Islamic reception of Greek astronomy

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Abstract. Research in Islamic science over the last half century or so has clearly established that such old myths as Islamic science being a preservation of Greek science, or that science was always in conflict with religion in Islamic civilization as it was in Europe, or that the European scientific Renaissance was independent of outside influences –a European phenomenon par excellence– are now all subjects of great dispute if not altogether dead. In what follows I will illustrate the evidence that has put such myths into question with only few examples, since time and space do not allow me to elaborate more.

Keywords. Greek astronomy, Islam, prayers, qibla, Copernicus, Ibn al-Shāṭir, Ṭūsī

1. Islamic religion and the reception of Greek science

Of all the ritual requirements imposed by religious Islamic practice, I will choose only two that seem to have made both a great difference in defining the rôle of science within the domain of religious thought, and to illustrate how those ritual practices allowed Islamic science to depart from the received Greek scientific legacy. In the apparently straight forward simple Qur'anic verses "*Guard strictly your (habit of) prayers, especially the* **middle prayer***; and stand before Allah in a devout (frame of mind)*"(emphasis added) [Cow 238], and "*Wherever you are, turn your face towards it (the holy sanctuary)*" [Cow 144], there was apparently much cause to inaugurate and sustain a serious scientific interest that burgeoned towards the beginning of Islamic civilisation and continued to flourish throughout the history of that civilisation. In the first verse, the believer is urged to pay special attention to the *middle prayer* over and above the other four prayers that s/he is supposed to perform daily.

There was a good reason why this prayer was singled out. The times for the other four Islamic prayers are all defined by such astronomical phenomena as the onset of dawn for the *fajr* prayer, the slight movement of a shadow eastwards after the Sun's crossing of the local meridian for the *ẓuhr* prayer, the onset of the evening twilight *maghrib* prayer, and the onset of the night *'ishā'* prayer, are all rather easy to determine without much effort. The *middle prayer*, the *'aṣr*, or afternoon prayer, however is not so simple.

1.1. *The time of prayer*

The beginning of the *'aṣr* prayer was originally defined by the time when the shadow of a gnomon, or any person, was equal to its length. And its end was determined by the time when the shadow length was equal to twice the height of the gnomon. Both of these measurements could be established with relative ease when the Muslim community was still confined to the two holy Islamic cities of Mecca and Medina in Arabia, and where the latitude of the northernmost of the two cities, Medina, did not exceed 24°30′ and that of the first did not exceed 21°30′. But when the Muslim community spread northwards into areas like Damascus, and later Baghdad, it was quickly noticed that in those northern climes there were many days of the year, during the winter season, when the length of

the shadow was never equal to the length of the gnomon. It was always longer, for days on end. It was during those later times, and most likely in those same northern climes, that the definition of the beginning and end of this particular prayer were re-stated, now to become as follows: the beginning of the 'aṣr prayer was redefined to become the time when the shadow of a gnomon was equal to whatever it was at noon, plus the length of the gnomon, and it was to end when the shadow became equal to whatever it was at noon plus twice the length of the gnomon.

In other words the beginning of the 'aṣr prayer was redefined by the shadow length $s_1 = s_n + g$ where s_1 is the shadow length, s_n is the length of the shadow at noon for the specific locality, and g is the height of the gnomon. The end of that prayer was accordingly defined to occur at the time when the shadow $s_2 = s_n + 2g$.

In regard to the relationship between religion and science it is important to note in passing that this mere redefinition of prayer times is an elegant testament of the ability of religious pronouncements to accommodate natural phenomena such as the varying shadow lengths at different terrestrial climes, rather than see this relationship as monolithically antagonistic. Once articulated in this fashion, where the shadow length s that determines the beginning of time of prayer and its end was expressed as a function of the variable length of the shadow at noon in the particular locality, then the problem was shifted from the actual determination of the shadow length of a gnomon to the determination of that gnomon's shadow at noon, for any clime one desired. And in turn the shadow length at noon s_n was itself expressed as a function of the height of the sun at noon h_n, as in $s_n = g \cot h_n$, which obviously meant that one had to determine the height of the sun at noon h_n for all localities before one could retrace his steps to calculate the beginning time of the 'aṣr prayer. But $h_n = \widehat{\Phi} \pm \delta$, where $\widehat{\Phi}$ is the complement of the latitude of the said locality, and δ is the declination of the Sun on a particular day. Now $\widehat{\Phi}$ is a constant value for the specific locality, which can be determined by direct observation, and all that was left to do was to determine the day to day variation in the declination of the Sun δ. The declination δ is itself a function of the variable longitude of the Sun λ, from day to day, and the constant maximum value of the solar declination as embedded in the application of the sine law $\sin \delta = \sin \lambda \sin \varepsilon$. This expression means that the determination of the instantaneous longitude of the Sun for noon of a specific day becomes now of paramount importance, and one required the one time direct observational measurement of the maximum declination, ε.

Ptolemy had already reported in the *Almagest* that the maximum declination, ε, can be determined directly by the use of instruments, and that it was already observed by Aratosthenes and Hipparchus before him, and was found to be, $\varepsilon = 23;51,20°$ [Almagest, I, 12, 16]. For reasons that are not yet completely clear, astronomers of the early Islamic civilisation decided, sometime around the first half of the ninth century, not to accept the accounts of the three sages of Greek astronomy, Aratosthenes, Hipparchus and Ptolemy, at face value, and to re-determine from scratch this particular parameter as well as others, as we shall see. In the case of this parameter, could it be that they had realised the importance of this value for the chain of variables just listed that were all needed to determine the limits of the religious injunction of the afternoon prayer? Or did they realise already that this specific parameter was so fundamental to all astronomical work that one would be better of making sure that it was observed properly?

Ptolemy had also described, in the same chapter, the two possible instruments that could be used to determine this parameter: a metal ring, or a masonry quadrant. Judging by the surviving documents, we can assert that astronomers working in the Islamic civilisation seem to have followed suit and attempted to construct both types of instruments. In the process, a serious discussion touching upon the relationship of instruments
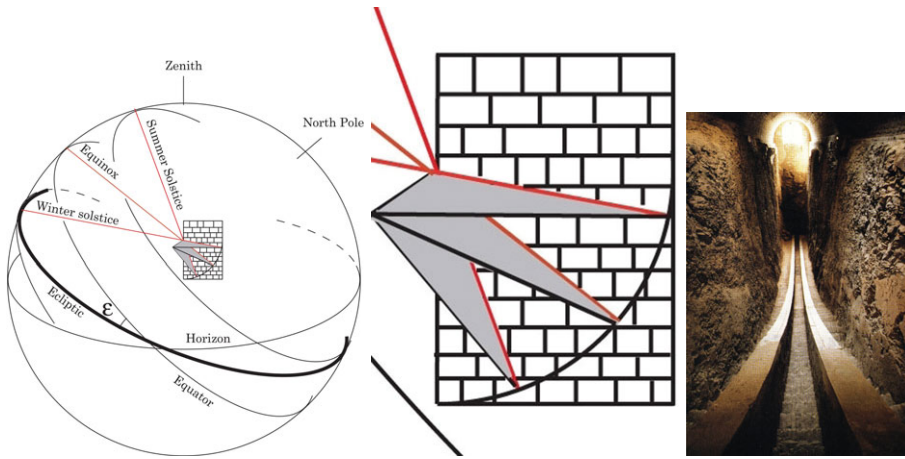
Figure 1. The depiction of the method for determining the maximum solar declination and the archeological remains of a quadrant from the observatory of Ulug Beg (d. 1449) at Samarqand (*right*) which was probably modeled after earlier quadrants.

to observational results, issues of size of instruments and their bearing on the precision of observations, as well as a discussion of the best kind of stable material of which the instruments were to be made and upon which they were set, all seem to have been initiated at that time. There are both textual and archeological extant sources to attest to this discussion.

The social and cultural need for this development in Islamic civilisation and its implication for the field of astronomy has been discussed elsewhere†. What concerns us here is to point to the fact that the net result of all those developments led to the establishment of a much more refined value for that parameter, where it was found that maximum declination of the Sun was in the range of 23°30′ to 23°33′, and definitely not more than 23°35′, instead of the grossly off-the-mark value in the Greek sources. This gross variation naturally led to questioning the validity of other earlier Greek values as well. For example, a similar variation was also noted in the value of precession, where the fresh observations of the early part of the ninth century found precession to be more like 1°/66 years or 1°/70 years instead of the 1°/100 years as was reported by Ptolemy. And again the position of the solar apogee, was also re-examined and it was found to be variable and not fixed at Gemini 5;30°, as was reported by Ptolemy.

All these results seem to have encouraged astronomers to undertake a comprehensive approach to the Greek legacy and to question its observational techniques as well. It was not enough to find a new value for a specific parameter, those astronomers wanted to know where did the discrepancy come from. Did it come from the type of instruments that were used in the Greek times? Or did it result from the faulty observational techniques that had been deployed in arriving at those results? In one instance, which was occasioned by the desire to get as good a value as possible for the beginning of the time of the afternoon prayer, which we saw above, it was noted that the equation $\sin \delta = \sin \lambda \sin \varepsilon$, which was used to determine the declination of the Sun and involved the observational value of the maximum declination, ε, we just saw, that equation also required the determination

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† For the discussion of instruments and their role, see Hartner (1977) especially p. 6, where he refers to the medieval Arabic sources in which this discussion of instrument sizes is mentioned; for the cultural significance of this activity see Saliba (2007), p. 133.
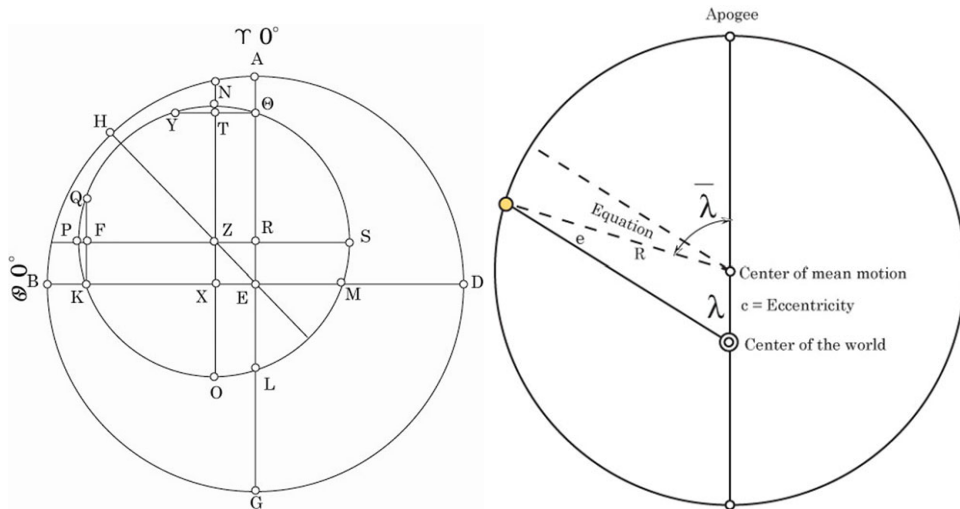
Figure 2. Ptolemy's determination of the solar eccentricity EZ [Almagest III, 4] required the observation of the Sun at the times when the Sun reaches the equinoxes at A,G, and the solstices at B, and D. Once determined the eccentricity could then be used to determine the solar equation, which in turn would yield the solar longitude.
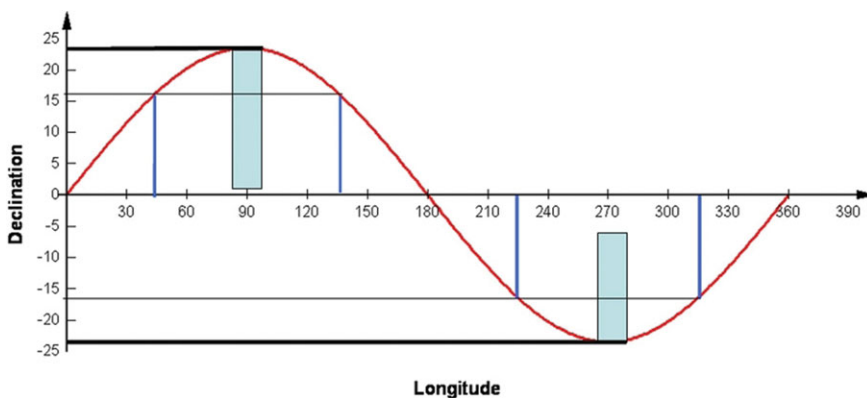

Figure 3. The determination of the solar declination at the times of solstices, i.e. at $90°$ and at $270°$ is extremely difficult as can be seen from the graph. In the middle of the seasons (the *Fuṣūl*), at $45°$, $135°$, etc. the variation in the declination can be just as easily detected as it would be around the equinoxes.

of the instantaneous longitude of the Sun, λ, at noon on a specific day. For that value of the longitude a new determination of the solar eccentricity, EZ in Fig. 2, was essential.

For if armed with a reliable value for the solar eccentricity, c, one could easily determine the solar equation, e, from $e = \tan^{-1}\left[(c \sin \bar{\lambda})/(R + c \cos \bar{\lambda})\right]$, and then use it to determine the instantaneous longitude, λ, from $\lambda = \bar{\lambda} \pm e$. In the process of determining the eccentricity, the Baghdad astronomers could easily notice the main flaw in Ptolemy's observational methodology. They quickly realised that although it was easy to detect a variation in the declination of the Sun from one day to the next around the times of the equinoxes, it was rather difficult to observe such variation when the Sun approached the solstices and when for about three days before, and three days after the solstice crossing, the variation in the declination of the Sun would be hardly noticeable as can be easily seen from Fig. 3.
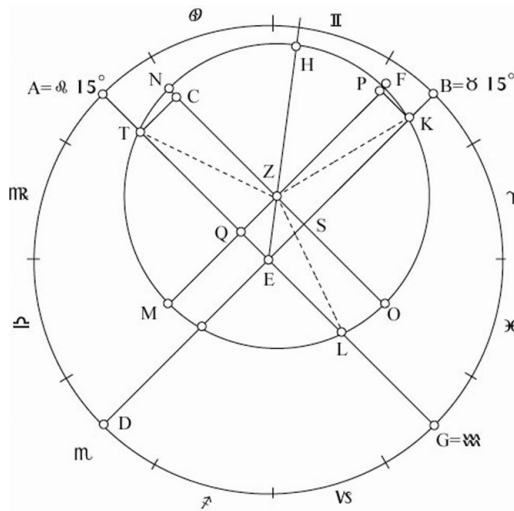
Figure 4. Using the same computational techniques, the *Fuṣūl* method only shifted the strategic observational positions to the mid seasons in order to achieve the higher precision.

This finding led to the realisation that Ptolemy's fault was not in the instruments he used, but rather it was in his strategy to observe at the specific times of the solstices. The alternative was to keep the same strategy of derivation and computation that was followed by Ptolemy but shift the observation points to fall in the middle of the seasons (*Fuṣūl*, sing. *faṣl* = season), that is to observe the declination of the Sun when the Sun crossed the 15° of Taurus, the 15° of Leo, the 15° of Scorpio and the 15° of Aquarius (Fig. 4).

This *Fuṣūl* method naturally gave a much higher precision for the declination values, on the basis of which the new solar eccentricity was determined to be in the vicinity of 2 parts of the same parts that made the radius of the Sun's eccentric sphere equal to 60 parts, rather than the Ptolemaic value of the eccentricity that was determined to be around 2;30 parts [Almagest III, 4]. Further consideration of the observational strategies led later astronomers to refine those strategies again and to devise techniques by which only three observations were required to determine the eccentricity, two of these observations had to be diametrically opposite but no longer restricted to points of the equinoxes or the solstices†.

Furthermore, the same risky method used by Ptolemy led him to compute the position of the solar apogee and to find that it was more or less at the same point where Hipparchus had found it some three centuries before him and thus concluded that the solar apogee was fixed at Gemini 5;30°‡. Consequently, Ptolemy's computation of the maximum solar equation, which is a function of the eccentricity as we have already seen, was also in error, about half a degree too large. All these variations from the Ptolemaic values led the Baghdad astronomers to conclude that although some of the theoretical aspects of Greek astronomy were sound, the results based on the observational aspects of that astronomy were extremely suspicious, and needed to be double checked over and over again. This very attitude of suspicion gave rise to other questions which we will touch upon in the sequel. For our present purposes it is enough to note how a simple religious

† See, for example, the techniques developed by Mu'ayyad al-Dīn al-'Urḍī (d.1266) in Saliba (1985).

‡ For the complex problem of Ptolemy's determination of this perimeter, see Petersen & Schmidt (1968).

requirement to pray at a specific time of the day when the length of a shadow is of a specific length forced astronomers to overhaul almost all of the basic parameters of the Greek astronomical legacy, discovering in the process not only the more precise values, but the theoretical and methodological problems that plagued that legacy. In that respect, it would be more than a slight exaggeration to claim that Arabic/Islamic astronomy attempted to preserve Greek astronomy, or that it simply continued the projects that were already set in the earlier Greek tradition. Rather it should be easy to notice that a new religious requirement, unknown to the Greeks, in fact led to the generation of new techniques, new results, and thus new and better astronomical foundations. From the declination of the ecliptic, to the value of precession, to the solar eccentricity, equation and apogee, at every turn we find the Baghdad astronomers politely criticising the older Greek tradition, and silently erecting a better edifice for their own astronomy. The fact that modern astronomy still uses values very close to the ones that were determined in Baghdad during the first half of the ninth century for such fundamental parameters as the inclination of the ecliptic, the precession and the solar equation is a testament to the rigor and seriousness with which those Baghdad astronomers took their job. At their hands, Greek astronomy was turned over, and on its ruins a new astronomy was born. This will become clearer too in what follows.

1.2. *The direction of prayer – birth of new trigonometry*

The second part of the religious requirement of prayers was to pray in a specific direction, the *qibla*, which is the direction of the holy sanctuary in Mecca in Arabia. And like the first requirement, as long as people were within sight of the holy sanctuary, facing that sanctuary did not constitute a problem. But as the community of Muslems grew and moved farther and farther away from Arabia, the holy sanctuary was no longer visible, and thus its direction had to be computed. That requirement immediately challenged the astronomers to find the precise angle along the local horizon of any city at which one should pray so that s/he would be facing Mecca during that prayer. This was true for individual obligations of prayers as it was for the building of the prayer niches (*miḥrāb*) in local mosques where the whole congregation would face the same sanctuary during the communal Friday prayers. In essence, this requirement forced the astronomers to compute the angle, q, marked along the spherical globe for any city, C, distinct from Mecca, in Fig. 5.

Solving spherical trigonometric triangles was possible to do in Greek, but the tools for such solutions were restricted to the applications of the Menelaus or the Ptolemaic theorems, both using chord functions and clumsy quadrilateral expressions; the Greeks did not have anything like the Sine or Cosine laws for that purpose. Once those new laws were developed in the Islamic civilisation, together with a whole battery of trigonometric identities and equations, the solution of the *qibla* problem, that is for the angle q, in Fig. 5, became a matter of solving the following equation:

$$q \ = \ \text{arc cot} \left[\frac{\sin \Phi \cos \Delta L \ - \ \cos \Phi \tan \Phi_M}{\sin \Delta L} \right] \ ,$$

where Φ is the latitude of the specific city, Φ_M the latitude of Mecca, and ΔL the difference in longitude between the two cities. One can easily imagine the clumsiness of the solution of such an expression if s/he were restricted to using the chord functions alone, as s/he would have had to do if limited to the Greek tradition. In any event, no such problems ever arose in the Greek tradition since this particular problem was a specific Islamic religious requirement. Once those trigonometric tools were developed in the Islamic civilisation to solve such problems, they could be applied at all occasions,
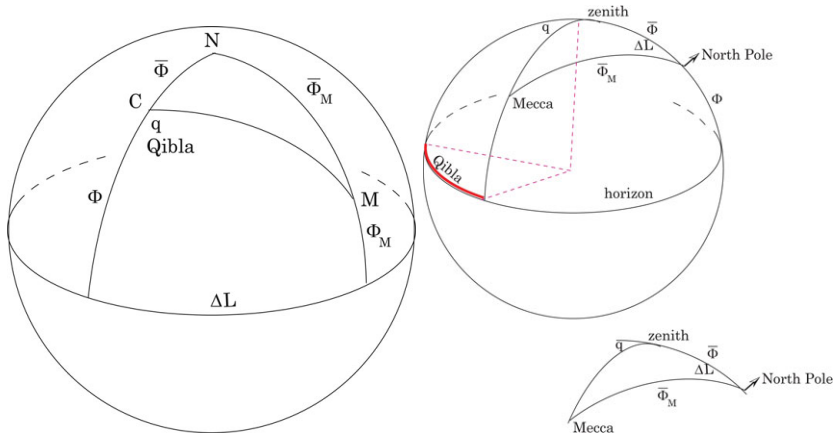
Figure 5. For any city on the terrestrial globe, the direction of Mecca, q, i.e. the *qibla*, is a function of the difference in longitude ΔL between that city, C, and Mecca, M, and the separate latitudes, ϕ, and ϕ_M of the city and of Mecca respectively. Expressed along the local horizon of that city, the *qibla* would be simply the arc along the local horizon between a geographical cardinal point, say the South, of that locality and the point of intersection, along the same horizon, of the meridian that passes through the zenith of that city's dwellers and the zenith of Mecca.

religious or not, where such solutions required the application of spherical trigonometric problems. This holds true for all other developments as well that may have been originally initiated in the Islamic civilisation to solve a particular religious problem but then found to apply to all other instances where the problems were encountered in religious contexts or not.

Moreover, the solutions of such problems as the height of the Sun at noon for different localities, and the direction of Mecca for a variety of cities, or the production of the trigonometric tables for the elementary functions of sine, cosine, tangent, cotangent and the like, all had to be solved once for the specific localities and thus their results written down once for all. And where could that be done better than to engrave all such results on the back of astrolabes, the medieval astronomical portable tool par excellence, as can be seen on the back of the astrolabe in Fig. 6.

The concern with such problems as the determination of the direction of prayers were then generalised to find the *qibla* directions for all cities anywhere on the globe, even in areas where there were no Muslims to pray. The general theoretical problem itself became attractive to mathematicians and astronomers alike, and in one instance, at the hands of the brilliant astronomer of the ninth century Ḥabash al-Ḥāsib (d.c. 870), it led to the production of a new mathematical projection of geographical maps (Fig. 7), centered around Mecca, where one could read from the map not only the direction of Mecca from that city, but the distance to Mecca as well; the latter was in response to the additional religious requirement for the believer to perform the pilgrimage to Mecca at least once in a life time.

This particular mapping has inspired a later seventeenth-century jewel-like plate which was recently published by David King (King 1999). In that plate, Mecca is located at the center of the plate; the rest of the circular plate represents the planispheric mappings of the locations of the major cities in the then known world. An attached alidade with scale markings pinned at the center rotates to give the direction of the *qibla* for the said cities, as its edge is aligned with any of the marked cities on the plane of the plate and its *qibla* is read in degrees at the rim of the plate. The scale along the alidade gives
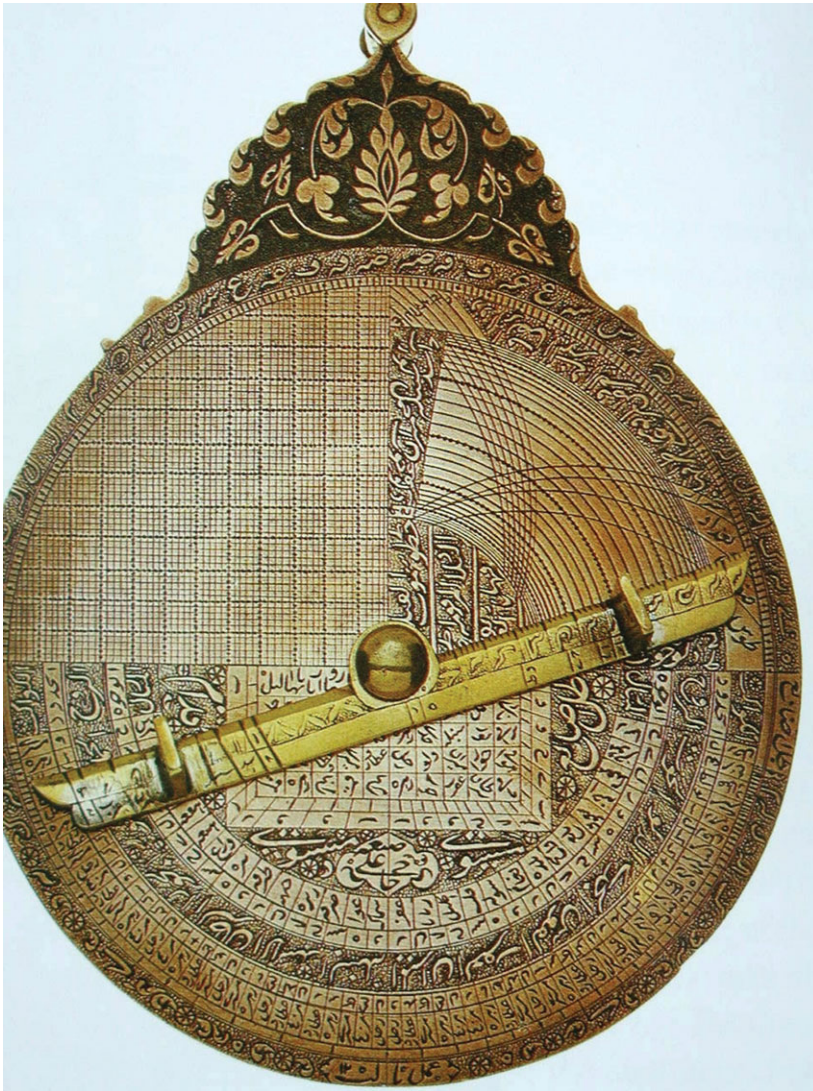
Figure 6. An astrolabe, once kept at the Time Museum in Illinois, where the upper right hand quadrant carries a series of upward bent curves used for the determination of the height of the Sun at noon for the various latitudes marked at the upper edges of the curves, and a series of downward bent curves that give the *qiblas* for the various cities whose names are engraved at the edges of those curves as well. From the upper left hand quadrant one could read the Sines (along the vertical axis) and the Cosines (along the horizontal). The tangents and cotangents could be read along the lower semi-circular edge of the astrolabe.

the distances of the specific cities from Mecca. The plate itself, according to King, was most likely produced in Safavid Iran. All of these developments had very little to do with the inherited Greek legacy. From their religious inspiration to their execution, to their deployment of new mathematical techniques –like the use of spherical trigonometric laws and functions, which were unknown from the Greek tradition– to the use of new observational techniques and strategies, all speak volumes to the Islamic reformulation of the Greek astronomical tradition so that it is made to fit the new cultural requirements, and in the process create a new form of science that has its own Islamic character.
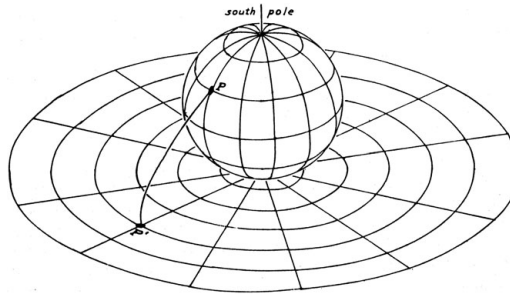
Figure 7. An azimuth projection that could be centered on Mecca, and thus the user could use it to read both the direction of Mecca as well as the distance to Mecca from any point P on the globe. Drawing by the late E.S. Kennedy (d. 2009, Kennedy *et al.* (1999)).

But that was not all. The sheer accumulation of mistakes, that were noted in the Greek traditional sources just like the ones we just enumerated, to others we have not mentioned yet, like the willingness of translators of the Greek texts into Arabic to correct the perceived textual errors in those Greek scientific sources, all indicate the creative and critical process with which Islamic civilisation approached the Greek sources. As an example of the textual corrections, one can note the effort of the Almagest Arabic translator, al-Ḥajjāj b. Maṭar (fl. c. 830) to correct what looked like a mistake in the original Greek text of the Almagest (IV, 3) where Ptolemy asserted that the length of the synodic lunar month of 29; 31, 50, 8, 20 days was determined by the division of the number of days separating two eclipses, 126007^d 1^h by the number of lunations 4,267. In fact, if one were to divide those two numbers, as al-Ḥajjāj apparently did, s/he would get the "correct" number 29; 31, 50, 8, 20 $=126007^d$ 1^h $/4,267$, which at face value must mean that Ptolemy had committed a textual error, and probably never carried out the division he reported that he did†. The important point to make here is that al-Ḥajjāj felt obliged to 'correct' the mistake as he was translating the text. This behavior does not happen with one who thought that the text he was translating was sacrosanct and needed to be preserved at any cost. Its translation was apparently executed for much more mundane reasons of utility and not for any desire the preserve the Greek tradition.

2. Questioning the deeper foundations of Greek astronomy

When all those blemishes, not to say outright mistakes and errors, are put together, and when one finds that the received Greek tradition was being deconstructed with such minute attention to details, one wonders what was left of the Greek astronomical tradition to be saved by a ninth-century Baghdadi astronomer? And to make matters worse, the theoretical cosmological basis of the Greek astronomical tradition, which was also being faced for the first time, as the Greek texts were being translated into Arabic, did not fare much better than the observational part of that astronomy, which, as we have seen, was erroneous on almost every count.

On the foundational level, Greek astronomy reached the Islamic world with a set of assumptions already embedded in it. In one of its mutations, the one that was admittedly adopted by Ptolemy in his *Almagest*‡, it conceived the world, in the Aristotelian style, as being made of a set of nested spheres, all centered on the Earth, which was by definition immobile and occupying the very center of heaviness of the cosmos. The spheres

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† For a possible explanation of the 'mistake' in the Ptolemaic text, see Aaboe (1995).
‡ From the very preface of the *Almagest*, Ptolemy declares his admiration of the Aristotelian epistemic vision of the universe and adopts it wholesale.

themselves carried the planets as well as the fixed stars and moved them in a uniform circular motion around an axis that passed through the earth, the north-south axis. And those spheres, together with the planets and stars that they carried, were all made of a simple fifth element, called ether by Aristotle. The natural motion of this element was circular, in contradistinction to the other four elements, of which the universe is made, and whose motions were all linear and in contrary directions. And because the celestial realm was immutable, it followed that all motions in the heaven had to be perceived as resulting from uniform circular motions for which there were no contraries to cause their decay and corruption.

All would have been fine, if the apparent motions of the universe indeed behaved as such. Starting with the Sun, Ptolemy quickly noticed that it did not describe uniform circles around the Earth, as would have been expected if the Sun were really being moved by a sphere that was concentric with the center of the Earth. The case of the planets was even worse for in addition to exhibiting varying individual motions, they displayed forward motion –that is, from west to east with respect to the starry background– stopped, reversed course, and then resumed their motion. In the case of the Sun, Ptolemy had to speculate that it moved on an eccentric circle, thus drawing sometimes closer to the Earth, while it moved farther at other times. The very notion of an eccentric was already a violation of the neat Aristotelian vision which stipulated the Earth as the center of heaviness around which all celestial elements had to move. In order to overcome this apparent handicap, Ptolemy resorted to the Apollonius theorem to equate the eccentric motion with an alternative concentric with an additional epicycle to perform the same resulting motion. That too violated the Aristotelian cosmology in its own ways, for it stipulated the existence of epicycles, moving on their own centers, which were relatively fixed centers of heaviness, out there in the celestial realm where no such variations were expected to be found. In the case of the other planets, Saturn, Jupiter, Mars and Venus, the situation got even worse still, for they all seem to exhibit motions that were best described by Ptolemy as taking place in such a way that the individual epicycles of those planets seemed to move across the starry background as if they were performing uniform circular motions, not around the Earth, nor around the deferent sphere that carried their epicycles, but around a fictitious point simply called the equator of motion by Ptolemy, and later summarily called the *equant*. The cases of the Moon and Mercury were even more complicated, and of course, more objectionable from the Aristotelian perspective, for they exhibited similar irregular behaviors, producing in the case of Mercury two perigees, and in the case of the Moon, a quarter Moon that was supposed to appear twice as big for an observer on the Earth, when the Moon would draw closer, almost halfway from where it would normally be when it was new or full Moon.

All these contradictions, abnormalities, and irregularities had to be explained away if one were to continue to adhere to the Aristotelian reigning cosmology; and it was the only cosmology to be had at the time. Starting with the very first two celestial spheres: (1) the one responsible for the daily motion of the whole heavens every twenty-four hours, and (2) the second much slower one that was responsible for the slow motion of the fixed stars that carried those stars in a motion of precession of about one degree every century according to Ptolemy and more like one degree every seventy-two years or so according to moderns. These two spheres had to be stipulated again because of the loyalty to the Aristotelian cosmology where every motion had to be accounted for by a mover, and in this case a sphere of its own.

As soon as the description of these motions reached the Islamic world through the Arabic translations of the Almagest, early ninth century astronomers, who were apparently active at the time of the translations and at times even patronising the translations

themselves, immediately noticed that it was not only the value of the motion of precession that was in question but that the nature of the motion of the eighth and ninth spheres that were responsible for the daily and precessional motions just mentioned. By direct observation, following methods and techniques already discussed earlier in connection with the re-examination of the Ptolemaic parameters in the observational part of this paper, and using more sophisticated and larger instruments as we have already noted, the ninth century astronomers found the value of precession to be more like one degree per approximately sixty-six or seventy years to be closer to what they saw with their own eyes than the provably erroneous value of one degree per century that was recorded in the Ptolemaic text.

More importantly, one of those ninth century astronomers and patron of translations, Muḥammad b. Mūsā b. Shākir (d. 873), noted the cosmological discrepancy in the description of the two spheres responsible for those motions. He noted that if there was a ninth sphere, stipulated to carry out the daily motion, and within it –and concentric with it– there was another sphere that was moved by it by the daily motion with which the whole apparent universe seems to move, but itself only responsible for the motion of precession, then there was a problem in the manner in which the ninth sphere could possibly move the eighth as long as both spheres were concentric. According to Muḥammad b. Mūsā the problem lies in the impossibility of one sphere moving another without having some form of friction, grabbing, or as he says intrusion (*nashabat*) of one into the other. The assumption is that both spheres, as well as the stars and planets they carried, were all composed of the simple element ether, and thus no such friction, grabbing, or intrusion would therefore be possible to occur between them. This situation led Muḥammad b. Mūsā to conclude "*Therefore, it has become clear that it is not in any way possible that there be beyond the orb of the fixed stars a circular body which moves by its own particular motion, and moves through that motion the orb of the fixed stars around the centre of the world.*" (Saliba 1994).

On cosmological grounds alone, the very first motions, that were rationalised by Ptolemy as he followed in the footsteps of Aristotle and assumed separate spheres for them to account for their different motions, those motions could not be justified on those very grounds, because their description had to violate the very ethereal nature of which those two spheres were supposed to be composed in order for the said motions to take place. This divorce between the Aristotelian cosmological assumptions which were accepted by Ptolemy and Ptolemy's own description of the bodies that were supposed to be responsible for the apparent motions in the celestial realm was a serious blow to the image of the Greek astronomical tradition, as it would have been a blow to any science that was supposed to adhere to a consistency between the original assumptions accepted in that science and the results that science proclaimed in its development.

Ptolemy was apparently aware of this divorce, which may account for his production of two separate books to describe more or less the same phenomena, namely the *Almagest*, where the observational aspects of astronomy and the uniform motions responsible for the observed phenomena are detailed and developed, with computational techniques, tables, equations and the like, and the *Planetary Hypothesis*, where he renders the cosmological accounts that were responsible for those phenomena. The two works were independently self contradictory, as we just saw in the case of the *Almagest*, but when they were read together, they became flagrantly contradictory, for it became obvious that the cosmology which was assumed and accepted in one was brazenly violated in the other. The situation became much worse, when attention was turned to the cosmological assumptions of the more complicated planetary motions that were described in the *Almagest*. As we just saw, all those motions assumed in one form or another the existence of arbitrary points,

called *equants*, around which either the deferent of the Moon and the director of Mercury or the epicycles of the other planets rotated at uniform circular motions. All of these irregularities shared one single feature, namely, that in all of them one would have to assume the existence of a sphere that could move uniformly in place about an axis that did not pass through its center, a sphere that is physically impossible to find in nature, and if one insists on calling such a body a sphere, then the word sphere would loose its meaning.

Exploration and exposure of those absurdities that were found lurking in almost all aspects of the Greek astronomical tradition became subjects of very hot debates throughout the Islamic civilisation, starting as we just saw as early as the ninth century, at the very same time when the masterpieces of the Greek astronomical tradition were being translated into Arabic. By the following century, there were full books devoted to those critiques, summarily titled as *shukūk* (doubts), or *istidrāk* (recapitulation) books, or the like. The best example of such books is Ibn al-Haytham's (d. 1040) text which was aptly called *al-Shukūk 'alā Baṭlamyūs* (*Dubitationes im Ptolemaeum*). In this work Ibn al-Haytham undertook the systematic project of exposing all the contradictions and faults of the Ptolemaic three major books, the *Almagest*, the *Planetary Hypothesis*, and the *Optics*, without even attempting to point the way as to how these problems and contradictions could be resolved.

About two centuries later, a Damascene astronomer by the name of Mu'ayyad al-Dīn al-'Urḍī (d. 1266) blamed Ibn al-Haytham just for that, namely that he only raised objections and did not offer solutions. In between the two astronomers the student of the famous Avicenna (d. 1037), by the name of Abu 'Ubayd al-Jūzjānī (d. c. 1070) attempted to resolve the problem of the equant by proposing a new non-Ptolemaic model of his own for the motion of the upper planets. His proposal can be categorised as amateurish at best, but its importance lies in the fact that it indicates how far afield this problem had already reached, even to the circles of the philosophers. With 'Urḍi the situation was quite different, for he did undertake to overhaul the whole of Greek planetary theories with an eye to rid their mathematical constructions of their contradictions with the physical cosmological realities†. But in order to do that, and in the case of resolving the problem of the equant, he had to propose a new mathematical lemma, now dubbed the 'Urḍi Lemma, which served the purpose adequately well. The Lemma became very attractive to later astronomers, and used by almost every serious theorist that followed, all the way and including Copernicus (d. 1543) himself. The statement of the Lemma (Fig. 8) was rather simple. It stated that if two equal lines formed equal angles, either internally or externally, with a third base line, then the line joining the other extremities of the two lines will always be parallel to the base line.

'Urḍi's colleague, collaborator, and one time boss, Naṣīr al-Dīn al-Ṭūsī (d. 1274) also had a theorem of his own to add which also solved the equant problem, both in the case of the lunar model as well as the model of the upper planets. Ṭūsī's theorem too was also used by Copernicus some two centuries later. The theorem itself, now dubbed the Ṭūsī Couple (Fig. 9) stipulated that if one had two spheres, one inside the other with a diameter half the size of the larger sphere, internally tangent, and if the larger sphere moved in a specific direction in place, and the smaller sphere moved in the opposite direction at twice the speed, and in place as well, then the point of tangency would

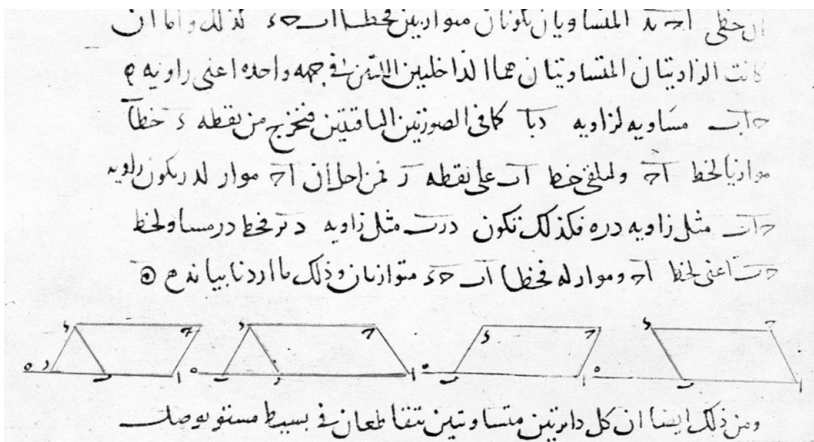† 'Urḍi's work has survived and was edited by Saliba (2001).

Figure 8. 'Urḍi's Lemma stated and proven for all the conditions of the angles between the two equal lines and the base line.

Figure 9. Statement of four different stages of the Ṭūsī Couple (*right*), and the proof of the general condition (*left*).

oscillate back and forth along the diameter of the larger sphere, thus producing a linear motion out of two uniform circular motions‡.

But the astronomer who lived a century after 'Urḍi and Ṭūsī, made the most use of the critiques of Greek astronomy, of course took full advantage of the mathematical innovations of his predecessors, and managed to systematically reformulate Greek astronomical planetary theories on much more rigorous basis, was Ibn al-Shāṭir (d. 1375) of Damascus. His major reform of Greek planetary theories included among other things a strict adherence to the principle of consistency between cosmological presuppositions of astronomical theory and the mathematical representations of the motions the theory described. This consistency went as far as systematising astronomy with its Aristotelian cosmological presuppositions, the only presuppositions available at the time, and at the same time taking issue with Aristotle himself whenever he found the latter contradicting himself. While he would insist that the Aristotelian conception of the centrality of the Earth in the general cosmological perspective should be taken seriously –and for that reason all the mathematical representations of the alternative planetary theories Ibn al-Shāṭir proposed to supersede the Greek ones were strictly geocentric– he still could not accept the Aristotelian presupposition that all the celestial bodies were made of the same

‡ For an edition, translation and commentary of the work of Ṭūsī in which this theorem was developed and applied in the construction of new planetary theories, see Ragep (1993).
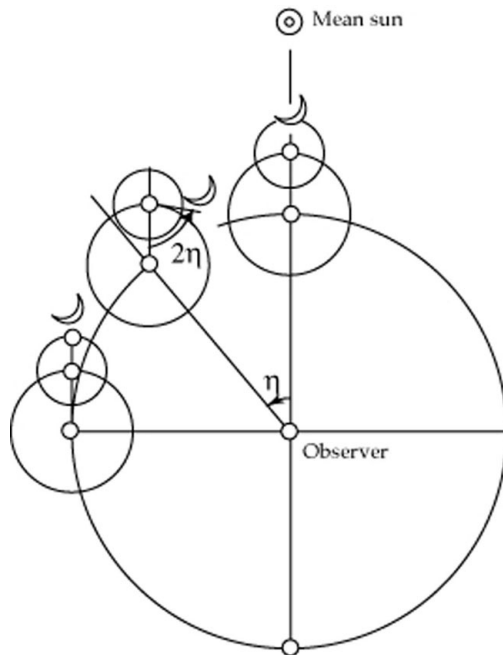
Figure 10. Depiction of the same model representing the motion of the moon in the works of Ibn al-Shāṭir (d. 1375) and Copernicus (d. 1543).

simple element ether. The reason for this discrepancy is that the planets and the stars were made of ether and emitted or reflected light that we can all see, while the spheres that carried those planets and stars, which were also made of the same ether did not emit such light. To him this must mean that there must be some form of composition in the celestial realm and that it was not all made of the simple element ether as Aristotle would assert. Using that loophole in the Aristotelian universe, he felt that he could make use of as many epicycles as he needed in order to get rid of the awkward Greek theoretical failings since he could classify those epicycles as being of the same kind of composition as the other compositions the Aristotelian element ether exhibited.

Ibn al-Shāṭir's remarkably successful project must have impressed Copernicus, who obviously knew of the works of his predecessor, so much that he took some of the mathematical constructions of Ibn al-Shāṭir and embedded them in his own. The most easily accessible and remarkable example was the mathematical representation of the motions of the Moon (Fig. 10) which reveals the direct dependence between the two astronomers, and the unlikelihood of mere coincidence, when vector for vector, to use anachronistic terms, angle for angle, epicycle for epicycle, double epicycle for double epicycle, and motion for motion were all identical in the works of both astronomers.

This was not all. Research of the last few decades has also unearthed other puzzling similarities between the works of the two men. Moreover, the same research also demonstrated that Copernicus may have had access to the works of other astronomers from the Islamic world, like Naṣīr al-Dīn al-Ṭūsī's, whose Ṭūsī Couple was already noted and used by Copernicus as well (Fig. 11).

This was not all. Ibn al-Shāṭir used the Ṭūsī Couple in his construction of the mathematical model that depicted the motions of the planet Mercury. And so did Copernicus. But there the similarities went even further and became more puzzling. In that instance, it was Noel Swerdlow (Swerdlow 1973), in his translation of Copernicus's first astronom-
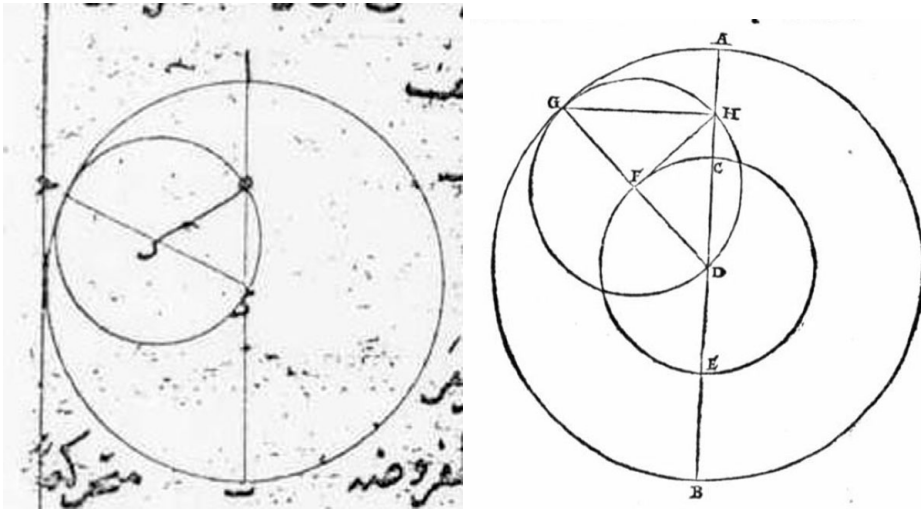
Figure 11. Proof of the Ṭūsī Couple. (*Left*) The proof that was offered by Ṭūsī in 1259, while that on the *right* was offered by Copernicus in 1543. It was the late Willy Hartner who first noticed that even the lettering in the two diagrams was for all intents and purposes identical, i.e. where Ṭūsī had "*alif*" in the Arabic diagram Copernicus had "*A*", where Ṭūsī had "*bā*", Copernicus had "*B*" and so on. The only difference between the two letterings is the point designating the center of the smaller epicycle, called "*zain*" in Ṭūsī's diagram and was misread as "*F*" in the Copernican diagram simply because of the very similar appearance of those two letters *Z* and *F* in the Arabic alphabet, especially in manuscripts.

ical work the *Commentariolus*, who first noticed that in Copernicus's own description of how the Mercury model worked he committed a revealing error which simply signaled that Copernicus did not fully understand all the intricacies of that model. When we know that the Copernican model was once again the same as that of Ibn al-Shāṭir (Fig. 12) the plot thickened, and ruled out all pretences of coincidental, accidental or independent discoveries of the two astronomers.

In the relatively complicated mathematical model depicting the motion of the planet Mercury, both Ibn al-Shāṭir and after him Copernicus resorted to the use of the Ṭūsī Couple in order to allow the orbit of Mercury to vary in size as the mean position of Mercury moved along the various points of its orbit. Ibn al-Shāṭir's purpose of his reconstruction of the Mercury model was to alleviate the Ptolemaic absurd use of the fictitious equant and replace its effect with a series of epicycles, Ṭūsī Couple included, so that it would still account for the Ptolemaic observations that gave Mercury two perigees at $\pm 120°$ away from the apogee. By definition, the elongation angle of Mercury is at its largest when Mercury is at one of its perigees, which means that the orbit of Mercury would appear at its largest when it is closest to the earth at those two perigees. Copernicus seems to have confused the absolute size of an object with its apparent size, and thus did not realise that the apparent size of Mercury's orbit would still be at its largest size at one of the two perigees of Mercury instead of its apparent size at quadrature when it is 90° away from the apogee, despite the fact that the absolute size of the orbit would be relatively smaller at perigee than at quadrature. One cannot explain this confusion away, and must conclude that Copernicus was working with a description of Ibn al-Shāṭir's model that he did not fully understand. In Swerdlow's own words:

> "*This misunderstanding must mean that Copernicus did not know the relation of the model to Mercury's apparent motion. Thus it could hardly be his own invention for, if it were, he would certainly have described its fundamental purpose rather*
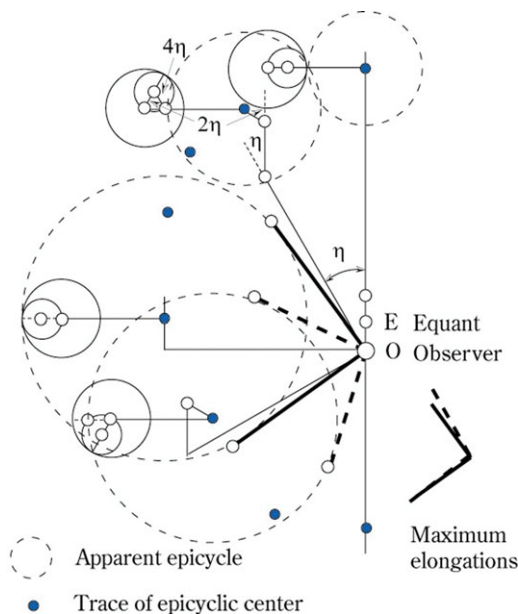
Figure 12. Ibn al-Shāṭir's model for the motion of Mercury which was used by Copernicus. In describing its workings Copernicus erroneously concluded that Mercury's orbit, in dashed circles, appeared largest at 90° away from the apogee, thus forgetting that the size of an object depends on its absolute size as well as on its distance from the observer. For an observer on Earth the relatively smaller orbit at ±120° from the apogee would still look bigger to the observer since it would be closer to Earth as marked by the elongation angles drawn to scale and where the angle drawn in dashed lines is markedly larger than the one drawn in continuous lines.

> than write the absurd statement that Mercury "appears" to move in a larger orbit when the Earth is 90° from the apsidal line. The only alternative, therefore, is that he copied it without fully understanding what it was really about. Since it is Ibn ash-Shāṭir's model, this is further evidence, and perhaps the best evidence, that Copernicus was in fact copying without full understanding from some other source, and this source would be an as yet unknown transmission to the west of Ibn ash-Shāṭir's planetary theory." (Swerdlow 1973)

3. Conclusion

All these similarities between the works of Copernicus and those of his predecessors from the Islamic world cannot *all* be simply brushed away as mere coincidences as is usually done by naïve and apologetic Copernican scholars. When the model of the Moon is identical to that of Ibn al-Shāṭir, and the model for the upper planets employs the same 'Urḍī Lemma, and now Mercury's model being identical to that of Ibn al-Shāṭir with a revealing puzzling misrepresentation by Copernicus of how the model worked, when *all* this evidence is added to the same use of alphabetic letterings in the proof of the Ṭūsī Couple –with its own misreading of a *Z* for *F* because the two letters are very similar in the Arabic script– then it becomes too far fetched, if not simply absurd, to argue that Copernicus was working independently of the Islamic astronomical tradition.

What makes things even more puzzling is the fact that none of Copernicus's predecessors have argued for a heliocentric universe, which would have been a cosmological oddity at the time, and which is admittedly Copernicus's claim to fame much more than the mathematical intricacies we have been pointing to. But as we have been arguing all along

that both the Greek tradition, as well as the Islamic astronomical tradition –with the sole exception of directly confronting Aristotle's conception of the element ether in the case of Ibn al-Shāṭir– were all well wedded to the Aristotelian cosmological vision of the universe as the only cosmology available at the time. All that was in the pre-Newtonian universe. And in that universe heliocentrism would have been completely out of tune. Furthermore, Copernicus himself objects in the introduction of the *Commentariolus* to the concept of the equant, and that objection makes sense only from an Aristotelian perspective, that is from the perspective of thinking of spheres as real bodies that moved in a cosmological universe well arranged by Aristotle and centered on the earth. So if Copernicus was dissatisfied with the equant –because it violated the Aristotelian cosmological presuppositions– then why would he turn around and intentionally fix all the mathematical models to fit the Aristotelian presuppositions exactly as was done by his predecessors from the Islamic world that he was so desperately mimicking, and at the very end hold the Sun fixed and move everything with all the mathematical models that he pieced together to adopt a heliocentric universe. This is probably the most important puzzle that the Copernican scholars still have to solve. Of all the apologetic attempts that have been put forward so far to rescue Copernicus from what looks like a flagrant violation of his own accepted principles, none of them makes any dent in the resolution of this cosmological puzzle. And this is not the place to review the inner contradictions and absurdities of those attempts.

Returning to the Islamic reception of the Greek scientific tradition I hope the reader can now see how this Greek tradition was indeed deeply questioned and dissected, and its constituent parts were first critiqued, modified, and then reconstructed, added to, mathematical rescued, and fully overhauled in order to fit a new cultural framework and a better perception of the inner consistency of science. And I hope that after this quick survey of the developments that took place in the Islamic tradition one can fully appreciate how this reconstruction made the Islamic scientific tradition particularly attractive to the Renaissance scientists, for it was correctly perceived as a rebellion against the antiquated fault-ridden older Greek tradition, and a fundamental reconstruction of that tradition on much sounder basis.

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