# THE SYMBOL FOR ZERO.

THE following letter and the reply explain themselves :

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REF. { *Yours* / *Ours* CT/I.S.

COUNTY BOROUGH OF TYNEMOUTH,
ELECTRICITY WORKS,
NORTH SHIELDS,
*Feb.* 1923.

SIR,

Much of the difficulty that learners find, when they begin higher mathematics, is due to the fact that the symbol "0" has two entirely different meanings, and it is not made clear to them which meaning is involved in the individual functions which they are trying to understand. In his early work the student's idea of "0" is expressed by the equation $x-x=0$. When he comes to higher mathematics, sometimes he is dealing with this 0 and sometimes with the other 0, which is the value of $\frac{x}{\infty}$, $x$ being finite. It would clear the air if the second 0 was expressed by a different symbol, say $\bar{0}$.

The learner would then be able to understand the different properties of the two nothings : such as that while $3\times 0=4\times 0$, $3\times\bar{0}$ does not necessarily equal $4\times\bar{0}$. Also $\frac{x}{0}$ has no meaning, but $\frac{x}{\bar{0}}=\infty$, $x$ being finite. The meaning of the equation $\sin\bar{0}=\bar{0}$ becomes quite clear. The equation given in so many text-books that $\sin\theta=\theta$ when $\theta=0$ has worried numberless students, and it is of course incorrect.

The equation $\frac{\theta-\sin\theta}{\tan\theta-\theta}=\frac{1}{2}$ when $\theta=0$ has a mysterious appearance, where the text-book has stated that when $\theta=0$, $\sin\theta$, $\theta$, and $\tan\theta$ are all identical, but it becomes simple when the symbol $\bar{0}$ is used.

It would also become clear that $\log 0$ has no meaning, as it stands for $\log(x-x)$; on the other hand $\log\bar{0}$ is equivalent to $\log x-\log\infty$, or to $\log\frac{x}{\infty}$, $x$ being finite, and therefore it has a meaning.

The statements in text-books that $1^{\infty}$ and $0^{\infty}$ have values other than I and 0 are very puzzling, and actually incorrect. On the other hand $(1+\bar{0})^{\infty}$ and $\bar{0}^{\infty}$ have values other than 1 and 0, quite evidently.

To the trained mathematician it may appear that students ought to understand these things, but from actual experience I can affirm that many men, possessed of sound engineering instincts, are quite unable to comprehend mathematics as presented by many standard text-books, but they do see the meaning when presented with the symbol $\bar{0}$.—Yours faithfully,

C. TURNBULL.

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BANGOR, 19*th Feb.*, 1923.

MY DEAR SIR,

Your letter greatly interested me, because I feel that your proposals for the different meanings of the symbol for zero arose from difficulties very

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