Airscrew Characteristics and Aeroplane Performance Prediction.

By Col J D Blyth, O B E , M I Ae E

IN approaching any problem in Aerodynamics the student, and the severely practical engineer, whose academic knowledge may be limited, are too frequently confronted with two difficulties The first of these is that though the literature on the subject is extensive, yet the information sought usually has to be collected from a number of sources and the second is that when found the information is frequently presented in such a manner as to be of little immediate practical value without a certain amount of simplification

The object of this paper is to collate and present in an immediately useful form the information contained in parts of the following publications —

R & M No 474
N A C A Reports Nos 101 and 168
Bairstow's "Applied Aerodynamics"

The method of performance prediction to be described involves the use of the Airscrew Torque Function curve The particular advantage of employing this curve lies in the fact that if the design conditions selected for the airscrew are such that at some point in the speed range of the machine the airscrew will not permit the engine to develop the correct number of revolutions per minute at full throttle corresponding with that speed, the fact at once shows in the calculations

We will commence by examining the Torque Function curve, *i e*, the curve of $K_Q Q_C$ against $\frac{V}{nP}$

The following symbols will be employed —

K_Q = torque coefficient of the airscrew
K_T = thrust coefficient of the airscrew
Q = torque absorbed by airscrew
T = thrust of airscrew
V = forward velocity of machine
D = airscrew diameter
P = experimental mean pitch of the airscrew
n = revs per second of airscrew
n_m = revs per second of airscrew at maximum efficiency
HP = B H P of the engine at n airscrew revs

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HP_m = B H P of the engine at n_m airscrew revs
HP_T = thrust horsepower
η = efficiency of airscrew
ρ = relative density of air
F(hp)= horsepower factor for altitude (often written ρ_e)

Q_C is a coefficient of such value that when V/nP = 0 5, $K_Q\,Q_C$ = 1 0, and T_C at the same value of V/nP makes $K_T\,T_C$ = 1 0

Where applicable, all measurements are in lbs /feet/sec units

Referring to R & M 474, Fig 14a, we find the equations

$$K_T\,T_C = \frac{4}{3}\left[1-\left(\frac{V}{n\rho}\right)^2\right] \qquad (1)$$

and

$$K_Q\,Q_C = 1\ 1042 - 0\ 833\left(\frac{V}{n\rho}\right)^3 \qquad (2)$$

The natural inference is that there is only one thrust function curve and only one torque function curve whatever may be the value of the ratio P/D On reference to Bairstow's "Applied Aerodynamics," page 321, we find that this conclusion is correct as far as the thrust function curve is concerned, but that for each different value of P/D there is a different torque function curve Our first problem, then, is to find the connection between $K_Q\,Q_C$, P/D, and V/nP

We know from R & M 474 that the equation of the torque function curve is of the form

$$K_Q\,Q_C = A - B\left(\frac{V}{n\rho}\right)^3$$

With this knowledge it is not difficult a matter to obtain the equation of each curve in Bairstow's "Applied Aerodynamics" If this is done it will be seen that the values of A for different values of P/D are connected by a straight line law, as also are the values of B Determining and applying these laws we arrive at the general equation connecting $K_Q\,Q_C$, P/D, and V/nP, which is

$$K_Q\,Q_C = 1\ 017 + 0\ 0738\frac{P}{D} - \left(0\ 14 + 0\ 587\frac{P}{D}\right)\left(\frac{V}{n\rho}\right)^3 \qquad (3)$$

It will be seen from this that the curve given in R & M 474 is correct only when P/D = 1 18

We are now in a position to draw the curve of $K_Q\,Q_C$ against V/nP for any value of P/D our next step, therefore, is to determine the value of P/D Here we probably meet with the difficulty that the aeroplane exists on paper only, and that the airscrew has not yet been designed The only information at our disposal is the values of V, n, and HP, at which maximum airscrew efficiency is required

By the use of Dr Watt's monogram, given on p 319 of Bairstow's "Applied Aerodynamics," or by using one of the various empirical formulæ for the purpose, we can find the value of D We now know V, n, and D, and can therefore find the value of V/nD at maximum airscrew efficiency

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Again referring to R & M 474 we see that

$$\eta = \frac{P}{2\pi D} \frac{K_Q}{K_T} F\left(\frac{V}{n\rho}\right)$$

where
$$F\left(\frac{V}{n\rho}\right) = \frac{K_T \; \Gamma_C}{K_Q \; Q_C} \left(\frac{V}{n\rho}\right)$$

Examining the equation for η we see that for any particular value of P/D the only variable on the right hand side of the equation is F (V/nP) The airscrew efficiency is therefore a maximum when F (V/nP) is a maximum

Combining the equation for F (V/nP) with equations (1) and (3) we get

$$F\left(\frac{V}{\rho}\right) = \frac{\frac{4}{3}\left[\frac{V}{n\rho} - \left(\frac{V}{n\rho}\right)^3\right]}{1\,017 + 0\,0738\,\frac{P}{D} - \left(0\,14 + 0\,587\,\frac{P}{D}\right)\left(\frac{V}{n\rho}\right)^3} \qquad (4)$$

For any particular value of P/D we can find, by using equation (4), the value of V/nP which makes F (V/nP) a maximum Multiplying this value of V/nP by the particular value of P/D for which it is found, we get the value of V/nD at which the efficiency of an airscrew having this particular value of P/D is a maximum

Repeating this calculation for a number of values of P/D we can plot the curve given in Fig 1

Since the value of V/nD at which the airscrew efficiency is required to be a maximum is known from design conditions, we can read from Fig 1 the value of P/D which fulfils these conditions and knowing D we can find P In the method of performance prediction to be described the value of P is not needed , but this curve will be found useful in determining the blade settings at various speeds and revs for variable pitch airscrews

An example may make matters clearer

Given a two-bladed airscrew, 10-ft in diameter, V/nD at maximum efficiency = 0 87 what should be the value of P/D, and the experimental mean pitch ?

From Fig 1, the value of P/D corresponding to this value of V/nD is 1 2 The value of P is therefore 12-ft

Having determined the value of P/D we can now substitute it in equation (3), and draw the curve of $K_Q \, Q_C$ against V/nP for use in our prediction calculations

We have now completed our examination of the airscrew characteristics and will leave them for a time whilst we consider the method of performance prediction

The method is based upon that described in N A C A Report No 101, the actual thrust horsepower and speed of advance being replaced by " reduced horsepower " and " reduced speed " The particular advantage of the method to be described over that given in N A C A No 101 is that the curves of " reduced horsepower available " for any number of altitudes can be derived at once from a single simple basic tabulation, while, as in N A C A No 101, a single curve of " reduced horsepower required " is employed

The method of drawing the "reduced horsepower required" curves differs but little from that described in the report mentioned but the method of finding the "reduced horsepower available" will require fuller explanation

Reduced Horsepower Required and Reduced Speed of Advance

The following additional symbols will be employed —

W = weight of machine in lbs
A = effective wing area in square feet
w = wing loading = W/A lbs /square feet
K_2 — overall lift coefficient at any angle of attack
K_D = corresponding overall drag coefficient
h = "reduced horsepower"
u = "reduced speed of advance"

For steady horizontal flight we have the fundamental equations —

$$W = 00237\,\rho\ AV^2\ K_2 \qquad (5)$$

$$\text{Drag} = 00237\,\rho\ AV^2\ K_d \qquad (6)$$

From these

$$\text{Drag} = W\,\frac{K_d}{K_2} \qquad (7)$$

From (7)

$$HP_1 = \frac{W\,V}{550}\ \frac{K_d}{K_2} \qquad (8)$$

From (5)

$$V = \sqrt{\frac{w}{00237\,\rho\,K_2}} \qquad (9)$$

From (8) and (9)

$$HP_1 = \frac{W}{550}\sqrt{\frac{w}{00237\,\rho\,K_2}}\ \frac{K_d}{K_2} \qquad (10)$$

From (9) and (10)

$$V\sqrt{\frac{00237\,\rho}{w}} = \sqrt{\frac{1}{K_2}} \qquad (11)$$

and

$$\frac{HP_1}{W}\sqrt{\frac{00237\,\rho}{w}} = \frac{1}{550}\ \frac{K_d}{K_2}\sqrt{\frac{1}{K_2}} \qquad (12)$$

Put

$$u = V\sqrt{\frac{00237\,\rho}{w}} \qquad (13)$$

and

$$h = \frac{HP_1}{W}\sqrt{\frac{00237\,\rho}{w}} \qquad (14)$$

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Substituting in (11) and (12) we get

$$u = \sqrt{\frac{1}{K_2}}$$

$$h = \frac{u}{550} \frac{K_d}{K_2}$$

Plotting these for various values of K_2 and K_d we get the curve of reduced horse power required against reduced speed of advance

Reduced Horsepower Available and Reduced Speed of Advance

The following data will be required —

(1) Engine power curve (B H P against r p m)

(2) η against V/nD This may be derived from the curves given in N A C A No 168 which are reproduced in Fig 2

(3) Values of ρ and F (hp) for different altitudes

Our first step is to find P/D and draw the curve of K_Q Q_c against V/nP as already described

Next find η_{max} on curve I, Fig 2, for the design value of V/nD, and employing this value in conjunction with Curve II, Fig 2 draw the curve of η for various values of V/nD

Now $$V = nD \frac{V}{nD} \qquad (15)$$

and $$HP_r = HP\,\eta\,F(hp) \qquad (16)$$

Combining with (13) and (14)

$$u = nD\frac{V}{nD}\sqrt{\frac{00237\,\rho}{w}}$$

and $$h = \frac{HP}{W}\,\eta\,F(hp)\sqrt{\frac{00237\,\rho}{w}}$$

Plotting these for various values of n at different altitudes we get a series of curves of reduced horsepower available against reduced speed The points at which these interesct the curve of reduced horsepower required give the reduced maximum speeds of advance, which are converted in true speeds by multiplying by $\sqrt{\frac{w}{00237\,\rho}}$

It should be borne in mind that the speeds so found are in ft /sec

The rate of climb in feet per minute is given by

$$33{,}000\sqrt{\frac{w}{00237\,\rho}} \times \text{maximum difference in ordinates}$$

The absolute ceiling may be found in the ordinary way by plotting rates of climb against altitude If the curve of reduced horsepower available at the absolute ceiling is plotted it will touch the curve of reduced horsepower required the

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point of contact will give the speed of advance at the ceiling, and the optimum speed for long range flight at any other altitude

It only remains to be shown how corresponding values of n, V/nD, and n are obtained These are found as follows —

Having found V/nD and P/D at η_{max} we can also find V/nP at η_{max} From the curve of $K_Q Q_C$ which we have drawn we find the value of $K_Q Q_C$ at this value of V/nP, *i e*, the value of $K_Q Q_C$ at η_{max}

Call this a

At any altitude

$$Q = \frac{550 \ HP_m \ F(hp)}{2 \ \pi \ n_m} \text{ for max efficiency}$$

$$\text{Since} \quad K_Q = \frac{Q}{00237 \ \rho \ \eta^2 \ D^5}$$

we get, for maximum efficiency

$$K_Q = \frac{550 \ HP_m \ F(hp)}{00237 \ \rho \ n^3_m \ D^5} \quad \frac{1}{2n} \tag{17}$$

$$\text{and} \quad Q_C = \frac{a \times 00237 \ \rho \ n^3_m D^5 \times 2\pi}{550 \ HP_m \ F(hp)} \tag{18}$$

For any value of n and corresponding value of HP

$$K_Q = \frac{550 \ HP \ F(hp)}{00237 \ \rho \ n^3 \ D^5} \quad \frac{1}{2\pi} \tag{19}$$

From (18) and (19)

$$K_Q Q_C = a \times \frac{n^3_m}{HP_m} \times \frac{HP}{n^3} \tag{20}$$

This gives the value of $K_Q Q_C$ for any value of n The corresponding value of V/nP is found from the $K_Q Q_C$ curve, and is multiplied by P/D to find V/nD The value of η is now read from the efficiency curve, and the thrust horsepower available can be found We are now in a position to proceed with our basic tabulation

First plot the curve of HP/n^3 against n on the engine power curve, and draw up the following list of constants —

$$a = K_Q Q_C \text{ at } \eta_{max}$$

$$b = \frac{n^3_m}{HP_m}$$

$$c = \sqrt{\frac{00237}{w}}$$

$$1/c = \sqrt{\frac{w}{00237}} \quad \text{(for use in finding true values of V)}$$

$$d = K_Q Q_C \text{ at } \frac{V}{nP} = O$$

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Substituting *a*, *b*, and *d* in equation (20) we get

$$\frac{HP}{n^3} = \frac{d}{a\,b}$$

Read off the value of n from the HP/n^3 curve which gives us this value of HP/n^3. This gives the minimum revs. per second at full power at ground level, and shows that if in our tabulation we use lower values of n, impossible values of $K_Q\,Q_C$ will be obtained.

The value of n so found gives the revs. per second which should be obtained at full throttle with the machine at rest if the airscrew has the value of P/D as found and the engine is developing its full rated B.H.P.

Having made the list of constants, the basic tabulation is proceeded with as follows:—

TABULATION

N (r.p.m.)					From engine power curve
n					
HP					
HP/n^3					
$K_Q\,Q_C = a\,b\,HP/n^3$					
V/nP					From $K_Q\,Q_C$ curve
V/nD = V/nP. P/D					
η					From η curve
nD					
$U = nD.V/nD.c$					
$H = \eta.c.HP/W$					

At any altitude

$$u = U \times \sqrt{\rho}$$
$$h = H \times \sqrt{\rho} \times F\,(hp)$$

It will be seen that the figures contained in the last two lines of this basic tabulation enable us to plot rapidly the curves of reduced horsepower against reduced speed of advance for any required series of altitudes.

These curves are the curves of reduced horsepower available at constant throttle. If the value of V at which n is made a maximum has been selected to give good rates of climb it will frequently happen that the curve of reduced horsepower available at the maximum value of n will not reach the curve of reduced horsepower required. This shows that to find the maximum speed attainable in horizontal flight we must develop a portion of the curve of reduced horsepower available at constant maximum airscrew revolutions, which is controlled by the maximum allowable r.p.m. of the engine.

To do this it is necessary only to start in the 7th line of the tabulation and take two value of V/nD rather higher than the greatest value found, and complete the tabulation, keeping n and HP constant and equal to the maximum allowable It will be found that the curve of reduced horsepower available now takes the form of two intersecting curves, which should be drawn as such, and no attempt made to fair off the peak

In the case of a Variable Pitch airscrew whose blade angle is variable during flight, it is possible to so set the blade that the revs and horsepower are constant throughout the speed range

In this case $V/nD \propto V$, and the basic tabulation becomes—

V					
V/nD					
η					From curve I, Fig 2
$U = cV$					
$H = \eta c$ HP/W					(HP/W is constant)

For both types of airscrew, by assuming values of V and n and working back to $K_Q Q_c$ we can find the horsepower available at these values of V and n The efficiency is found as usual, which gives us the thrust horsepower available

Working in this way we can determine forward speeds and rates of climb with the engine throttled

It will be noted that the curve of reduced horsepower available is independent of wing loading while in the basic for reduced horsepower available the factor for weight and loading appears only in the last two lines

To compare the complete performances of the same machine with difference loads therefore necessitates alteration in the last two lines only of the basic tabulation, with very little additional work

This completes the description of the method As in general principle it is similar to all methods of performance prediction, *i e*, curves of horsepower required and horsepower available are plotted against forward speed, it has not been considered necessary to work out an example in detail

FIG 1

RELATIONSHIP BETWEEN E M P & D FOR MAXIMUM EFFICIENCY

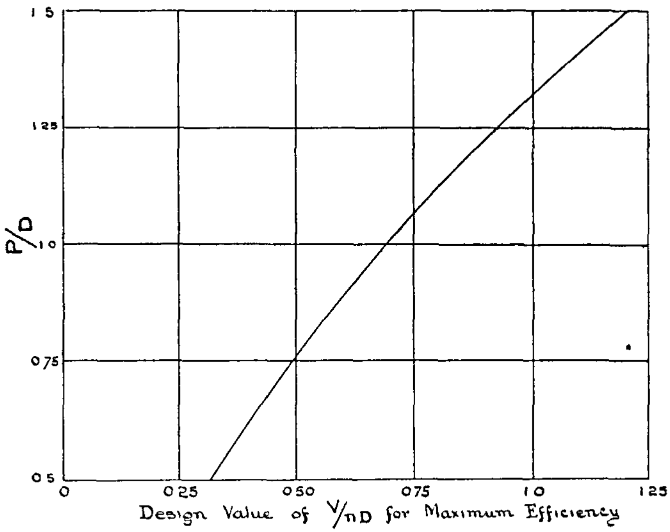
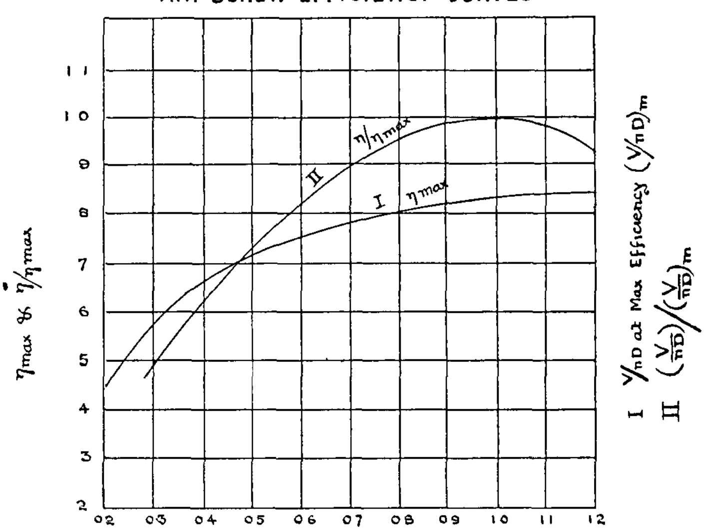


FIG 2

AIR SCREW EFFICIENCY CURVES



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