# A UNIFIED THEORY OF CORONAL HEATING

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This presentation focuses upon the coronal heating problem and reports the results of Ionson's (1984) unified theory of electrodynamic heating. This generalized theory, which is based upon Ionson's (1982) LRC approach, unveils a variety of new heating mechanisms and links together previously proposed processes. Specifically, Ionson (1984) has derived a standing wave equation for the global current, I, driven by emfs that are generated by the $\beta \gtrsim 1$ convection. This global electrodynamics equation has the same form as a driven LRC equation where the equivalent inductance, $L=4\ell/\pi c^2$, scales with the coronal loop length and where the equivalent capacitance, $C=c^2\ell/4\pi v_A^2$, is essentially the product of the free space capacitance, $\ell/4\pi$, and the low frequency dielectric constant, $c^2/v_A^2$. The driving emf, $\mathcal{E}=vBa/c$, is a formal integration constant associated with the convective stressing of $\beta \gtrsim 1$ magnetic fields. Since the transition from the $\beta \gtrsim 1$ driver to the $\beta<1$ coronal loop is typically small compared to the "wavelength" of the associated magnetic fluctuation, this integration constant is not sensitive to details of the transition zone. The total resistance, $R_{tot} = L(1/t_{diss}+1/t_{phase}+1/t_{leak})$, represents electrodynamic energy "loss" from dissipation, magnetic stress leakage out of the loop and phase-mixing. These three processes have been parameterized by appropriate timescales. Note that $R_{leak}=L/t_{leak}$ and $R_{phase}=L/t_{phase}$ do not result in resistive heating but do participate in limiting the amplitude of the global current, I. This is fairly obvious with regard to magnetic stress leakage but not for phase-mixing. The phase-mixing resistance, $R_{phase}$, represents coupling between the global current and the local current density. Since the global current is essentially an integration of the local currents, the degree of coherency between the local currents can play an important role in determining the ultimate amplitude of I. The rate at which coherency between the local currents is lost is given by the phase-mixing time, $t_{phase}$. A loss of coherency implies a corresponding reduction in the amplitude of I. In this sense, $R_{phase}$ measures the phase-mixing contribution to the global current limitation process.



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The time averaged coronal heating flux, $F_H$, is given by

$$F_H = \frac{2\langle I^2 R_{diss}\rangle}{(\pi a^2/4)} \equiv F_{max}\,\epsilon \qquad (1)$$

where the factor of two implies two footpoint drivers per loop and where $\pi a^2/4$ represents the cross sectional area of a loop within the corona. The maximum flux, $F_{max}$, of energy available to the coronal portion of a loop is given by

$$F_{max} = 4\left(\frac{B}{B_{\beta \gtrsim 1}}\right)\left(\frac{4\,v_A^{\beta \gtrsim 1}}{v_A}\right) v_A^{\beta \gtrsim 1}\left(\tfrac{1}{2}\rho\, v_{tot}^2\right)_{\beta \gtrsim 1} \qquad (2)$$

which can be interpreted by noting that the first factor of four accounts for two footpoints per loop and two excitation polarizations; the $B/B_{\beta \gtrsim 1}$ term results from magnetic expansion into the corona; and the third term represents a transmission coefficient which in this case is given by the Fresnel relation, $4v_A^{\beta \gtrsim 1}/v_A$. The fourth term, $v_A^{\beta \gtrsim 1}(1/2\rho v_{tot}^2)$, represents the maximum available flux of electrodynamic energy within the $\beta \gtrsim 1$ region. The fraction of $F_{max}$ that actually does enter the loop, heating the contained coronal plasma is denoted by an electrodynamic coupling efficiency, $\epsilon$. The electrodynamic coupling efficiency is a convolution of the normalized convection velocity spectrum with a frequency dependent velocity-magnetic field interaction parameter, $e(\nu;\, t_A,\, Q_{tot},\, Q_{diss})$, i.e.,

$$\epsilon \equiv 2\int_0^\infty \left(\frac{\langle V_\perp^2\rangle_\nu}{V_{tot}^2}\right)_{\beta \gtrsim 1} e(\nu; t_A, Q_{tot}, Q_{diss})\, d\nu \qquad (3)$$

$$e(\nu;\, t_A,\, Q_{tot},\, Q_{diss}) = \frac{Q_{tot}^2}{Q_{diss}\left[1 + \left(\nu t_A - \frac{1}{\nu t_A}\right)^2 Q_{tot}^2\right]} \qquad (4)$$

with the coronal loop quality factors, $Q_{diss}=2\pi t_{diss}/t_A$ and $Q_{tot}=2\pi/(t_A/t_{diss}+t_A/t_{phase}+t_A/t_{leak})$. The velocity-magnetic field interaction parameter peaks at the resonance frequency, $\nu_{res}=t_A^{-1}$, with a maximum value $e_{max} = Q_{tot}^2/Q_{diss}$ which can be thought of as a measure of the magnitude of the magnetic stressing rate spectrum with respect to the $\beta \gtrsim 1$ mechanical energy spectrum. The characteristic width of $e(\nu;\, t_A,\, Q_{tot},\, Q_{diss})$ about this maximum can be thought of as an interaction bandwidth, $(\Delta\nu)=\pi/t_A Q_{tot}$.

A general form for the normalized velocity spectrum that parameterizes the details is assumed, viz.,

$$\frac{\langle V_{\perp}^{2}\rangle_{\nu}}{V_{tot}^{2}} = \left(\frac{t_c}{\pi}\right)\left[\frac{1}{1 + (\nu t_p - \frac{1}{\nu t_p})^2 (\frac{t_c}{t_p})^2}\right] \tag{5}$$

where $t_C$ represents the correlation time and where $t_p^{-1}$ is the frequency at which the spectrum peaks. Using the above parameterized form for the convection spectrum in equation (3) results in the following general expression for the electrodynamic coupling efficiency, $\mathcal{E}$:

$$\epsilon = \left(\frac{t_c}{t_A}\right)\left(\frac{Q_{tot}}{Q_{diss}}\right) \begin{cases} [1 + (t_p/t_A - t_A/t_p)^2 (\frac{t_c}{t_p})^2]^{-1} ; & Q_{tot} > 1 \quad \text{HIGH-QUALITY COUPLING} \\ \frac{Q_{tot}}{(t_c/t_A) + Q_{tot}} ; & Q_{tot} < 1 \quad \text{LOW-QUALITY COUPLING} \\ 1 ; & t_c < t_A, t_p, t_A Q_{tot} \quad \text{STOCHASTIC COUPLING} \end{cases} \tag{6}$$

Equation (6) for the electrodynamic coupling coefficient highlights three general coupling categories.

If $Q_{tot}>1$, than a coronal loop responds to external driving as a high quality resonance cavity in the sense that the velocity-magnetic field interaction parameter is sharply peaked bout the resonance frequency. Therefore, a high quality coronal loop with $Q_{tot}>1$ interacts only with a small portion of the convection spectrum centered about the resonance frequency. This, however, can still lead to significant power absorption because the corresponding magnetic stressing rate is quite large within ($\Delta\nu$). Specifically, the maximum value of the velocity-magnetic field interaction parameter, $e_{max}>1$.

If $Q_{tot}<1$ then there is no preferred excitation frequency because $t_A\Delta\nu>1$. A large interaction bandwidth does not necessarily imply a large absorption of power because the magnetic stressing rate is small within $\Delta\nu$ since $e_{max}<1$. It's also important to note that although $\Delta\nu$ is large compared to $t_A^{-1}$ it could be small compared to the correlation width, $t_C$, of the convection spectrum. In this regard, the maximum value of $\Delta\nu$ is given by $t_C^{-1}$ and the loop interacts with the entire convection spectrum. In fact, when $t_C$ is smaller than all other timescales, both the high and low quality coupling categories degenerate into a third general category describing stochastic coupling.

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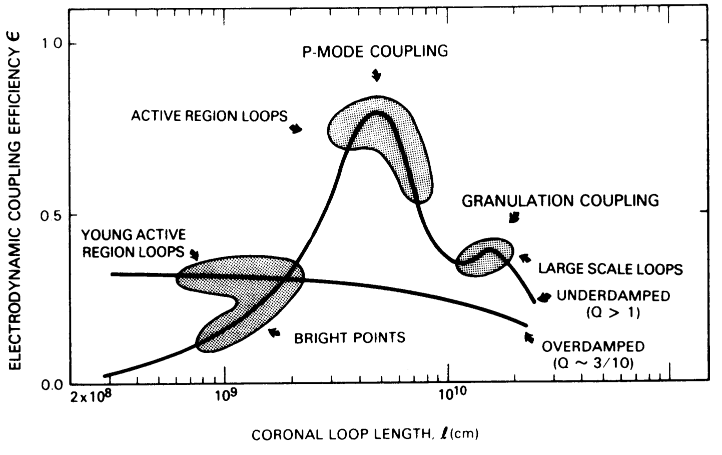
Each of the above general representations embodies three special cases. These special cases highlight the dominant "loss" process responsible for limiting the magnetic stressing rate with respect to the available convection energy. As discussed earlier, $e_{max}=Q^2_{tot}/Q_{diss}$, represents a measure of the magnetic amplitude limitation process. The three special cases of interest are "dissipation limited" when $Q_{tot} \simeq Q_{diss}$, phase-mixed limited" when $Q_{tot} \cong Q_{phase}$ and "leakage limited" when $Q_{tot} \cong Q_{phase}$. This classification scheme consolidates a variety of new heating mechanisms and also illustrates that existing mechanisms are actually special cases of a much more general formalism. Specifically, Alfvenic surface wave heating is a high-quality, phase-mixed limited mechanism, resonant electrodynamic heating is a high-quality, dissipation-limited mechanism, dynamical dissipation is a low-quality dissipation-limited mechanism and stochastic magnetic pumping is a stochastic, dissipation-limited mechanism.

An application of this theory to solar coronal loops indicates that the total quality is essentially equal to the dissipative quality (i.e., dissipation limited); and that the general coupling category changes as loops age and hence increase in length. Specifically, young active region loops and possibly bright points are overdamped systems (i.e., $Q_{diss}<1$), heated at a rate that depends upon the dissipation time $t_{diss}$, while older active region and large scale loops are underdamped systems (i.e., $Q_{diss}>1$), heated at a rate that is quantitatively independent of the dissipation time. Furthermore, it appears that active region loops are heated by electrodynamically coupling to $\beta \gtrsim 1$ p-mode oscillations, while large scale loops are heated by coupling to the solar granulation.




Ionson, J. A., 1982, Ap. J., 254, 318.
Ionson, J. A., 1984, Ap. J., Jan. 1.

DISCUSSION

*Rust:* I do not think there are definitive studies to indicate that coronal loop lengths are "quantized" as indicated by your theory. Probably the visible loop lengths are determined by the magnetic field distribution, not by the heating mechanism. That is, coronal pictures suggest that any closed loop will be heated.

*Ionson:* The theory does not predict quantized coronal loop lengths. The observations do, however, as indicated by the blocks of data illustrated in the last figure. In other words, coronal pictures do not suggest that any closed loop will be heated in a thermally stable sense.

*Bratenahl:* Your driving spectrum did not include the supergranulation time scale ∿ 20 hours. It is a strong peak in the overall spectrum.

*Ionson:* Such long period power only resonates with loops with lengths larger than the solar radius. There are very few loops with such large sizes.

*Krishan:* How do you make contact with the heating theories invoking flow of current in thin sheaths?

*Ionson:* The physics of thin sheaths is absorbed within $t_{diss}$ which does not appear in the final result for the heating flux, $F_H$. Therefore, in many cases one can predict heating rates without delving into the details of current sheaths.