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Fractional iteration of functions of two variables

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Let $X \in \{\mathbb{R}^2, \mathbb{C}^2\}$ and

$$T : x_1 = \sum a_{kl}x^k y^l \ , \quad y_1 = \sum b_{kl}x^k y^l \quad (k, l \geq 0 \ , \quad k+l \geq 1)$$

be an invertible holomorphic function from a neighbourhood of $0 \in X$ to X. Denote by $A = \begin{pmatrix} a_{10} & a_{01} \\ b_{10} & b_{01} \end{pmatrix}$ the linear part of T. In this thesis, the fractional iteration of T is examined when

(1) $X = \mathbb{C}^2 \ , \quad A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \ , \quad 0 < |b| \leq |a| < 1 \ ,$

(2) $X = \mathbb{C}^2 \ , \quad A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} \ , \quad 0 < |a| < 1 \ ,$

(3) $X = \mathbb{R}^2 \ , \quad A = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix} \ , \quad b^2 < 4a \ , \quad 0 < a < 1 \ ,$

(4) $X = \mathbb{R}^2 \ , \quad A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \ .$

In the first three cases the fractional iterates are obtained from the algorithm

$$T(s) = \lim_{n \to \infty} T^n \circ B^s \circ T^n$$

where B is the linear or almost linear part of a suitable normal form of T.

In case 4, an asymptotic expansion is obtained for the natural iterates T^n of higher order near the fixpoint 0, and this leads to an algorithmic solution of the functional equations

$$\lambda(Tx) = \lambda(x) - 1 \ , \quad \mu(Tx) = \mu(x) \ , \quad x \in X \ .$$

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The fractional iterates are obtained by solving for $T(s)$ the (invertible) equations

$$\lambda\big(T(s)x\big) = \lambda(x) - s \ , \quad \mu\big(T(s)x\big) = \mu(x) \ , \quad x \in X \ .$$