CHAOTIC ASTEROIDAL TRAJECTORIES EXHIBITING MULTIPLE BURSTS OF ECCENTRICITY: A STATISTICAL ANALYSIS

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Abstract. For chaotic asteroidal trajectories exhibiting multiple bursts of eccentricity in the 3/1 Jovian resonance, we derive the distribution function of time intervals between such bursts. At the onset, the resulting distribution decays exponentially. In the tail, an algebraic decay is observed, with the power-law index for the integral distribution in the range -2 to -1.

1. Introduction

Wisdom (1982) found a now-well known behaviour of chaotic asteroidal trajectories in the 3/1 Jovian resonance, namely bursts in the time evolution of orbital eccentricities. During a long period of time (hundreds of thousands of years) the eccentricity of an asteroidal trajectory may remain small (< 0.1), then it jumps suddenly to a high value (> 0.3) and after a comparatively short period jumps back to small values. During this mode, an asteroid is planet-crossing. The bursts in eccentricity repeat and appear to be separated randomly in time. In the following, we call such orbits with chaotic transitions between two modes, intermittent. We recover the shape of the distribution of lengths of time intervals between eccentricity bursts of intermittent orbits. Especially we are interested in the shape of the long time part of this distribution.

S. Ferraz-Mello et al. (eds.), Dynamics, Ephemerides and Astrometry of the Solar System, 183–186.

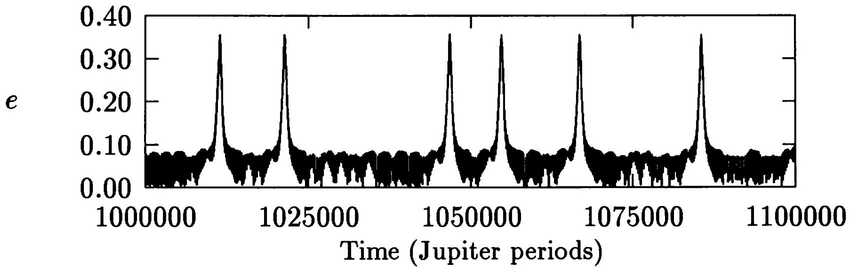

Figure 1. Eccentricity versus time for an intermittent trajectory.

2. Statistical analysis of intermittent trajectories

We used Wisdom's mapping (1983) for the planar-elliptic problem Sun–Jupiter–asteroid in order to obtain intermittent trajectories at the 3/1 mean motion commensurability. There exists a class of intermittent orbits for which the eccentricity bursts are self-similar in shape and the time interval which an asteroid spends in the high eccentricity mode is short as compared to the time spent in the low eccentricity one (see Fig. 13 in Wisdom, 1983).

Intermittent trajectories with various values for Jupiter's eccentricity and initial conditions were analysed. We show the results for a trajectory with the following starting values: the semimajor axis $a = 0.4806$, the eccentricity $e = 0.05$, the longitude of perihelion $\varpi = 0$, and $\varphi \equiv \ell - 3\ell_J = \pi$, where ℓ and ℓ_J are the mean longitudes of an asteroid and Jupiter. For Jupiter, $e_J = 0.05$, $a_J = 1$. Fig. 1 shows the bursts of eccentricity. For all other studied orbits they look very similar, the maximum eccentricity not exceeding $e = 0.3 \div 0.4$. The integral distribution for the length of time intervals between bursts is shown in Fig. 2 in logarithmic scales. The duration D of interburst intervals is measured in Jupiter periods. The quantity F at the vertical axis is the relative fraction of interburst intervals with duration greater than D in the total set of observed intervals for a given orbit. The computation of the orbit spanned 10^9 Jupiter periods. This provided statistics for ≈ 50000 eccentricity bursts. Even with Wisdom's mapping, the computation time needed to obtain these statistics for a single trajectory is several days for a Sun 4 Sparc station. The irregular drop in the distribution function near $\log D = 6$ in Fig. 2 is due to insufficiency of the statistics at high values of D.

For all orbits, the tail fits a power law D^a with a between -2 and -1. For smaller time intervals, not exceeding the limit $D \approx 10^5$, the distribution is of Poisson type, i.e. the decay is exponential. Theoretical interpretation of these dependences will be given elsewhere.

The duration of interburst intervals can be measured e.g. by the number

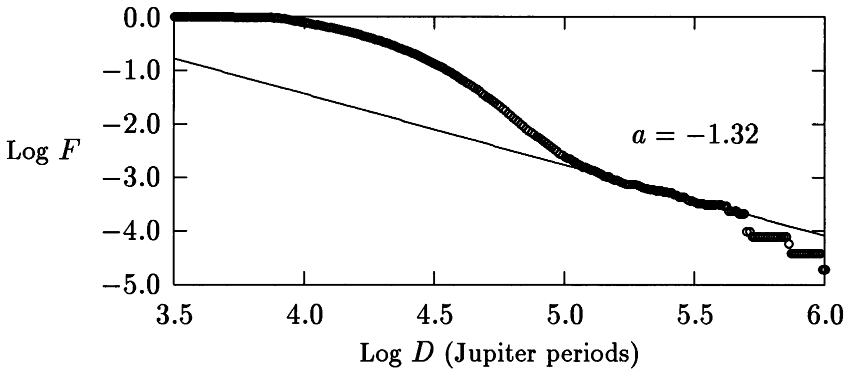

Figure 2. Integral distribution of intervals between eccentricity bursts in decimal logarithmic scales. A best-fit straight line for the tail is plotted, and the corresponding power-law index is indicated. The trajectory is the same as in Fig. 1.

N of periods of rotation of the longitude of the asteroid's perihelion. The resulting distribution of lengths is qualitatively the same as in the case of measuring in real time units: it is a junction of an exponential decay and a slow algebraic one. The transformation from exponential into algebraic behaviour takes place at $N = 50 \div 100$. An influence of the choice of units is that the distribution of intervals measured in Jupiter periods starts to decay beginning from $\log D > 3.5$ (see Fig. 2), while in the case of measuring in perihelion revolutions the distribution starts to decay immediately at $N = 1$.

A somewhat complicating factor, inflicting the very onset of the distributions, occur when an intermittent orbit exhibits not only single random eccentricity bursts, but also bursts forming periodic sequences. For the trajectory with starting values given above the presence of such sequences is negligible, but for some orbits they are abundant. It is easy to exclude them from the analysis, because the quantity N for an interval between bursts in a periodic sequence is less than one. Recovering the shape of distributions of lengths of such sequences constitutes an interesting separate problem.

3. Conclusions

The distribution of the lengths D of time intervals between eccentricity bursts follows a power-law decay for $D > 10^5$ Jupiter periods. For shorter intervals, $D < 10^5$, we find a distribution of Poisson type.

The qualitative character of the over-all distribution is independent of the choice of the time variable (real time or e.g. perihelion revolutions).

The value of the power-law index for the integral distributions for $D > 10^5$ is usually confined between −2 and −1.

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