5. H. HEYER, *Probability measures on locally compact groups* (Springer-Verlag, 1977).

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RAMSAY, A. and RICHTMYER, R. D., *Introduction to hyperbolic geometry* (Universitext, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong 1995), xii+287 pp., soft-cover, 3 540 94339 0, £30.

This is indeed a very nice book on hyperbolic geometry. It clarifies the axiomatic and logical development of the subject, describes its traditional geometrical features and rounds off with a differential geometric setting.

After a useful introduction Chapter 1 deals with the axiomatic method (based on Hilbert's ideas), summarises the relevant properties of the real numbers and discusses categoricalness. The choice of parallel axiom, distinguishing Euclidean and hyperbolic geometry, is introduced. In Chapter 2 'neutral geometry' and the usual 'neutral theorems' are studied. A quite thorough discussion of the hyperbolic plane H^2 is given in Chapter 3, including asymptotic features, isometries, tilings and horocycles. Three-dimensional hyperbolic space H^3 is the subject of Chapter 4 and again an instructive section on isometries is included. In Chapter 5 the differential geometry of surfaces is introduced and the discussion includes metrics, parallel transport and geodesics, vectors and tensors and the relation between isometries and preservation of the metric (line element). Here H^2, as such a surface, is described and continued further in Chapter 6. In Chapter 7 the classical models of H^2 are clearly described and its isometries interpreted within them. The categoricalness of the axioms is established. Isometries are revisited in Chapter 8 and the link with fractional linear transformations and $SL(2,\mathbb{R})$ is shown for H^2 and compared to the isometry group of the Euclidean plane. The differential geometry of H^3 is studied in Chapter 9 where the idea of a manifold is introduced. Lorentz metrics are introduced in Chapter 10 and the corresponding Lorentz and Poincaré groups are discussed. Special relativity is briefly mentioned and the 'symmetries' of Maxwell's equations used to introduce Lorentz transformations. The realisation of H^2 in 3-dimensional Minkowski space is shown, as is the relation between the isometries of H^2 and the associated Lorentz transformations. A similar discussion of H^3 is presented. Chapter 11, the final chapter, is devoted to straightedge and compass constructibility in H^2.

This book represents a most commendable attempt to introduce hyperbolic geometry in a little under 300 pages. It clarifies the subject axiomatically, describes it differentially and exhibits it within Minkowski space. There is much discussion of isometries and comparisons with Euclidean space are usually at hand. The book is well laid out with no shortage of diagrams and with each chapter prefaced with its own useful introduction. The mathematical prerequisites are developed *ab initio*. Also well written, it makes pleasurable reading.

G. S. HALL

SEYDEL, R. *Practical bifurcation and stability analysis: from equilibrium to chaos* (2nd edition) (Interdisciplinary Applied Mathematics, Vol. 5, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong 1994), xv+407 pp., 3 540 94316 1, £34.50.

According to the Preface this book is a new version of a previous text, *From Equilibrium to*

Chaos, with drastic revisions and extensive additions. The author aims the book at the needs of scientists and engineers but also wishes 'to attract mathematicians'.

The outcome is impressive. The book is beautifully written in a style that seeks not only to develop the subject matter but also to expose the thought processes behind the mathematics. It provides a very readable account of nonlinear bifurcation phenomena and of analytical and computational methods for studying them. Throughout, examples are discussed from an elementary level, which will give comfort to newcomers to the field and casual readers, to a level of sophistication and detail which should be pleasing and useful to experts. The commentary throughout the text and the extensive list of references give evidence of the author's detailed knowledge and understanding of his subject.

The book is mainly aimed at people who are comfortable with computers and who will learn from the text by experimenting with algorithms. Throughout, there are examples and exercises that urge the reader to turn to numerical simulation to gain insight into the nonlinear phenomena. However, the text does not get lost in discussing and explaining numerical techniques. It takes a 'black box' approach, assuming the reader has access to appropriate numerical software to carry out specific numerical tasks (solve equations, integrate numerically, and so on). But even without a background or interest in numerical methods the reader will still find this a stimulating book, both for its analytical content and for the author's ability to make one comfortable with the idea that the computations, while an issue in their own right, need not distract from the flow of understanding.

An excellent summary of the book's contents chapter by chapter appears in the Preface. Chapters 1 and 2 deal respectively with required applied mathematical concepts and basic nonlinear phenomena leading to bifurcations. Chapters 3 to 7 deal with practical aspects of studying bifurcations. Chapters 8 and 9 deal with more qualitative aspects: singularity theory, catastrophe theory and chaotic behaviour. There are guidance notes for those who may wish to avoid the more computational aspects of the text.

A couple of specific comments are worth making. I found the discussion of the Lorenz equations in §2.8 fascinating and much more convincing than other elementary treatments that I have read. Secondly the author is refreshingly candid about the experimental nature of much of the numerical work involved. To quote from the end of Chapter 4: '. . . . a continuation algorithm should offer the option of switching between extrapolation yes or no—that is, between an optimistic view and a more pessimistic view' (a bifurcation problem!) and '. . . a parameter study of a difficult problem is a venturous exploration.'

This book, with its many examples from scientific and engineering sources (usefully listed on page 371), should be of great interest to researchers who have to make practical studies of bifurcation phenomena. It should also be inspirational to mathematicians who wish to develop a knowledge and understanding of nonlinear phenomena from either an analytic or a computational viewpoint.

D. F. McGHEE

DiBenedetto, E., *Partial Differential Equations* (Birkhäuser, Basel–Berlin–Boston 1995), xiv+416 pp., 3 7643 3708 7, £39.

This book is an impressive account of many important aspects of the theory of partial differential equations. As acknowledged in the preface, the extensive nature of the subject matter means that a number of topics are unavoidably omitted, most noticeable being a treatment of some numerical methods. Nevertheless, the material which has been included is sufficiently wide-ranging to be of interest to many involved in the theoretical analysis of partial differential equations.

A substantial part of the book is devoted to linear equations, with the Laplace, heat and wave equations very much to the fore. Chapters II and III deal with the Laplace equation and related