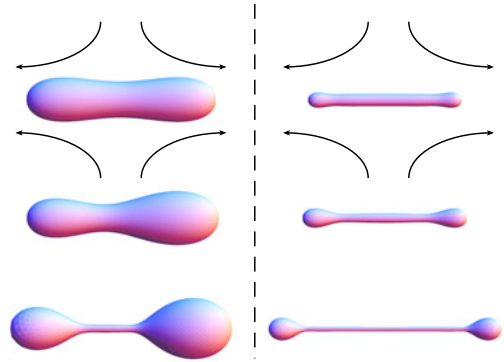


Asymmetric shapes and pearling of a stretched vesicle

Petia M. Vlahovska†

School of Engineering, Brown University,
Providence, RI 02906, USA



Closed bilayer membranes (vesicles) display a plethora of non-spherical shapes under equilibrium conditions, unlike drops and bubbles which are kept spherical by surface tension. Even more complex behaviour arises under applied flow. Intriguingly, a vesicle can adopt asymmetric shapes even under symmetric forcing such as uniaxial extensional flow. Narasimhan, Spann & Shaqfeh (*J. Fluid Mech.*, vol. 750, 2014, pp. 144–190) explain the mechanism of this peculiar instability and trace its origin to the tension which develops in the area-incompressible membrane in response to the applied stress. The authors also show that this mechanism is relevant to the pearling of tubular vesicles. This study raises many questions, e.g. whether other soft particles with load-dependent tension such as capsules can undergo similar shape transformations.

Key words: flow, instability, vesicle

1. Introduction

The biological cell is, in essence, a lipid bilayer membrane encapsulating the cellular content. Giant vesicles made of lipid membrane serve as simple cell mimics, especially for the red blood cell (Vlahovska, Podgorski & Misbah 2009; Li, Vlahovska & Karniadakis 2012). In simple shear, a vesicle deforms into an ellipsoid which can either tank-tread, keeping the orientation steady, or tumble. In Poiseuille flow, vesicles typically migrate towards the flow centreline where they assume axisymmetric bullet- or parachute-like shapes (Couplier *et al.* 2012).

Linear and quadratic shear flows have received most attention because of their relevance to the blood microcirculation. Only recently have strain-dominated flows such as the extensional flow begun to be explored. Experiments by Kantsler, Segre & Steinberg (2008) revealed that a stretched tubular vesicle can transform into a sequence of beads (‘pearls’) connected with thin tethers, see figure 1(a), similarly

† Email address for correspondence: petia_vlahovska@brown.edu

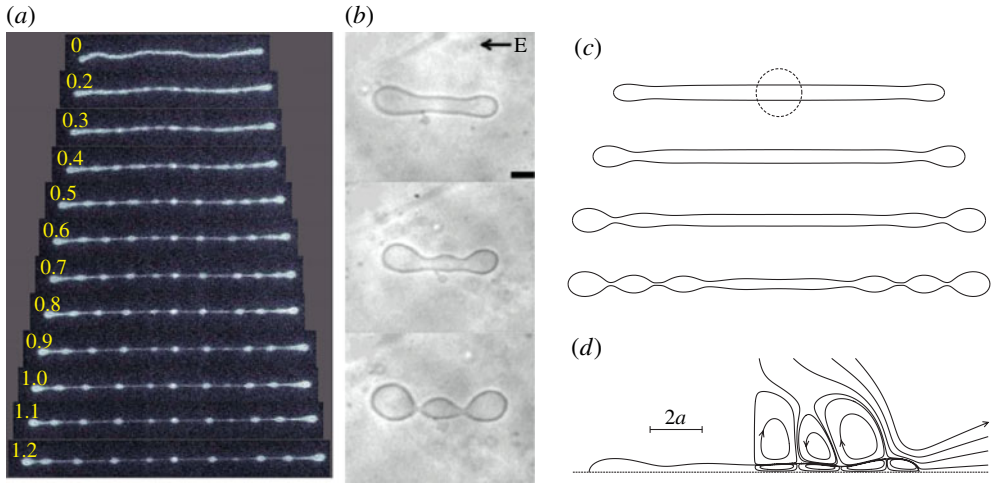


FIGURE 1. (a) Stretching of a tubular (high aspect ratio) vesicle in hyperbolic flow (Kantsler *et al.* 2008); scale bar is $10\ \mu\text{m}$, numbers are accumulated strain, $t\dot{\epsilon}$. (b) Pearling of a tubular vesicle in a uniform AC electric field; scale bar is $20\ \mu\text{m}$, times 0 s (top) 35 s (middle) 50 s (bottom). The field amplitude is $E = 25\ \text{V cm}^{-1}$ and the frequency is 60 kHz (from Sinha, Gadkari & Thaokar 2013: used with permission). (c) Drop ‘pearling’ in a uniform DC field. (d) Sketch of the flow streamlines around a stable four-lobed drop (from Lac & Homsy 2007: used with permission).

to the classic Rayleigh–Plateau instability of a fluid cylinder (uniform electric field also drives similar transformation, see figure 1*b*). Numerical simulations by Zhao & Shaqfeh (2013) discovered that a vesicle can transition to pear-like shapes. Narsimhan, Spann & Shaqfeh (2014) explain the mechanism of this counterintuitive symmetry breaking and provide a comprehensive analysis of vesicle instabilities in straining flows.

Vesicles are fascinating particles: even in the absence of external fields, vesicles display a spectrum of equilibrium non-spherical shapes, unlike drops and bubbles whose equilibrium shape is a (plain) sphere (Seifert 1997). The diversity of shapes arises from the unique mechanics of the bilayer. The molecularly-thin membrane resists bending but behaves as a two-dimensional incompressible fluid. The mechanical equilibrium is described by a generalized Laplace’s equation, which for the simplest membrane model (due to Helfrich) is

$$\Delta p = 2\sigma H - \kappa (4H^3 - 4HK_G + 2\nabla_s^2 H) \quad (1.1)$$

where σ is the membrane tension, κ is the bending modulus, and H and K_G are the mean and Gaussian curvatures. Equation (1.1) is a nonlinear equation for the surface profile and yields a plethora of solutions as a function of the vesicle deflation, which characterizes the departure of the vesicle shape from a sphere; it is quantified by the reduced volume $v = V/((4\pi/3)a^3)$, where the vesicle equivalent radius is defined as $a = \sqrt{A/4\pi}$. As an example, the equilibrium shapes predicted by (1.1) for vesicles with an intermediate aspect ratio e (defined as the ratio of the long and short axes) possess both axial and top-down mirror symmetry: prolate-dumbbell for $v > 0.652$ ($e \lesssim 4$), and oblate-discocyte (the red blood cell shape) for $0.591 < v \leq 0.651$.

In addition to the non-spherical equilibrium shape, a fundamental physical difference between drops and vesicles is the tension. The tension of the incompressible bilayer adapts itself to the forces exerted on the membrane in order to keep the local and total area constant. Hence, under flow the membrane tension varies with forcing. In contrast, the surface tension of surfactant-free fluid surfaces is a constant material property and the surface area is allowed to change. The unique interfacial mechanics of vesicles, in particular their non-spherical rest shape, fixed area, and forcing-dependent tension, give rise to the multiple steady states in shear, asymmetric slipper shapes in Poiseuille flow (Kaoui & Misbah 2009; Farutin & Misbah 2011) and underlie the observed instabilities as explained by Narsimhan *et al.* (2014).

2. Overview

In this overview, I focus on the asymmetric instability of vesicles with intermediate aspect ratio. For these particles, the essential physics of the instability is unaffected by the bending rigidity of the membrane. Note, however, that this is not the case for tubular vesicles: the high curvature of the interface sets a minimum tension for the onset of pearling. Following the analysis of the Rayleigh–Plateau instability, (1.1) shows that a perturbation in the steady-state vesicle shape, quantified by the change in the interface curvature δH , induces a change in pressure proportional to $\delta p \sim 2\sigma\delta H + 2\delta\sigma H$, where σ and H are the tension and curvature in the unperturbed state, and $\delta\sigma$ is the change in tension. If in the regions of swelling the resulting pressure is lower than in the regions of contraction, the induced flow will amplify the shape perturbation leading to instability. Unlike the classic Rayleigh–Plateau instability, in which the surface tension is constant, the curvature-induced pressure δp depends indirectly on the external forces through the tension. A somewhat similar situation arises with the ‘pearling’ of elongated drops in an electric field shown in figure 1(c): in this case, the applied electric pressure acts as tension.

Narsimhan *et al.* (2014) analyse the dynamics of ellipsoidal (intermediate aspect ratio) and tubular (high aspect ratio) vesicles both theoretically and numerically. In the case of vesicle with intermediate aspect ratio, the analytical theory considers a small perturbation around a spheroidal shape, while the base shape for a tubular vesicle is a cylinder. The solution for the flow is derived using an expansion in a basis of solutions of the Stokes equation in spheroidal or cylindrical geometries. The numerical simulations are done with the boundary integral method, which allows for very precise computation of the shape evolution.

Detailed examination of the energy contributions from pressure, surface tension, curvature, and bending shows that the main factor in the destabilization of the vesicle, in both uniaxial and planar extensional flows, is the changes in curvature $2\sigma\delta H$, which suggests that the vesicle’s initial geometry plays the most important role in the instability. Indeed, the results show that only for vesicles with reduced volume below 0.75 is the most unstable mode the one corresponding to an asymmetric, pear-like shape. Furthermore, the instability is sensitive to the vesicle aspect ratio: the more deflated the vesicle, the lower the flow strength needed to trigger the instability. The onset of the instability is insensitive to the viscosity ratio; this is related to the fact that the flow in the base state is independent of the viscosity contrast because the interface is immobilized (due to area incompressibility).

3. Future

Vesicle dynamics has proven an exciting source of challenging problems. Membrane inextensibility engenders nonlinearity that gives rise to unexpected behaviours:

multiple dynamical states of vesicles in shear flow, slipper-like shapes in Poiseuille flow, and asymmetric shapes in straining flows. Narsimhan *et al.* (2014) highlights the importance of forcing-dependent tension and non-spherical rest shape in destabilizing the symmetric dumbbell in extensional flows, both uniaxial and planar. It would be interesting to study whether other soft particles that can develop non-uniform surface tensions in response to applied flow, e.g. surfactant-covered drops or capsules, could also exhibit such instability. Although there is some experimental evidence that surfactant-covered drops can undergo asymmetric deformation and breakup, the phenomenon has never been rigorously studied. An experimental validation of the steady asymmetric vesicle shapes also needs to be systematically carried out, although this may be a difficult task; for example, the vesicle ‘slippers’ in Poiseuille flow have proven elusive to experimental observation (Coupier *et al.* 2012). Finally, external fields other than flow, e.g. electric field (Salipante 2013) (see also the supplementary movie available at <http://dx.doi.org/10.1017/jfm.2014.373>), or membrane activity can drive novel non-equilibrium dynamics awaiting discovery.

Supplementary movie

Supplementary movie is available at <http://dx.doi.org/10.1017/jfm.2014.373>.

References

- COUPIER, G., FARUTIN, A., MINETTI, C. & MISBAH, C. 2012 Shape diagram of vesicles in Poiseuille flow. *Phys. Rev. Lett.* **108**, 178106.
- FARUTIN, A. & MISBAH, C. 2011 Symmetry breaking of vesicle shapes in Poiseuille flow. *Phys. Rev. E* **84**, 011902.
- KANTSLEER, V., SEGRE, E. & STEINBERG, V. 2008 Critical dynamics of vesicle stretching transition in elongational flow. *Phys. Rev. Lett.* **101**, 048101.
- KAOUI, B. & MISBAH, C. 2009 Why do red blood cells have asymmetric shapes even in a symmetric flow? *Phys. Rev. Lett.* **103**, 188101.
- LAC, E. & HOMSY, G. M. 2007 Axisymmetric deformation and stability of a viscous drop in a steady electric field. *J. Fluid Mech.* **590**, 239–264.
- LI, X., VLAHOVSKA, P. M. & KARNIADAKIS, G. EM. 2012 Continuum- and particle-based modeling of shapes and dynamics of red blood cells in health and disease. *Soft Matt.* **9**, 28–37.
- NARSIMHAN, V., SPANN, A. & SHAQFEH, E. S. G. 2014 The mechanism of shape instability for a lipid vesicle in extensional flow. *J. Fluid Mech.* **750**, 144–190.
- SALIPANTE, P. S. 2013 Electrohydrodynamics of simple and complex interfaces. PhD thesis. Brown University.
- SEIFERT, U. 1997 Configurations of fluid membranes and vesicles. *Adv. Phys.* **46**, 13–137.
- SINHA, K. P., GADKARI, S. & THAOKAR, R. M. 2013 Electric field induced pearling instability in cylindrical vesicles. *Soft Matt.* **9**, 7274–7293.
- VLAHOVSKA, P. M., PODGORSKI, T. & MISBAH, C. 2009 Vesicles and red blood cells: from individual dynamics to rheology. *C. R. Phys.* **10**, 775–789.
- ZHAO, H. & SHAQFEH, E. S. G. 2013 The shape instability of a lipid vesicle in a uniaxial extensional flow. *J. Fluid Mech.* **719**, 345–361.