

PHYSICAL PROCESSES AND MODELS OF INTERPLANETARY  
RESPONSES: SUGGESTED THEORETICAL STUDIES

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ABSTRACT

Three ways to improve the theory and therefore the understanding of the physical processes in the interplanetary medium (during both quiet and disturbed periods of solar activity) are suggested. They are: (1) the development and consequently the use of *higher order moments (fluid) equations* as well as of more realistic *closure conditions* and *transport coefficients* for the macroscopic description of the solar wind; (2) the undertaking of *computer simulation experiments* on the nonlinear collective relaxation process through particle-wave-particle interaction due to the plasma electromagnetic instabilities which may develop under conditions prevailing in the solar wind; and (3) the consideration of collective interactions in the evaluation of the transport coefficients, as deduced from the quasi-linear theory and computer simulation experiments, and their incorporation into the higher order moment equations.

1. INTRODUCTION

It is now recognized that the solar wind phenomenon represents that part of the solar corona which is not confined by the solar magnetic field, and therefore escapes into interplanetary space. Essentially, the solar wind is a warm, magnetized, almost collisionless multicomponent plasma whose space and time behaviour depends on the activity of the sun's surface. The physical processes occurring in such plasma systems are rather complex.

Since the pioneering work by Parker (1958), a large observational and theoretical effort has been directed toward the understanding of the solar wind phenomenon. The theoretical research followed two main ways of investigation, namely the *macroscopic* (fluid) approach based on the continuity, momentum and energy equations and the *microscopic* (kinetic) approach based on Vlasov (correlationless Boltzmann) equation. Significant progress has been achieved by these two methods; they are summarized in the comprehensive review papers by Parker (1968, 1969) Dessler (1967), Scarf (1970), Brandt (1970), Holzer and Axford (1970),

Hundhausen (1972), Dryer and Cuperman (1972), Barnes (1975), Dryer (1975, 1978), Burlaga (1975), Völk (1975), Hollweg (1975, 1978), Holzer (1976), Cuperman (1977, 1979) and Suess (1978).

To further advance the understanding of the solar wind phenomenon, besides additional detailed spacecraft observations, more advanced theoretical methods are required. For example, the familiar fluid theory used so far treats the particle density, streaming velocity and temperature on an equal footing; however, it uses for the heat flux an approximate expression which is invalid over most of the interplanetary medium; also, it neglects higher order moments of the distribution function which is equivalent to taking it to be a Maxwellian distribution function, in contradiction to many of the actual observations (c.f. Feldman et al., 1976).

On the other hand, the kinetic (*collisionless*) approach, which correctly accounts for the shape of the particle distribution function was developed for the case of infinite homogeneous plasmas. Even in that case, it adequately treats the *linear* stage of evolution and only approximately the later (e.g., quasi-linear) stage. It is not able to predict the dynamical behaviour of an unstable plasma system through its nonlinear stage, as observed in the interplanetary medium (e.g. Abraham-Shrauner et al., 1979).

This paper discusses three ways in which the theory of the solar wind plasma can be advanced. They are: (1) the development and consequently the use of *higher order moments (fluid) equations* as well as of more realistic *closure conditions* and *transport coefficients* for the solar wind; (2) the undertaking of *computer simulation experiments* on the nonlinear collective relaxation process through particle-wave-particle interaction due to the plasma electromagnetic instabilities which may develop under conditions prevailing in the solar wind; and (3) the use of "hybrid-models" in which collective contributions to the transport coefficients as deduced from computer simulation experiments are incorporated into the higher order moments equations in order to describe the actual physical processes in the solar wind.

## II. A HIGHER ORDER FLUID THEORY

Recently, a generalized fluid theory which is required for the description of time-dependent, spatially nonhomogeneous, anisotropic, multi-species spherically systems of particles obeying an inverse-square law of interactions was derived by Cuperman et al. (1979).

The generalization consists of the derivation—starting from the Boltzmann equation—of a higher order, closed system of equations for the moments of the particle velocity distribution  $f_a$  ( $a=e, p, \alpha$ , etc.). Thus, in addition to the familiar equations for the  $a$  particle density,  $n_a$ , streaming velocity,  $\langle \underline{v} \rangle_a$  and temperature,  $T_a$ , this *closed* set of

equations also includes equations for the heat flux,  $q_a$  as well as for the fifth moment,  $\zeta_a \equiv \langle (\underline{v} - \langle \underline{v} \rangle)^4 \rangle$  which characterizes the particles in the tail of the distribution function.

The particle-particle interaction terms in the Boltzmann's equation were calculated in the way indicated by the Fokker-Planck relaxation theory. Thus, the velocity distribution functions in the collisional terms were represented by expansions in Legendre polynomials corresponding to the particle distribution functions being functionals of all the moments to be considered, i.e.

$$f_a = f_a(n_a, \langle v_r \rangle_a, \langle v_r^2 \rangle_a, \langle v_\perp^2 \rangle_a, q_{a,r}, \xi_a, r, t) \tag{1}$$

( $r, \perp$  indicate the radial and tangential directions, respectively).

This set of closed equations is relatively simple and mathematically tractable. Thus, the first five<sup>1</sup> moments of the distribution function  $f_a$  are defined as follows ( $v_r \equiv u, v_\theta \equiv v, v_\phi \equiv w$ ):

$$\begin{aligned} n_a(r, t) &\equiv \int f_a(r, u, v, w, t) d^3v \\ \langle u(r, t) \rangle_a &\equiv n_a^{-1} \int u f_a(r, u, v, w, t) d^3v \\ \alpha_a(r, t) &\equiv \langle (u - \langle u \rangle_a)^2 \rangle \rightarrow kT_{a,r} / m_a \\ \beta_a(r, t) &\equiv \langle v^2 \rangle_a = \langle w^2 \rangle_a \rightarrow kT_{a,\perp} / m_a \\ \varepsilon_a(r, t) &\equiv \langle (u - \langle u \rangle_a)^3 \rangle \rightarrow 1.2 q_{a,r} / n_a m_a \\ \xi_a(r, t) &\equiv \langle (u - \langle u \rangle_a)^4 \rangle - 3\alpha_a^2 \rightarrow 0 \end{aligned} \tag{2}$$

In these definitions,  $\alpha_a$  and  $\beta_a$  are the mean squared random velocities in the radial and tangential directions, respectively;  $\varepsilon$  is related to the radial heat flow;  $\xi_a$  represents the excess or deficiency of high velocity particles in the tail of the distribution function relative to a Maxwellian. In the case of a Maxwellian particle distribution function, the quantities (2) take the values indicated by the arrows.

Following the procedure indicated above, Cuperman et al. (1979) obtained the following closure conditions for the "mixed" higher order moments

$$\begin{aligned} \langle (u - \langle u \rangle) v^2 \rangle &= \varepsilon / 3 \\ \langle (u - \langle u \rangle) v^2 v^2 \rangle &= \zeta / 3 - B(\alpha - \beta) \end{aligned} \tag{3}$$

$$\langle (u - \langle u \rangle)^5 \rangle = 10 B \epsilon$$

$$\langle (u - \langle u \rangle)^3 v^2 \rangle = 2B \epsilon$$

where

$$B \equiv (\alpha + 2\beta) / 3.$$

Finally, the closed set of moment equations read:

$$\frac{\partial n_a}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n_a \langle u \rangle_a) = 0$$

$$\frac{\partial \langle u \rangle_a}{\partial t} + \langle u \rangle_a \frac{\partial \langle u \rangle_a}{\partial r} + \frac{1}{n_a} \frac{\partial}{\partial r} (n_a \alpha_a) + \frac{2}{r} (\alpha_a - \beta_a) + \frac{\partial \Phi}{\partial r} = 0$$

$$\frac{\partial \alpha_a}{\partial t} + \langle u \rangle_a \frac{\partial \alpha_a}{\partial r} + 2\alpha_a \frac{\partial \langle u \rangle_a}{\partial r} + \frac{1}{n_a} \frac{\partial}{\partial r} (n_a \epsilon_a) + \frac{2}{3} \frac{\epsilon_a}{r} = \left( \frac{\partial \alpha_a}{\partial t} \right)_c$$

$$\frac{\partial \beta_a}{\partial t} + \langle u \rangle_a \frac{\partial \beta_a}{\partial r} + 2\beta_a \frac{\partial \langle u \rangle_a}{\partial r} + \frac{1}{3n_a} \frac{\partial}{\partial r} (n_a \epsilon_a) + \frac{4}{3} \frac{\epsilon_a}{r} = \left( \frac{\partial \beta_a}{\partial t} \right)_c \quad (4)$$

$$\frac{\partial \epsilon_a}{\partial t} + \langle u \rangle_a \frac{\partial \epsilon_a}{\partial r} + \frac{1}{n_a} \frac{\partial}{\partial r} (n_a \xi_a) + 3\alpha_a \frac{\partial \alpha_a}{\partial r} - \frac{4}{r} (\alpha_a - \beta_a)^2 +$$

$$+ 3\epsilon_a \frac{\partial \langle u \rangle_a}{\partial r} = \left( \frac{\partial \epsilon_a}{\partial t} \right)_c$$

$$\frac{\partial \xi_a}{\partial t} + \langle u \rangle_a \frac{\partial \xi_a}{\partial r} + 6\epsilon_a \frac{\partial \beta_a}{\partial r} - \frac{8}{3} \epsilon_a \frac{\partial}{\partial r} (\alpha_a - \beta_a) + 4B \frac{\partial \epsilon_a}{\partial r}$$

$$+ 4\xi_a \frac{\partial \langle u \rangle_a}{\partial r} - 4(\alpha_a - \beta_a) \frac{\partial \epsilon_a}{\partial r} - \frac{20}{3} \frac{\epsilon_a}{n_a} (\alpha_a - \beta_a) \frac{\partial n_a}{\partial r}$$

$$- \frac{32}{3} \frac{1}{r} (\alpha_a - \beta_a) \epsilon_a = \left( \frac{\partial \xi_a}{\partial t} \right)_c$$

In Eq. (4)

$$\frac{\partial \phi_a}{\partial r} = - F_{a,r} = \frac{GM_o}{r^2} - \frac{Z_a E}{m_a} \quad (5)$$

where E is the electrostatic field<sup>2</sup>, G - the gravitational constant, M<sub>o</sub> - the solar mass, Z<sub>a</sub> - the particle charge.

The r.h.s. of Eq. (4), representing the rate of change with time of the moments of f<sub>a</sub> due to particle-particle interactions (i.e. the transport coefficients) are given by the following expressions<sup>3</sup>:

$$\begin{aligned} \left(\frac{\partial \alpha_a}{\partial t}\right)_c &= - \frac{8}{15} \frac{k(T_{a,r} - T_{a,l})}{m_a \tau_{aa}} \left\{ 1 + \sum_{b \neq a} \frac{n_b}{n_a} F_{b,2} \right\} \\ &- \frac{2}{m_a} \sum_{b \neq a} \frac{m_{ab}}{m_a + m_b} \frac{k(T_a - T_b)}{\tau_{ab}} + \frac{m_a Q_a}{2^{1/2} k T_a \tau_{aa}} \sum_{b \neq a} \frac{n_b}{n_a} F_{b,0} \end{aligned} \quad (6)$$

$$\left(\frac{\partial \epsilon_a}{\partial t}\right)_c = - \frac{174}{400} \frac{q_{a,r}}{n_a m_a \tau_{aa}} \left\{ 1 + \sum_{b \neq a} \frac{n_b}{n_a} L_{b,1} \right\} \quad (7)$$

$$\begin{aligned} \left(\frac{\partial \xi_a}{\partial t}\right)_c &= \frac{6}{7} \frac{k^2 T_{a,r}}{m_a^2 \tau_{aa}} (T_{a,r} - T_{a,l}) \{1 + \Delta \xi_{a,2}\} - \frac{6}{15} \frac{Q_a}{\tau_{aa}} \{1 + \Delta \xi_{a,0}\} \\ &+ 12 \sum_{b \neq a} \frac{m_{ab}}{m_a + m_b} \frac{k^2 (T_b - T_a)}{m_a^2 \tau_{ab}} \left( \frac{0.4 T_a}{1 + \Gamma_b} - T_{a,r} \right) \end{aligned} \quad (8)$$

The expression for  $(\partial \beta / \partial t)_c$  differs from that for  $(\partial \alpha / \partial t)_c$  only by the fact that  $(-8/15)$  is replaced by  $(+4/15)$  in the term proportional to  $(T_{a,r} - T_{a,l})$ . Consequently, the following equation giving the rate of change of the thermal anisotropy  $(\alpha_a - \beta_a)$  due to particle-particle interaction is easily obtained:

$$\left\{ \frac{\partial}{\partial t} (\alpha_a - \beta_a) \right\}_c = - \frac{4}{15} \frac{n_a k}{\tau_{aa}} (T_{a,r} - T_{a,l}) \left\{ 1 + \sum_{b \neq a} \frac{n_b}{n_a} F_{b,2} \right\} \quad (9)$$

In the Eqs. (6)-(9), the "collision" times are defined as  $(m_{ab} = m_a m_b / (m_a + m_b))$

$$\tau_{ab} = \frac{3 m_{ab}^2 [(k T_a / m_a) + (k T_b / m_b)]^{3/2}}{4 (2\pi)^{1/2} Z_a^2 Z_b^2 n_b \ell n \Lambda} \quad (10)$$

and

$$\tau_{aa} = 3m_a^{-1/2} (kT_a)^{3/2} / 8\pi^{1/2} Z_a^4 n_a \ln \Lambda \tag{11}$$

The quantities  $F_{b,2}$ ,  $F_{b,0}$ ,  $L_{b,1}$ ,  $\Delta\xi_{a,2}$  and  $\Delta\xi_{a,0}$  and  $Q_i$  represent simple algebraic expressions and are given in Cuperman et al. (1979).

At this point two remarks are in order:

(i) In Eqs.(4) there exist three types of deviations from an isotropic Maxwellian which are able to drive the system to a relaxed state namely, the anisotropy factor ( $T_{a,r} - T_a$ ), the heat flow  $q_{a,r}$  and the non-thermal tail,  $\xi_a$ . To these, one has to add the differences in the temperatures of the various species which act in the same direction (i.e.  $T_b - T_a$ ).

(ii) The evolution of a physical system described by such equations exhibits a global non-local behaviour.

For isotropic plasma components ( $T_{i,r} = T_{i,\perp}$ ,  $i = a, b$ ), with the notation  $T \equiv (T_r + 2T_\perp) / 3 = T_r = T_\perp$ , the equations read:

$$n_a m_a \frac{\partial \langle u \rangle}{\partial t} + n_a m_a \frac{\partial \langle u \rangle}{\partial r} + k \frac{\partial}{\partial r} (n_a T_a) + \frac{GM_o m_a n_a}{r^2} - n_a Z_a E = 0 \tag{12}$$

$$\begin{aligned} \frac{3}{2} n_a k \frac{\partial T_a}{\partial t} + \frac{3}{2} n_a \langle u \rangle k \frac{\partial T_a}{\partial r} + n_a k T_a \frac{\partial \langle u \rangle}{\partial r} + 2 n_a k T_a \frac{\langle u \rangle}{r} \\ + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 q_{a,r}) = - 3 n_a \sum_{b \neq a} \frac{m_{ab}}{m_a + m_b} \frac{k}{\tau_{ab}} (T_a - T_b) \\ + \frac{3}{2^{2/3}} \frac{n_a m_a^2 Q_a}{k T_a \tau_{aa}} \sum_{b \neq a} \frac{n_b}{n_a} F_{b,0} \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\partial}{\partial t} q_{a,r} + \frac{\langle u \rangle}{r^2} \frac{\partial}{\partial r} (r^2 q_{a,r}) + 4 q_{a,r} \frac{\partial \langle u \rangle}{\partial r} + \frac{5}{6} m_a \frac{\partial}{\partial r} (n_a \xi_a) \\ + \frac{5}{2} \frac{n_a k^2}{m_a} T_a \frac{\partial T_a}{\partial r} = - \frac{29}{80} \frac{q_{a,r}}{\tau_{aa}} \{ 1 + \sum_{b \neq a} \frac{n_b}{n_a} L_{b,1} \} \end{aligned} \tag{14}$$

$$\frac{\partial \xi_a}{\partial t} + \langle u \rangle \frac{\partial \xi_a}{\partial r} + 4 \xi_a \frac{\partial \langle u \rangle}{\partial r} + \frac{36}{5} \frac{q_{a,r} k}{n_a m_a^2} \frac{\partial T_a}{\partial r} + \frac{24}{5} \frac{k T_a}{m_a^2} \frac{\partial}{\partial r} \left( \frac{q_{a,r}}{n_a} \right)$$

$$= -\frac{2}{5} \frac{Q_a}{\tau_{aa}} \{1 + \Delta \xi_{a,o}\} - 12 \frac{T_a}{m_a^2} \sum_{b \neq a} \frac{m_{ab} k^2}{m_a + m_b} \frac{T_b - T_a}{\tau_{ab}} \frac{0.6 + \Gamma_b}{1 + \Gamma_b} \quad (15)$$

Finally, to recover the standard three-moment equations case, one has to use the following simplifications:

(i)  $\xi_a \equiv n_a^{-1} \int (u - \langle u \rangle_a)^4 f_a d^3v - 3(kT_{a,r}/m_a)^2 = 0,$

that is neglect the effect of the non-maxwellian tail of the particle velocity distribution. Then, no equation for  $\xi_a$  is required;

(ii) in the Eq.(14), neglect in the l.h.s. all the terms but the fifth. One obtains

$$q_{a,r} = - \frac{6.9}{1 + \sum_{b \neq a} (n_b/n_a) L_{b,1}} \frac{n_a}{m_a} \tau_{aa} k^2 T_{a,r} \frac{\partial T_{a,r}}{\partial r}$$

which is the familiar approximate relation used for the radial heat flux. The simplifications implied by this expression are obvious.

### III. COMPUTER SIMULATION EXPERIMENTS OF COLLECTIVE INTERACTIONS IN MAGNETIZED PLASMAS

Computer simulations are able to provide information about the time-evolution and the final state of an initially unstable plasma system. They consist of the simultaneous solution of the Lorentz equation for each one of the many thousands of the plasma particles acted upon by the external and collective electromagnetic fields (see, e.g.: Haber et al., 1970; Morse and Nielson, 1971; Cuperman and Salu, 1972). In the following two examples (relevant for the interplanetary medium) are considered, namely one in which electromagnetic interactions are dominant and another in which electrostatic interactions are dominant.

#### 1. Ion-cyclotron (Alfvén) electromagnetic instability.

For bi-Maxwellian particle distribution functions, the linearized dispersion relation for *electromagnetic* ion-cyclotron waves (left-hand polarization) propagating along a static and homogeneous magnetic field  $B_0$  may be written as (e.g. Cuperman and Landau, 1974):

$$n^2 \equiv c^2 k^2 / \omega^2 = 1 + \sum_j \frac{\omega^2}{\omega k} \frac{p_{j\parallel}}{f_{\parallel,j}} \frac{[1 + (kv_{\parallel} / \omega) A_j]}{v_{\parallel} - s_j} dv_{\parallel}, \quad (17)$$

where the summation is overall + and -, warm and cold plasma components. The notation used is as follows: complex frequency  $\omega_r + i\omega_i$ ; real wave-number of disturbance  $k$ ;  $v_{\parallel}, v_{\perp}$  are particle velocity components

parallel and perpendicular to  $B_0$ , respectively; plasma frequency  $\omega_p$ ; light velocity  $c$ ;  $f_{||,j}$  is the particle distribution function defined by

$$f_0 \equiv f_{\perp} f_{||} = \frac{1}{2\pi v_{\perp,th}^2} \exp\left(\frac{-v_{\perp}^2}{2v_{\perp,th}^2}\right) \cdot \frac{1}{(2\pi)^{1/2} v_{||,th}} \exp\left(\frac{-v_{||}^2}{2v_{||,th}^2}\right) \quad (18)$$

where  $v_{\perp,th} \equiv (KT_{\perp}/m)^{1/2}$  and  $v_{||,th} \equiv (KT_{||}/m)^{1/2}$  represent the

thermal velocity in the directions perpendicular and parallel to  $B_0$ , respectively. Other notations are  $A_j \equiv (T_{\perp}/T_{||})_j - 1$  and  $s_j = (\omega - \Omega_j)/k$ ,

where  $\Omega_j$  is the cyclotron frequency defined by  $\Omega_j = q_j B_0 / m_j c$  and  $q_j = Z_j e$  is the particle charge. Expressing the integral part in (17) in terms of the plasma dispersion function  $Z(\xi_j)$  one may rewrite (17) as

$$c^2 k^2 = \omega^2 + \sum_j \frac{\omega_p^2}{\Omega_j} \left\{ A_j - \frac{1}{\sqrt{2} v_{||,th,j} k} Z(\xi_j) [(A_j + 1)(\Omega_j - \omega) - \Omega_j] \right\} \quad (19)$$

For the case of a warm anisotropic plasma consisting of warm ions and cold electrons, assuming (i)  $\omega_i \ll \omega_r$ , (ii)  $v_r \equiv [(\Omega_p - \omega)/k] \gg v_{||,th,p}$ , (iii)  $k^2 c^2 / \omega^2 \gg 1$ , and (iv)  $k$  is given by the 'cold' plasma dispersion relation (Kennel and Petschek, 1966), from (19) one obtains the approximate analytical expressions ( $\beta_{||} \equiv 8\pi n_p k T_{||,p} / B_0^2$ ),  $v_A$  - Alfvén speed)

$$\gamma \equiv \omega_i / \Omega_p = \frac{1}{\alpha^{3/2}} \frac{1(1-x)^5}{x^2(2-x)} \frac{[(A+1)(1-x)-1]}{(\beta/\pi)^{1/2}} \exp\left\{-\frac{(1-x)^3}{x^2 \beta_{||,p} \alpha}\right\} \quad (20)$$

$$(kv_A / \Omega_p)^2 = x^2(1-x)^{-1}, \quad x \equiv \omega_r / \Omega_p \quad (21)$$

The maximum growth rate is (Cuperman et al., 1975)

$$\gamma_m = \left(\frac{\omega_i}{\Omega_p}\right)_m = \left(\frac{\pi}{\beta}\right)^{1/2} \frac{A_p(1-x_1) - x_1}{x_1^2(2-x_1)} (1-x_1)^{5/2} \exp\left\{-\frac{(1-x_1)^3}{\beta x_1^2}\right\}, \quad (22)$$

where  $x_1$  and  $Q_m$  are simple algebraic expression given in Cuperman et al. (1975). Notice that as a consequence of the assumptions (i)-(iv), the results (20)-(22) only hold for plasma values such that  $\beta_p \equiv \beta_p A_p^2 (A_p + 1) \ll 1$

As it is seen, even though significant information is provided by the approximate theory based on the linearized Eq. (19), its use is rather limited. Thus, it is only for small  $\beta_p$  (i.e., small  $T_{\perp}/T_{||}$  and small  $\beta$ ) values that the results hold. Moreover, the linear predictions hold only for the initial stage of the instability, which represents a relatively small time period of evolution. Although additional important information can be achieved from a quasi-linear



treatment, the only way to relieve the mathematical restrictions and physical approximations required by analytical treatments is by performing computer simulation experiments. This important point has now been established by detailed examination of individually-measured proton velocity distribution functions in space (Abraham-Shrauner et al., 1979) as discussed below.

Figures (1) and (2) give results obtained in the computer simulation of the electromagnetic ion cyclotron instability in homogeneous plasma systems with parameters  $kT_{\perp p} = 25$  keV,  $\beta=1$  and  $T_{\perp p}/T_{\parallel p} = 100$  (Cuperman and Sternlieb, 1975; see also Cuperman et al., 1976). As seen, a strong electromagnetic ion cyclotron instability develops, as expected from theoretical (linear) considerations. The initial low electromagnetic noise ( $W_B(0)/W_{\text{tot}} \sim 0.0004$ ) develops into a significant electromagnetic wave energy level which, at its maximum represents about 10% of the total energy in the system; this represents an increase by a factor of 230!

Inspection of  $\Delta \bar{W}_{\perp,p}$  and  $\bar{W}_B$  curves shows that  $\Delta \bar{W}_{\perp,p}$  is always larger than  $\bar{W}_B$  by at least a factor of three. This indicates that significant non-linear processes occur starting from the very beginning of the instability. Thus, the electromagnetic field developed during the instability heats (non-resonantly) the protons (mainly) in the plasma system and, consequently appears to possess less energy than expected from pure 'linear' considerations (which do not take into account such processes). This suggests the true (nonlinear) measure of the instability developed to be the change in the total proton transverse kinetic energy,  $W_{\perp,p}(0)$  (which is the actual energy source available in the system), rather than the enhancement of the electromagnetic ion cyclotron waves.

A natural result of the instability should be the tendency of the system towards thermal equilibrium,  $(T_{\perp p}/T_{\parallel p}) \rightarrow 1$ . This is indeed observed to occur in the experiment as indicated in Figure 2. Thus, after a time about  $800\omega_{pp}^{-1}$ , the transverse kinetic energy  $W_{\perp,p}(0)$  has decreased by about 25% and the parallel kinetic energy  $W_{\parallel,p}(0)$  has increased by a factor of about 26! Consequently, the kinetic anisotropy  $0.5W_{\perp,p}/W_{\parallel,p}$  which at  $t = 0$  represents the thermal anisotropy,  $(T_{\perp p}/T_{\parallel p})_p$  has decreased from '100' to '2.9'. The fact that the last value is different from '1' (as expected on the grounds of linear theory) may be due to the fact that the quasi-final (from a collective point of view) state in the experiment is one in which a significant amount of electromagnetic wave energy is present. This suggests the establishment of a non-linear quasi-equilibrium state for which the instability criterion may well be different from that predicted by the linear theory.

This result is of special interest for the solar wind. Indeed, recently, Abraham-Shrauner et al. (1979) have analyzed electromagnetic

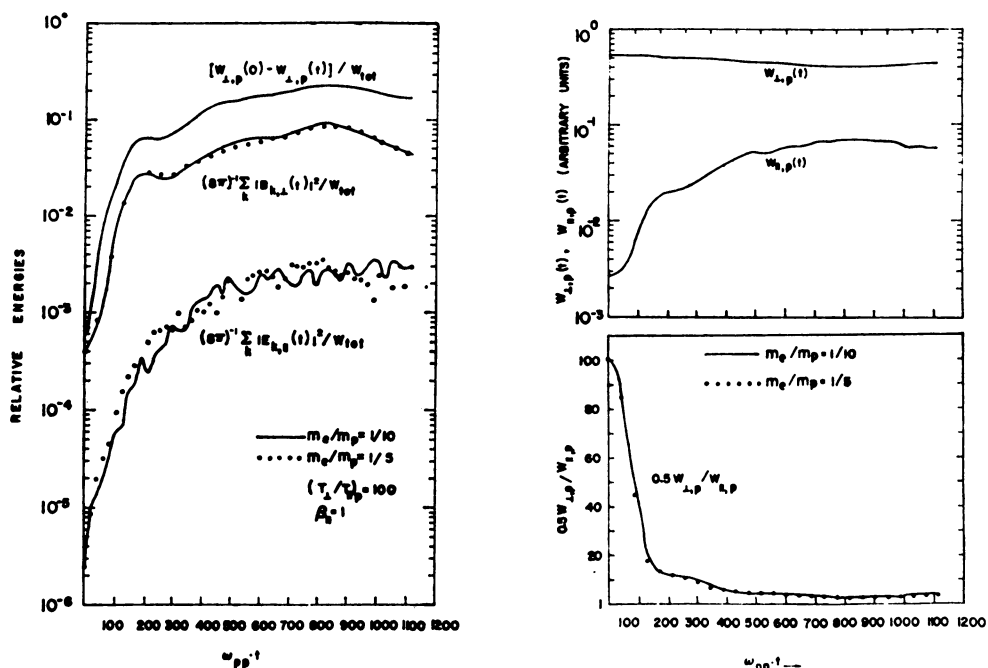


Fig. 1 The evolution in time of  $\Delta \bar{W}_{\perp,p}$ , the relative change in the total transverse kinetic energy of the warm anisotropic protons,  $\bar{W}_E \equiv (8\pi)^{-1} \sum_k |B_{k,\perp}(t)|^2 / W_{tot}$  the relative transverse magnetic wave energy and  $\bar{W}_E \equiv (8\pi)^{-1} \sum_k |E_{k,\parallel}(t)|^2 / W_{tot}$  the relative longitudinal electrostatic energy.

Fig. 2 The evolution in time of the total proton kinetic energies  $W_{\perp,p}$  and  $W_{\parallel,p}$  (top) and of the anisotropy ratio  $(0.5W_{\perp,p}/W_{\parallel,p})_p$  (bottom). At  $t=0$ ,  $(0.5W_{\perp,p}/W_{\parallel,p})_p \equiv (T_{\perp}/T_{\parallel})_p = 100$ .

instabilities of field-aligned right-hand circularly polarized magneto-sonic waves and left-hand circularly polarized Alfvén waves driven by two drifted proton components for model parameters determined from Imp 7 solar wind data measured during high-speed flow conditions. The authors found that (i) measured distributions are linearly *unstable with respect to Alfvén waves*; (ii) the characteristic of the proton velocity distributions in the high speed solar wind that is primarily responsible for driving the Alfvén instability is the large thermal anisotropy of the main proton component observed by Feldman et al. (1976) in high speed streams  $(T_{\perp}/T_{\parallel} = 3.1 + 0.7)$ ; and (iii) the instability of the Alfvén wave is *inconsistent* with the persistence of the model fits to the measured proton velocity distribution function, since the calculated e-folding times are very short - of the order of 1 min.

To explain the reason why linear stability theory appears to fail for the Alfvén instability the authors adopted the resolution suggested by Cuperman and Sternlieb (1975) to explain the saturation value,  $\sim 3$

in the simulation experiment described above. Namely, the instability criterion of the linear theory ( $T_{\perp}/T_{\parallel} > 1$ ) may not apply to the composite wave-particle plasma state. Instead, stabilization may result from the establishment of a nonlinear quasi-equilibrium state which necessarily includes the wave field.

2. Beam-plasma instability.

The *electrostatic* dispersion relation for homogeneous unbounded plasmas in a constant magnetic field  $\underline{B} = B_0 \underline{oz}$  and for disturbances with wave vectors  $\underline{k}$  along  $\underline{B}$ , may be written in the following form (Montgomery and Tidman, 1964)

$$R_{zz} = -\omega^2 + 2\pi\omega \sum_j \omega_{pj}^2 \int_{-\infty}^{+\infty} v_{\parallel} dv_{\parallel} \int_0^{\infty} v_{\perp} dv_{\perp} \frac{\partial f_{oj} / \partial v_{\parallel}}{kv_{\parallel} - \omega} = 0, \quad (24)$$

where the summation is over the plasma components. Here  $\omega$  is the complex frequency of the disturbance,  $f_{oj}$  is the equilibrium distribution function and  $\omega_{pj}$  is the plasma frequency of the  $j$ -plasma component. The notations  $\parallel, \perp$  refer to parallel and transverse direction with respect to  $\underline{B}_{oz}$ . The longitudinal electrostatic waves propagating parallel to  $\underline{B}_0$  are unaffected by the magnetic field.

In the following we will not elaborate on the linear (or quasi-linear) theories, as done for the case discussed in Section III.1. Rather, we will discuss the non-linear simulation results. Computer simulation experiments of the interaction between electron beams and background plasmas have been carried out by a number of authors (see, e.g. Cuperman et al., 1976). Both electrostatic and electromagnetic interactions were simultaneously considered. The relative beam concentrations considered were  $\epsilon = n_b/n_p = 1, 0.1$  and  $0.01$  (cases A, B and C, respectively). In all cases, the background plasma (1 eV thermal energy) was penetrated by an electron beam of 1 keV streaming energy and 5% thermal spread in the streaming direction. The following results emerged (see also Fig. 3):

(1) All the systems were strongly unstable against the electrostatic beam-plasma instability; the electromagnetic interaction was negligible. In the linear stage, the measured growth rates were in satisfactory agreement with the linear predictions for cold beam-plasma interacting systems.

(2) The maximum relative electrostatic energy developed,  $\overline{W}_{e.s.}^{max} \equiv W_{e.s.}^{max} / W_{tot}$  was 5.7, 8.8 and 6.3% for cases A, B and C, respectively. Thus, although  $\epsilon$  varied by a factor of 100,  $\overline{W}_{e.s.}^{max}$  was almost unchanged. This indicates an almost linear relationship between  $\overline{W}_{e.s.}^{max}$  and  $W_{tot}$ :  $\overline{W}_{e.s.}^{max} \sim 0.7 W_{tot}$  for  $10^{-2} < \epsilon < 1$ . These  $\overline{W}_{e.s.}^{max}$  values were reached after a time  $t_{sp} \sim 18, 30$  and  $75$ , respectively.

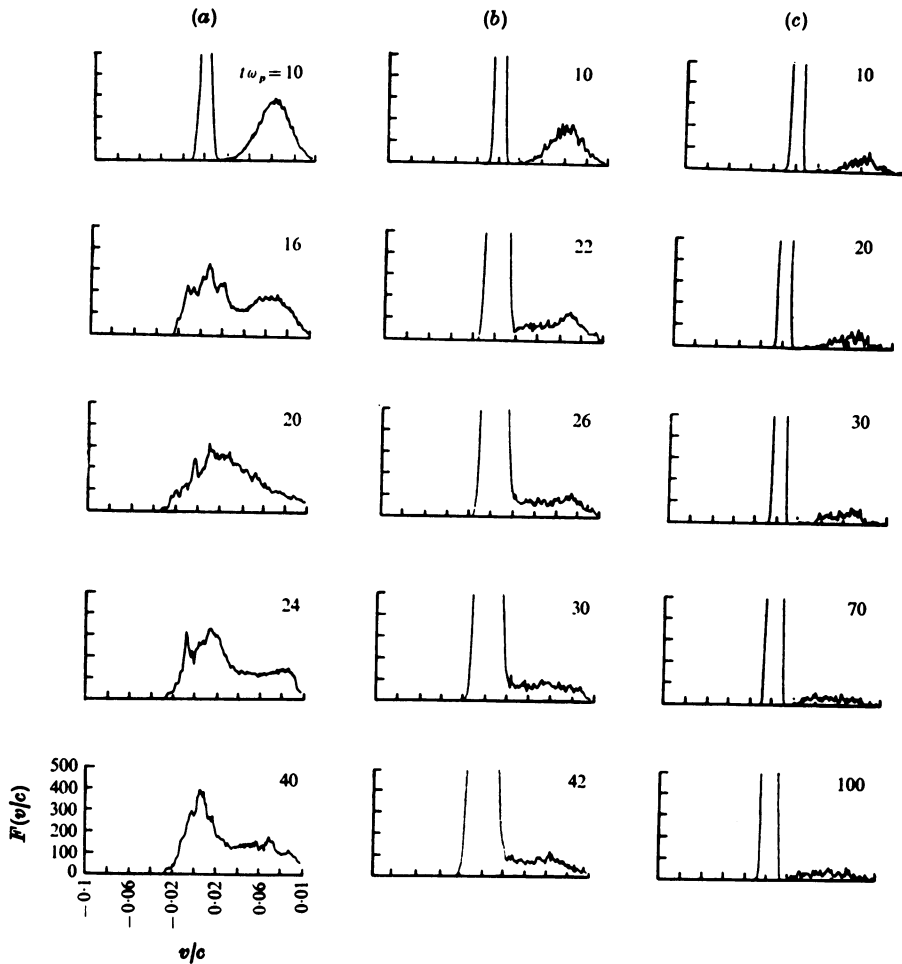


Fig. 3 The velocity distribution functions of the beam-plasma systems at several times of evolution.

(3) At the same time  $t_s$ , the streaming energy of the beam represented the following fraction of its initial value: 49.5%(A), 67.7%(B) and 76.7%(C). The parallel thermal energy in the beam increased by a factor of 4.5, 3 and 1.8, respectively. The largest change occurred in the parallel thermal energy of the background plasma which increased by a factor of 279, 136 and 2.1, respectively.

(4) At the end of the run,  $t_{end}$  in all three cases the kinetic states of the beam and plasma electrons were almost the same as at the saturation time, in spite of periodic changes which occurred between  $t_s$  and  $t_{end}$ . (In case A, for example, at the time  $t\omega_p \approx 25$  the

streaming energy in the beam decreased below the plasma thermal energy:  $W_{b,s} = 330\text{eV}$  and  $W_p = 430\text{ eV}$ .)

(5) As a result of nonlinear wave-particle and wave-wave interactions, electromagnetic waves were generated in the unstable beam-plasma configuration. However, the relative importance of electromagnetic to electrostatic activity was small,  $W_{e.m.}/W_{e.w.} \sim 10^{-5}$ .

(6) At the end of the run, particle distribution functions which were "nonlinearly" stable were obtained.

These results are relevant for the solar wind. Lemons et al., 1979, performed a linear analysis of plasma configurations measured with the Los Alamos instruments on Imp 7 and 8 during or close to times when *electrostatic* fluctuations have been observed. The authors concluded that the *ion-beam instability* is more likely the cause of the observed e.s. fluctuations. Since the time required by the Imp particle analyzers to make a complete measurement of the particle distribution is much larger than the periods of enhanced electrostatic fluctuations, one expects the observed distribution functions to be stable.

The suggestions by Lemons et al. (1979) are consistent with the results of the computer experiments of Cuperman et al., 1976, described above. The experiments provide complete quantitative information on the evolution of the beam-plasma systems through their nonlinear and final stages, which are the ones mostly observed by the instruments; this is so because of the short duration of the linear stages. Indeed, the particle distribution functions in the last row of Figure 3 (corresponding to the end of the computer simulation experiments) are representative for the "stable" distribution functions discussed by Lemons et al. (1979) in their work.

#### IV. HYBRID-MODELS

Both approaches indicated above are very useful for theoretical solar wind studies. However, each one has its limitations. Indeed, the fluid approach does not include the collective interactions which could play a significant role. The computer simulations, on the other hand assume specially uniform plasma systems<sup>5</sup>. A hybrid method exploiting the capabilities of both approaches is obviously desired, and such a method is being used for fusion plasmas in which electromagnetic turbulence plays an important role (e.g., Krall and Liewer, 1973)<sup>6</sup>. The model consists of a system of continuity, momentum and energy equations to be integrated numerically and including the effects of turbulence, self-consistently considered through anomalous transport coefficients. The last ones depend on the unstable modes which can develop in time and space as the macroscopic parameters evolve; they are based on the quasilinear theory in conjunction with nonlinear bounds as obtained in computer simulation experiments. This method has been successfully used for the investigation of theta pinches, for example.

In the above models only, the case of collisionless plasmas was treated. That is, the transport coefficients were purely anomalous, as no particle-particle interactions were considered.

For the solar wind case in which collisions can play a non-negligible role over a significant part of the interplanetary range, a generalization of the hybrid-model is indicated. Thus, the higher order moment equations described in Section II (in which particle-particle interactions are considered) should be complemented by anomalous contributions to the collisional transport coefficients. The last ones should be obtained from quasilinear theories bounded by the results of computer simulation experiments as described in Section III.

## V. NOTES

1. Actually the first six moments, if the radial and tangential random kinetic energies are considered to be different moments.
2. This electric field is responsible for the maintenance of both the equality in the particle fluxes (electrons and positive ions) in the solar wind, and the charge neutrality of the sun.
3. Here, as in the previous equations,  $kT$  is used to denote the mean random kinetic energy, rather than "thermal" energy (which is only defined in the Maxwellian case).
4. For comparison, for the case  $a=e$ ,  $b=p$ , from (9) one recovers the following particular expression used by Braginskii (1965)
 
$$\tau_{ep} = 3m_e^{1/2} (KT_e)^{3/2} / 4(2\pi)^{1/2} e^4 n_p \ln \Lambda.$$
5. Particle-particle interactions can be also considered simultaneously in computer simulation experiments.
6. Sometimes, hybrid models consist of "fluid" electrons and "particle" ions (e.g., Hamasaki et al., 1977).

ACKNOWLEDGEMENT. The author would like to thank M. Dryer for helpful discussions and reading of the manuscript.

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