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FINITE ALGORITHMS FOR LINEAR OPTIMISATION PROBLEMS

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In this thesis we investigate certain linear optimisation problems,

$$\text{minimise } f(\mathbf{x}) \text{ subject to } g_i(\mathbf{x}) \geq 0 \text{ , } i = 1, \ldots, n \text{ ,}$$

where the Kuhn-Tucker conditions

$$\text{(i)} \quad g_i(\mathbf{x}) \geq 0 \text{ , } i = 1, \ldots, n \text{ ,}$$

$$\text{(ii)} \quad \text{for some } \mathbf{u} \geq 0 \text{ , } \nabla f(\mathbf{x}) = \sum u_i \nabla g_i(\mathbf{x}) \text{ ,}$$

$$\text{(iii)} \quad \mathbf{u}^T \mathbf{g}(\mathbf{x}) = 0 \text{ ,}$$

comprise a set of simultaneous linear equations.

Chapter 1 introduces the problems, the restricted least squares, M-estimator, and least absolute deviations problems, and places them in their context.

In Chapter 2, the restricted least squares problem is examined, and pruning rules developed which transform a rather inefficient branch and bound algorithm into an essentially iterative one. The implementation of the resulting algorithm is considered in Chapter 3 and, by working with dual variables and using orthogonal transformations, the algorithm in its final form is at least competitive with existing algorithms for this problem. An error analysis is also given, showing that the use of dual variables has led to superior numerical properties.

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Chapter 4 considers the structure of the M-estimator function. Several speculations are raised and these are answered either negatively by means of a counter example, or positively by proving a theorem. The broad areas covered by these speculations include the question of non-uniqueness, the connection between the M-estimator and the least absolute deviations estimator, what might be called the "proper behaviour" of the function and the function value itself.

Chapter 5 deals with algorithms for calculating the M-estimator. Existing algorithms are surveyed, and two new ones developed. One of them, a continuation algorithm, is examined in detail and numerical results presented. Finiteness is proved for them both.

Finally, in Chapter 6, the least absolute deviations problem is considered. The existing algorithms are reviewed and a new one presented which, although proven finite, did not perform competitively on a particular class of example. The thesis concludes with a discussion of why the algorithm failed, how it differs from algorithms which succeeded for that type of example, and how the algorithm may be improved.

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