Colloques internationaux du C.N.R.S.
N° 263 - L'ÉVOLUTION DES GALAXIES ET SES IMPLICATIONS COSMOLOGIQUES

ANOMALOUS HUBBLE EXPANSION AND INHOMOGENEOUS COSMOLOGICAL MODELS

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Résumé

Les amas ou superamas de galaxies sont assimilés à des condensations en expansion, plongées dans l'univers cosmologique. L'application des résultats déduits de modèles non homogènes en Relativité Générale au cas particulier du Superamas Local permet de rendre compte des observations. Elle montre aussi l'existence de modèles d'amas tels que la lumière qui les traverse présente (pour un observateur extérieur à l'amas) un décalage vers le rouge supplémentaire.

The different kinds of so-called "anomalous" extra-galactic redshifts could prove, if they are confirmed, that the Hubble expansion is not the same all over the sky :

1. The value of Hubble's parameter would be different inside the Local Supercluster of galaxies (LSG) of its value outside.
2. Some evidence for an anisotropy in the Hubble expansion has been given by Rubin et al. (1973). Karoji, Nottale and Vigier (1975) have shown that this result can be related to the peculiar distribution of compact clusters of galaxies.

Many authors claim that these results, deduced from observations of redshifts, can be interpreted only by a new theory of tired light involving a process still unknown.

We have shown that it is also possible to account for these phenomena in the classical framework of relativistic cosmology, but with a non-homogeneous model.

Clearly, our universe, on a certain medium scale, is not homogeneous and uniformly dust-filled. The Friedmann models can only be a first step to a more elaborate description.

We have studied a non-homogeneous cosmological solution which is consistent with the observational data, at least those that were known to us before this Colloquium.

1. Let us first consider the case of the L.S.G. To simplify the mathematical treatment, we assume spherical symmetry and the following scheme :
1 a central condensation, with radius R_1, described by an expanding Friedmann model. This is a very crude scheme of the LSG which, in fact, is flattened, and rotating as well as expanding.
2 an empty intermediate region surrounding the preceding material distribution. Being empty, its dimensions will be minimum. This is the vacuole-model, already introduced by many authors.
3 the expanding universe in which the preceding vacuole is embedded.
It is possible, for this non-homogeneous model, to get a global, exact and complete solution of Einstein's equations. This solution is perfectly continuous, the junction conditions being satisfied at the two interior boundaries.

The solution

It is well known that the field inside the vacuole 2 is given by the static Schwarzschild solution. What becomes time-dependent, in this model, are the two boundaries R_1 and R_2 of this region.
As usual, the material in 1 and 3 will be described as an ideal fluid without pressure $(T^{\alpha\beta} = \rho u^\alpha u^\beta)$. Among the numerous spherically symmetric solutions of Einstein's equations for dust-filled inhomogeneous models, a very convenient one is Tolman's solution (1934). In a co-moving coordinate system, it may be written

(1) $ds^2 = dt^2 - [R'^2/1+f(r)]\,dr^2 - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$.

$f(r)$ is an arbitrary function of r only. $R(r,t)$ must satisfy, according to the field equations

(2) $\dot{R}^2 = f(r) + F(r)/R \quad , \quad (\dot{R} \equiv \partial R/\partial t \; ; \; R' \equiv \partial R/\partial r)$.

$F(r)$ is another arbitrary function of r .

The proper density is related to the metric by

(3) $8\pi\rho = F'/R'R^2$.

We shall limit ourselves to the peculiar case $f(r) = 0$, which corresponds to the parabolic models. The exact solution corresponding to the scheme adopted has been determined, in this case, by Papapetrou (1976). Its main features are the following :

(4)
$$\begin{cases} \text{1} \quad 0 \leq r < r_1 \;; \quad 4R^3 = 9\beta r^3 [t - T_0]^2 \;; \quad (T_0 = c^{te} > 0) \\ \qquad 8\pi\rho_1 = \dfrac{4}{3(t - T_0)^2} \\ \text{2} \quad r_1 \leq r < r_2 \;; \quad 4R^3 = 9\beta r_1^3 [t - T(r)]^2 \;; \quad (T'(r) < 0) \\ \qquad \rho_2 = 0 \,. \\ \text{3} \quad r_2 \leq r \;; \quad 4R^3 = 9\alpha r^3 t^2 \;; \\ \qquad 8\pi\rho_0 = \dfrac{4}{3t^2} \,. \end{cases}$$

The interior condensation 1 and the embedding exterior universe 3 are two Einstein-de Sitter models. The matching conditions that must be satisfied at the two boundaries ($r = r_1$ and $r = r_2$)of the vacuole, as shown by Mavridès (1976), reduce to

(5) $$\beta r_1^3 = \alpha r_2^3$$

The mass of the condensation 1 is then such that

(6) $$M(r_1) = F(r_1)/2 = \beta r_1^3/2 = (4/3)\pi R_1^3 \rho_1 = (4/3)\pi R_2^3 \rho_0 \,.$$

Application to the LSG.

Tolman (1934) had already shown that, in such a combination of uniform distributions, the dust in each zone behaves independently, without reference to the other parts. So, the general relations of the Friedmann models are still valid here for each zone, and in particular the equation

(7) $$(4\pi G/3)\rho = q_0 H^2 \,,$$

where H is Hubble's parameter and q_0 the deceleration factor. For dust-filled homogeneous models with flat 3-space - as are regions 1 and 3 - we know that (when $\Lambda = 0$)

(8) $$q_0 = 0,5 \,.$$

Let us admit, with Pecker (1975), that inside the LSG, the diameter of which is roughly 30 Mpc, the value of Hubble's parameter is

(9) $$H_{LSG} \sim 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \,.$$

Outside the LSG, H would reach its cosmological value

(10) $$H_0 \equiv H_{cosmological} \sim 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \,.$$

Consequently, we get from (7), (8), (9) and (10)

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(11) $$\frac{(\rho_1)_{LSG}}{\rho_0} = \frac{(H_{LSG})^2}{(H_0)^2} = 4 .$$

With the value (10) of H_0, we get from (7) and (8) for the density of matter in the universe ($\Lambda = 0$)

(12) ρ_0 (cosmological) $\sim 0{,}5 . 10^{-29}\ g.cm^{-3}$

Let us suppose that the mass of the Virgo cluster is about

(13) $M_{V.C} \sim 2 . 10^{14}\ M_\odot$.

We can then consider that the mass of the LSG is approximately

(14) $M(r_1) \equiv M_{LSG} \sim 2 . 10^{15}\ M_\odot$.

The mean density ρ_1 of the LSG, as deduced from (6) is in good agreement with that obtained taking into account (11) and (12) if

(15) $R_1 \sim 12\ Mpc$,

a value which fits well de Vaucouleurs' estimation (1970). The junction conditions (5) and (6) then lead to the following value for the radius of the vacuole :

(16) $R_2 \sim 19{,}2\ Mpc$.

Bondi (1947) has shown that the variable $R(r, t)$, in these inhomogeneous models, is equivalent to the luminosity distance, from the origin, of a source (r, θ, φ) at time t. The study of the propagation of light in the model justifies the assimilation of observed redshifts with velocity shifts :

(17) $z_1 \sim v_1$, (in relativistic units : c=1).

We can thus write, using the habitual definition $(z \equiv v = HR)$

(18) $$H = \frac{\text{velocity}}{\text{distance}} = \frac{\dot{R}}{R} .$$

In region 1 , we have

(19) $$H_1 = \frac{2}{3(t - T_0)} \sim 100\ km\ s^{-1}\ Mpc^{-1} .$$

In region 3 , we obtain

(20) $$H_0 = \frac{2}{3t} \sim 50\ km\ s^{-1}\ Mpc^{-1} .$$

Consequently, when one takes into account (4) and (12), one gets

(21) $$T_0 = t/2 = (1/2)\,(6\pi G \rho_0)^{-\frac{1}{2}} \sim 2 . 10^{17}\ s .$$

t is the present time of observation.

So, in spite of its great simplification, and the controversial values (9) and (10), this scheme is in quite good agreement with the numerical data adopted for the Local Supercluster. A better fitting would be obtained with a more realistic model.

ℓ. With the same type of relativistic solution, but slightly modified, it is also possible, as shown by A. Tarantola (1976) to account for the effect found by Karoji, Nottale and Vigier (1975) : an excess of redshift for light going through very compact clusters. A definite cluster of supposed spherical symmetry is there considered as a region where the density is much larger than that of the cosmological fluid on which it is superimposed. The other condensations are smoothed out in this first step. The central part of the cluster can be expanding, in equilibrium, or contracting. In every case, it is possible to account for the "anomalous" redshifts within the classical theory of General Relativity and expanding universe. But this universe, with uniformly distributed condensations, would be non-homogeneous on the scale of clusters or superclusters of galaxies.

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DISCUSSION

R.C. ROEDER: If Miss Mavridis were to start her model off at high densities, she would have the situation of the classical White Hole.

Also, C.C. Dyer has recently used a similar model to discuss gravitational effects on the 3°K cosmic background. His paper was published in Mon. Not. Roy. Astr. Soc. this spring.

S. MAVRIDIS: I agree. In the complete mathematical solution, the inner condensation behaves like a White Hole exploding at $t = T_o$. But this model is only a very simplified scheme for a cluster or supercluster of galaxies.

A. VIGNATO: What about the time evolution of the vacuole model? As Bonnor pointed out if you have 2 regions (0,1) described by a parabolic metrics you have an asymptotical tendency to have $\rho_o = \rho_1$.
Is that true for your model?

S. MAVRIDIS: Yes, it is. As can be seen from table (4), the ratio $\rho_1/\rho_o \to 1$ when $t \to \infty$, whatever the value of T_o may be.

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