BULL. AUSTRAL. MATH. SOC. 90B35
VOL. 67 (2003) [351–352]

Scheduling with Precedence Constraints: Worst-case Analysis of Priority Algorithms

GAURAV SINGH

In this thesis we present tight performance guarantees for the models with precedence constraints that range from a unit execution time (UET) task system and models with arbitrary processing times with preemptions, through models with a unit communication delay (UCT) and multiprocessor tasks. Indeed, for the problems with a UET task system and precedence constraints we present tight performance guarantees for the classical Hu's algorithm [3] and its extension to the maximum lateness problem, the Brucker–Garey–Johnson algorithm [1]. We present a new and simplified proof for the worst-case performance of Hu's algorithm, which enables us to obtain a tight performance guarantee for Hu's algorithm and therefore, improves the classical Chen's [2] asymptotically achievable bound.

For the models with arbitrary processing times and preemptions, we analyze the worst-case performance of the preemptive counterpart of the Brucker–Garey–Johnson algorithm [4] and the algorithm presented in [9]. We present performance guarantees, which are tight for arbitrarily large instances of the problem.

For the UET multiprocessor task system, where a task requires either one or all available processors in order to complete its processing, we present tight performance guarantees for the Brucker–Garey–Johnson algorithm and Hu's algorithm. We extend this result to the case where a task may require an arbitrary number of processors in order to complete its processing and present performance guarantees for critical path type algorithms.

We also consider the model with a UET-UCT task system and arbitrary precedence constraints. We first present a generalization of the Brucker–Garey–Johnson algorithm and Hu's algorithm for this model and analyze their worst-case performance. In the case when the graph representing the partially ordered set of tasks is an outforest, we show that if the number of processors are restricted this generalization of the Brucker–Garey–Johnson algorithm is a 2-approximation algorithm, and it produces an optimal schedule if the number of processors is sufficiently large. We also present another polynomial time

Received December 11, 2002
Thesis submitted to the University of Technology, Sydney May 2002. Degree approved, September 2002. Supervisors: Dr Yakov Zinder and Associate Professor Graeme Cohen.

algorithm, which enables us to obtain an optimal schedule when number of processors are two and precedence constraints are in the form of an outforest.

Many performance guarantees, found in the literature, present an upper bound on the value of the criterion corresponding to the approximation algorithm expressed in terms of its optimum value. In contrast, in this thesis, we present several performance guarantees in the form of bounds on the deviation of the criterion value from its optimum expressed in terms of the parameters characterizing the partially ordered set of tasks. Also, a distinctive feature of all presented performance guarantees is their tightness for arbitrarily large instances of the respective problem, in contrast to most of the bounds found in the literature, which are either asymptotically achievable or tight only for some restricted instances of the problem.

The results in this thesis have been a part of three refereed publications [**7**, **8**, **5**], one research report [**10**], and one paper submitted for publication [**6**].

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Department of Mathematical Sciences
University of Technology, Sydney
Broadway, NSW 2007
e-mail: Gaurav.Singh@uts.edu.au