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Disc Modes and Orbital Eccentricity Growth

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Abstract. An eccentric protoplanetary gaseous disc provides a slowly precessing non axisymmetric mass distribution. As a result of self-gravity, the precession frequency may resonate with that of a near circular protoplanet orbit within an inner cavity. A large eccentricity might then be produced.

Introduction

The recent discovery of a number of extrasolar giant planets orbiting around nearby solar–type stars (Marcy & Butler 1998, 2000) reveals that they have orbital semi-major axes in the range 0.04 AU $\lesssim a \lesssim 2.5$ au, and orbital eccentricities in the range $0.0 \lesssim e \lesssim 0.67$.

Protoplanet disc interactions occuring immediately post formation could potentially produce eccentric orbits but wide gaps are required in order to avoid damping by coorbital material (Artymowicz, 1992, Lin & Papaloizou 1993). Wide enough gaps are only likely to be produced for masses in the brown dwarf regime (Nelson et al. 2000, Papaloizou et al. 2000).

A possible mechanism for producing an eccentric orbit from a near circular one is to have a secular resonance between the precession frequency of the orbit and some nonaxisymmetric disc mass distribution. Torques can then act on a long timescale, removing angular momentum from the orbit to make it eccentric. To avoid damping the orbit has to be in a cavity interior to the disc. This may occur at the disc clearance phase when secular resonances could sweep through the system.

Slowly varying modes with m=1 We consider the linear normal modes of an axisymmetric disc that make it eccentric. In a cylindrical coordinate system (r, φ, z) centred on the central star, the dependence of perturbations on time, t, and azimuthal angle, φ, is through a factor $\exp i(\varphi - \omega_p t)$ henceforth taken as read. Here the azimuthal mode number $m = 1$ and the mode precession frequency ω_p is such that $|\omega_p| \ll \Omega$, where Ω is the angular velocity in the unperturbed disc in which the surface density is Σ. In the low eigenfrequency limit the gravitational potential perturbation takes the form

$$\Psi' = G \int \int_0^{2\pi} \cos(\varphi) \left(\frac{1}{\sqrt{(r^2 + r'^2 - 2rr' \cos(\varphi))}} - \frac{r}{r'^2} \right) \frac{d(\Sigma e(r'))}{dr'} (r')^2 d\varphi dr' \quad (1)$$

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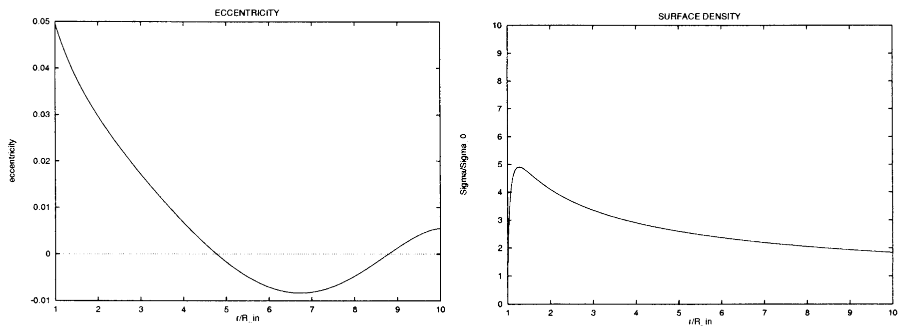

Figure 1. Surface density (left panel) and mode eccentricity (right panel) plotted as functions of r.

The normal mode equation to be solved for the disc eccentricity $e(r)$ and precession frequency (eigenvalue) ω_p is

$$2\left(\Omega_p - \omega_p\right)\Omega r^2 e(r) = -\frac{1}{r}\frac{d}{dr}\left(\frac{r^3c^2}{\Sigma}\frac{d[\Sigma e(r)]}{dr}\right) + \frac{1}{r}\frac{d\left(r^2\Psi'\right)}{dr} \qquad (2)$$

Here , c, is the local sound speed amd Ω_p is the free particle precesion frequency. The inclusion of self-gravity in the eigenvalue problem is essential if potentially resonant modes with prograde precession frequency are to be obtained. For typical protoplanetary disc models, self-gravity is strong enough to induce prograde precession for long wavelength modes.

An example of a disc mode We present here results obtained for a disc model such that $\Sigma = \Sigma_0\left(1 - (R_{in}/r)^{10}\right)(R_{in}/r)^{1/2}$, where R_{in} is the inner boundary radius. The arbitrary scaling factor Σ_0 was chosen such that the disc contained 7.5 Jupiter masses when $R_{in} = 1$au, $M_* = 1M_\odot$ and for $R_{in} < r < R_{out}$, with $R_{out} = 10R_{in}$ being the outer boundary radius. The sound speed is given by $c^2 = H^2\left(1 - R_{in}/r\right)R_{in}/r$. Here $H = 0.05$ is the disc aspect ratio at large distance. The surface density for this model is plotted in figure 1. It has a sharp edge which is favourable for inner orbit eccentricity generation. The eccentricity distribution of a $m = 1$ disc mode is also shown. The prograde precession frequency was $\omega_p = 6.75 \times 10^{-5}\sqrt{GM_*/R_{in}^3}$. This resonates with the precession of a near circular orbit at $R_{in}/3$, produced by the same disc.

Eccentricity excitation through secular resonance The evolution of the coplanar orbit of an interior protoplanet due to the gravitational interaction with the slowly precessing disc can be found using secular perturbation theory.
On performing a time average on obtains the Hamiltonian system:

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$$dh/dt = -\partial H/\partial \alpha, \qquad d\alpha/dt = \partial H/\partial h,$$

where $h = \sqrt{GM_* a(1-e^2)}$ is the specific angular momentum, a is the constant semi-major axis, and to leading powers in the eccentricity

$$H = -e^2a^2\left(\frac{3\pi G}{4}\int_{R_{in}}^{R_{out}}\frac{\Sigma(r')}{r'^2}dr' - \frac{\omega_p n}{2}\right) + \frac{e^4a^2\omega_p n}{8} + \frac{15a^3\pi Ge}{16}\int_{R_{in}}^{R_{out}}\frac{\Sigma_1(r')}{r'^3}dr'\cos\alpha. \qquad (3)$$

Here $n = \sqrt{GM_*/a^3}$, and $\alpha = \varpi - \omega_p t$, with ϖ being the longtitude of periapse.

Non Resonant Case When the term in H proportional to e^2 is non zero, the typical eccentricity generated is obtained by equating the first and third terms in H, thus

$$e = e_0 \sim (5a/4)(\textstyle\int_{R_{in}}^{R_{out}} \Sigma_1(r')(r')^{-3}dr' / \int_{R_{in}}^{R_{out}} \Sigma(r')(r')^{-2}dr').$$

For $a/R_{in} \sim 1/3$, and disc edge eccentricity ~ 0.25, this gives for the normal mode described above a modest $e \sim 0.07$.

Resonant case However, when the term in H proportional to e^2 vanishes we have secular resonance with matched precession periods for the disc and orbit. Then the characteristic eccentricity that can be generated from an initially circular orbit is found by equating the second and third terms in H, giving

$$e \sim (4e_0)^{1/3} \sim 0.6.$$

However, this is very much an uppper limit.

Discussion It is possible for an orbit with moderate eccentricity $e \sim 0.5$ to be produced if there is a secular resonance with an eccentric external disc. A configuration with an inner cavity might be produced during the disc clearance phase on a 10^5 y timescale when secular resonances could sweep through the system. However, more work is needed to see if special tuning is needed.

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