these loading curves, the effect of stalling is shown in Fig 10 which has been reproduced from R & M 1878 It is thus apparent that no great accuracy is usually achieved in the estimation of the transverse loads to which even an inflexible blade might be subjected

The bending deflexion arising in the case of the C 30 blades at 0 4 radius varies between zero and minus one inch and at the blade tip between zero and two inches If the whole of this bending deflection is at the frequency of rotation Ω, then at most the value of the inertia load due to bending of these amplitudes is, for the C 30, 1 5 and 3 lb /ft run, quantities which are of the order at least of the inaccuracies in the estimated loads on an idealized blade which is rigid in bending Again if the second harmonic terms, following Lock, are expected to be of the order of μ times the first harmonic terms then the inertia loads expected at the tip speed ratio of 0 3 are 0 3 × 4 — 1 2 times the above values, quantities which are still small compared with inaccurately estimated transverse loads on an inflexible blade which rise to the order of 30 lb /ft run in the regions considered

Harmonic analyses of the above types of equations (36), (37) and (38) have usually stopped at the second harmonic Had they proceeded to the third harmonic for the C 30 autogiro at least, somewhat disturbing results might well have been obtained and a deficiency in the procedure might have become apparent Proceeding on the lines of the above argument a third harmonic component of peak magnitude of about 1 2 and 2 4 lb /ft run at 0 4 radius and the blade tip would be expected This is probably an over estimate of the loading as the third harmonic of the aerodyanmic loads due to the first harmonic component of the flapping motion is only about 1 lb /ft run It is evident, however, that third harmonic loads of the order of a pound or so per foot run are to be expected at high forward speed But from Fig 11 in which the shaded area shows the amplitude of a distributed load applied at a frequency of about three times rotor frequency it will be observed that considerable deflection is produced Resonant conditions have been approached, but the analysis from which this figure was drawn, like equations (38), is such that no damping terms have been included Inclusion of inertia terms and omission of damping terms may then give a false impression of the importance of higher harmonic components and it seems desirable to consider the relative magnitude of inertia and damping terms

From (37) the velocity of elastic deflection is

$$\frac{\partial z}{\partial t} = \dot{z} = z_1\Omega \cos \Omega t + z_2 2\Omega \cos 2\Omega t \text{ —}$$
$$\text{— } z_1\Omega \sin \Omega t \text{ — } z_2 2\Omega \sin 2\Omega t \text{ —} \qquad (39),$$

the general term of which gives a velocity amplitude of $z_n n\Omega$ The maximum amplitude of the aerodynamic damping force brought into play is then in the usual notation

$$\tfrac{1}{2}\rho(\Omega r + V \cos i \sin \psi)^2 ca\dot{z}/(\Omega r + V \cos i \sin \psi)$$
$$= \tfrac{1}{2}\rho ac\Omega^2 Rnz_n(x + \mu \sin \psi) \qquad (40),$$

for the nth term This amplitude of the damping force may be compared with $m(n\Omega)^2 z_n$ the amplitude of the inertia forces associated with elastic deflection The ratio of the amplitude of the damping and inertia loads is then

$$\tfrac{1}{2}\rho acR(x + \mu \sin \psi)/mn = 1\ 9(x + \mu \sin \psi)/n \qquad (41),$$

for the C 30 autogiro The bracket has extreme values $x \pm \mu$ The damping is always positive when x is greater than μ, i e, at fractional distances x from the root greater than the ratio of the forward speed of the machine (parallel to the plane of the rotor disc) to the blade tip speed Considering motion in the first harmonic only, i e, $n = 1$, from (41) it is evident that the damping term may rise to 1 9 (0 4 + 0 3) = 1 3 times the inertia loading associated with bending at a radius of 0 4 and a tip speed ratio of 0 3 At the blade tip the damping loading amplitude rises to 2 5 times the inertia loading Considering third harmonic components however their relative magnitude falls to 0 44 and 0 82 at the 0 4 point and the blade tip respectively

Over considerable portions of the rotor disc damping loads will then be of the same order as the inertia loads , but, particularly at high speed in the root region on a retreating blade the damping values may become negative

Aerodynamic damping of drag bending deflection is negligible and when estimations of the resultant rigid blade loads w_y in drag are accurate there may be some justification for introducing inertia load corrections due to elastic deflections If these become appreciable, however, it is possible that the drag damper will come into play and then elastic deflections will need to be considered in the equations of blade motion about the drag hinge

When higher harmonic distortion is being investigated there is, however, no *justification for introducing elastic inertia terms without also introducing aerodynamic* damping terms in considering blade bending in the flapping plane The region in which damping of motion in the flapping plane is least is that region in which the blade is moving downwind and here the air flow over the blade may be from trailing to leading edge This region increases with the forward speed of the helicopter In this region the aerodynamic forces may not be simply related to blade incidence and possibly step by step calculations are desirable to establish whether appreciable higher harmonic distortion is likely

CONCLUSION

From the preceding illustrations and arguments it will be appreciated that even in steady forward flight rotor blades are subjected to fluctuating stresses These tend to cause fatigue Now the fatigue life of almost identical structures is somewhat variable and affected considerably by incidental imperfections Considerable accuracy in the estimation of the fluctuating stresses is then hardly necessary Transverse load calculations need be no more elaborate than those arising in estimating the loads on an inflexible blade, using simple assumptions Stresses may then be estimated by fitting a simple type solution approximately to represent the essential features of the estimated load system This procedure breaks down if there is appreciable dynamic response to higher harmonic components in the transverse loading system More work on this problem may be necessary Step by step calculations of the motion resulting after a blade has been given an arbitrary deflection in its primary mode may indicate whether the problem is of practical significance If this motion is rapidly damped out, then the exact calculation of the magnitude of the higher harmonics of the loading on a flexible blade, which will be a tedious process, would not seem physically essential

Fatigue conditions may not be those which are of the greatest significance in designing rotor blades Peak load conditions associated perhaps with a jerky start of the rotor or with taxying over rough ground with the rotor stopped may be more critical In these cases, while the design might be criticised, a rough estimate only of the blade stresses in forward flight is essential

It might be that peak loading conditions in forward flight, combined with a sudden up-gust, may be critical for a rotor blade Then more accurate calculation of blade loads would seem desirable, as well as a more accurate assessment of the stresses they cause by combining together several type solutions or otherwise

If uncertainties in the maximum load to which a rotor blade may be subjected in forward flight force one to assume arbitrarily, for example, that the blade stalls all along its length, then great accuracy in stressing is hardly justifiable One type solution may indicate the order of the stresses with sufficient accuracy

DISCUSSION

Opening the discussion on the two papers given at the morning session, **Mr R Hafner** (*Member—Bristol Aeroplane Co*) said that he had listened to both papers with great interest