ON THE TECHNIQUE OF THE INVERSION OF HELIOSEISMOLOGICAL DATA

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ABSTRACT. The asymptotic inversion technique is developed, including independent determination of the sound speed profile and frequency dependence of the effective phase shift. Numerical results are presented.

Application of the asymptotic theory to the study of the solar five-minute oscillations leads to the equation

$$\int_{r_1}^{r_2} \left(1-N^2/\omega^2\right)^{1/2}\left(\omega^2/c^2-L^2/r^2\right)^{1/2} dr = \pi n, \tag{1}$$

where r_1 and r_2 are inner and outer turning points and $L^2=(\ell+1/2)^2$. This equation differ from those given by Unno et al.(1979) in that we use $L^2=(\ell+1/2)^2$ instead of $\ell(\ell+1)$. It provides the significant improvement of JWKB analysis in the central regions (Kemble, 1937; Langer, 1937). Because JWKB analysis is invalid near the surface, an additional phase shift must be added to the right-hand side of eq.(1). Since N^2 in interior is usually small compared with ω^2, we have (Gough, 1984)

$$F(w)-\frac{1}{\omega^2}\Psi(w)-\pi\frac{\alpha(\omega)}{\omega} \simeq \pi\frac{n}{\omega}, \qquad w=\omega/L, \tag{2}$$

$$F(w)=\int_{r_1}^{R_\odot}\left(\frac{r^2}{c^2}-\frac{1}{w^2}\right)^{1/2}\frac{dr}{r}, \tag{3}$$

$$\Psi(w)=\frac{1}{2}\int_{r_1}^{R_\odot} N_i^2\left(\frac{r^2}{c^2}-\frac{1}{w^2}\right)\frac{dr}{r}. \tag{4}$$

Here $N_i(r)$ denotes buoyancy frequency in the interior and the upper limits in integrals are taken to be the solar surface. If $F(w)$ is known, eq.(3) can be inverted to determine r as a function of c/r :

$$r = R_\odot \exp\left\{-\frac{2}{\pi}\int_{c_s/R_\odot}^{c/r}\left(\frac{1}{w^2}-\frac{r^2}{c^2}\right)^{-1/2}\frac{dF}{dw}\,dw\right\} \tag{5}$$

(Brodsky and Levshin, 1979; Gough, 1984). If $\Psi(w)$ is also known from the experimental data, eq.(4) can be inverted to determine $N_i^2(r)$:

$$N_i^2(r)=-\frac{4}{\pi}\frac{d}{d\ln r}\int_{c_s/R_\odot}^{c/r}\left(\frac{1}{w^2}-\frac{r^2}{c^2}\right)^{-1/2}\frac{d\Psi}{dw}\,dw. \tag{6}$$

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The eq.(5) have been used by Christensen-Dalsgaard et al.(1985) for the determination of the sound speed in the solar interior, using $L^2 = \ell(\ell+1)$ and neglecting both the effects of buoyancy and the frequency dependence of α. For the calculation of dF/dw from the experimental data, they assumed the constant value of α which minimizes the scatter of experimental data around a single curve $F(w)$.

Three terms in the left-hand side of eq.(2) contain the information about the sound speed, buoyancy frequency in the interior and the structure of the outermost reflecting layers, respectively. Due to the different functional dependence on w and ω, these terms can be determined from the experimental data separately. Eigenfrequencies $\omega_{n\ell}$ may be considered as distinct values of unique continuous function $\omega(n,L)$. Then

$$\frac{dF(w)}{dw} = \pi\frac{L^2}{\omega^2}\frac{\partial\omega/\partial L}{\partial\omega/\partial n} + \frac{1}{\omega^2}\frac{d\Psi(w)}{dw}, \tag{7}$$

$$\frac{d\Psi(w)}{dw} = \frac{\pi L^2}{2(\partial\omega/\partial n)^2}\left[\frac{\partial\omega/\partial L}{\partial\omega/\partial n}\left(L\frac{\partial\omega}{\partial L}-\omega\right)\frac{\partial^2\omega}{\partial n^2} + \left(\omega-2L\frac{\partial\omega}{\partial L}\right)\frac{\partial^2\omega}{\partial n\partial L} + L\frac{\partial\omega}{\partial n}\frac{\partial^2\omega}{\partial L^2}\right], \tag{8}$$

$$-\omega^2\frac{d}{d\omega}\left(\frac{\alpha}{\omega}\right) = \frac{\omega - L\frac{\partial\omega}{\partial L} - n\frac{\partial\omega}{\partial n}}{\partial\omega/\partial n} - \frac{2}{\pi\omega}\Psi(w). \tag{9}$$

If we neglect the effects of buoyancy, only the first terms in the right-hand sides of eqs.(7,9) appear. The eq.(7) provides a simple and efficient way to calculate dF/dw, avoiding any reduction of the experimental data on to a single curve. It takes into account the frequency dependence of α, which can be calculated separately from eq.(9).

To test the inversion procedure, the standard solar model 1 of Christensen-Dalsgaard (1982) have been used as a reference model. Its eigenfrequencies were computed using a second-order difference scheme with 1000 mesh points in Cowling approximation; the corrections due to the gravity perturbation were then computed using the variational principle. We estimate the resulting accuracy to be about 0.1 per cent at higher frequencies and much better at lower frequencies. The computations were done for the 481 frequencies of the same five-minute modes with $0 \leqslant l \leqslant 200$ as those of the experimental data we use (see below).

The derivatives $\partial\omega/\partial n$ and $\partial\omega/\partial L$ were computed directly from the frequency table by central differences. Extrapolation to small values of w was similar to those used by Cristensen-Dalsgaard et al.(1985).

The results are shown in Fig. 1. They are improved significantly when we take into account the finite values of $N^2(r)$ in the solar interior (curve 3). The results are further improved when the effects of gravity perturbation on the eigenfrequencies are taken into account (the asymptotic theory is constructed in Cowling approximation). The remaining difference between the results of the inversion and C^2 in the model is close to the accuracy of our numerical computations.

Bccause the quality of the experimental data currently available is insufficient to compute the second derivatives in frequency tables, all further calculations were done neglecting thc effects of buoyancy.

To test the stability of the inversion procedure, the ± 10 mkHz noise was imposed on the eigenfrequencies (vertical bars in Fig.2).

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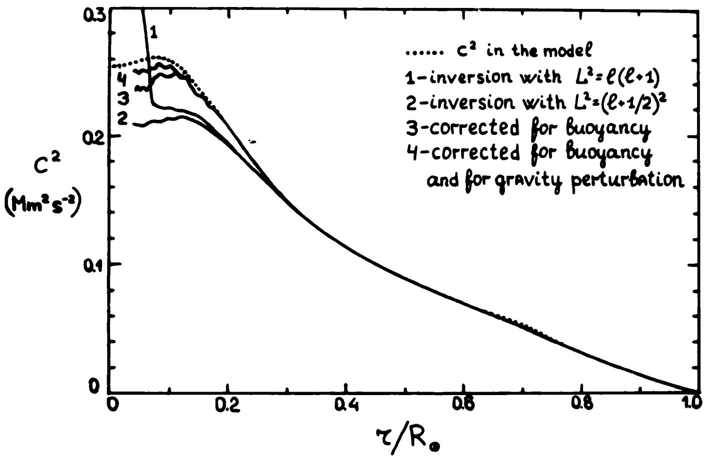

Fig.I. Sound speed obtained from the frequencies of the solar model.

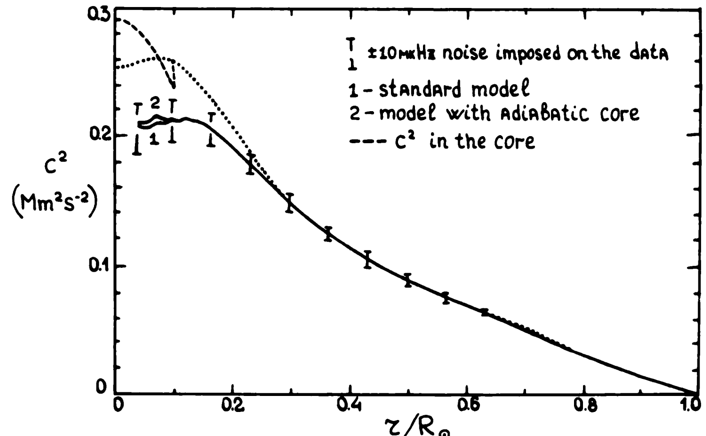

Fig.2. Inversions with $L^2=(\ell+1/2)^2$, buoyancy is neglected.

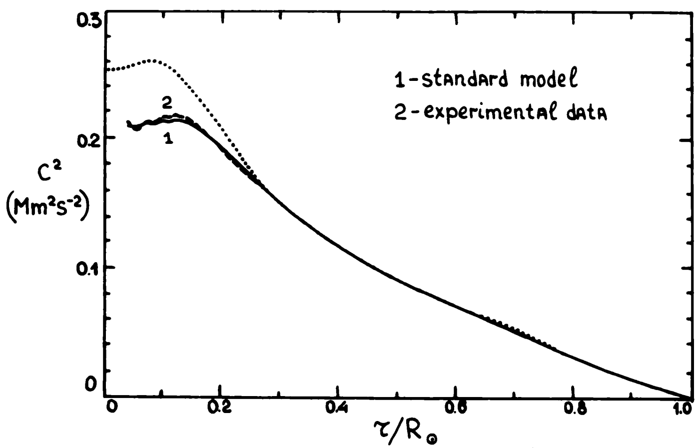

Fig.3. Sound speed obtained from the standard model and from the experimental data.

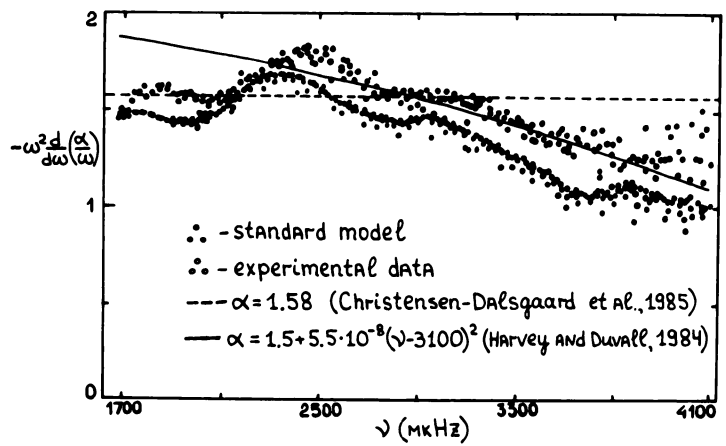

Fig.4. Frequency dependence of the effective phase shift.

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To test the sensitivity of the inversion procedure to the possible unusual solar core (mixing or different chemical composition), the core of the model with radius of $0.1\ R_\odot$ was replaced by the adiabatic polytropic core (Vorontsov and Marchenkov, 1983). The largest effect on the frequencies has been found to be 3 mkHz at higher frequencies of $l = 1$ modes and decrease rapidly for lower frequencies and larger l values. We conclude that with the accuracy of the experimental data currently available it is not possible to resolve such a core.

For the inversion of the experimental data, 481 frequencies for $0 \leqslant l \leqslant 200$ were used (Harvey and Duvall, 1984a; Libbrecht and Zirin, 1985; Woodard and Hudson, 1983; Hill, 1985). The result is shown in Fig.3 and very close to that obtained for the model. The only difference of marginal significance is that the squared sound speed in the Sun may be ~ 1 per cent lower around $r \approx 0.22\ R_\odot$ and ~ 2 per cent larger around $r \approx 0.12\ R_\odot$ than c^2 in the model.

The frequency dependence of α has been computed neglecting $N_i^2(r)$, for $5 \leqslant l \leqslant 20$ (Fig.4). The constant value $\alpha = 1.58$ adjusted by Christensen-Dalsgaard et al.(1985) is shown for the comparison, as well as the parabolic fit obtained by Harvey and Duvall (1984b).

The difference between the results obtained for the experimental data and for the model is clearly seen and confirm the conclusion that the main source of discreapancy in the five-minute oscillations is connected with the structure of the upper part of the solar convective envelope. The information contained in $\alpha(\omega)$ is relevant to different treatments of turbulent convection and requires further study.

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REFERENCES

Brodsky M.A. and Levshin A.L. *Geophys.J.Roy.Astron.Soc.*,1979,**58**,631.

Christensen-Dalsgaard J. *Mon.Not.Roy.Astron.Soc.*,1982,**199**,735.

Christensen-Dalsgaard J.,Duvall T.L.,Gough D.O.,Harvey J.W. and Rhodes E.J.,Jr. *Nature*,1985,**315**,378.

Gough D.O. *Phyl.Trans.Roy.Astron.Soc.Lond.*,1984,**A313**,27.

Harvey J.W. and Duvall T.L.,Jr. In: *Solar Seismology from Space*, ed.R.K.Ulrich; JPL: Pasadena,1984a,p.165.

Harvey J.W. and Duvall T.L.,Jr. In: *Theoretical Problems in Stellar Stability and Oscillations*,eds.M.Gabriel and A.Noels; Obs.de Liege:Liege,1984b,p.209.

Hill H.A. *Astrophys.J.*,1985,**290**,765.

Kemble E.C. *The Fundamental Rrinciples of Quantum Mechanics*, New York-London,1937.

Langer R.E. *Phys.Rev.*,1937,**51**,669.

Libbrecht K.G. and Zirin H. Submitted to *Astrophys.J.*,1985.

Unno W.,Osaki Y.,Ando H. and Shibahashi H. *Nonradial Oscillations of Stars*,Univ.of Tokyo Press,1979.

Vorontsov S.V. and Marchenkov K.I. *Solar Phys.*,1983,**82**,215.

Woodard M. and Hudson H.S. *Nature*,1983,**305**,589.