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ABSTRACT. Integrating backwards in time in the circular restricted threebody problem Galaxy-Sun-Comet, for both the real long-period comets and fictitious random sets of orbital elements, we have confirmed van Flandern's conclusion that there is a statistically-significant clustering of the orbits of real long-period comets, in heliocentric direction, some 5×10^6 years ago. The clustering is also significant in heliocentric distance, and is more marked if it is assumed that the comets have gone round the Sun more than once since the epoch of maximum clustering. We suggest that the "event" discovered by van Flandern is not the explosive disruption of a planet formerly in the asteroid belt, but the latest in a series of minor catastrophies, such as the collisional break-up of a pair of large asteroids.

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In a series of papers (1,2,3) one of us has developed dynamical arguments leading to the conclusion that there once existed, in the region of the asteroids, a massive planet which subsequently disrupted. T.C. van Flandern (4,5) has claimed that the statistics of the orbital elements of longperiod comets gives direct evidence that such a planetary disruption took place about 6 million years ago. His conclusion was based on (a) the distribution of orbital elements at the present time, and (b) the clustering of long-period comet orbits at a critical time in the past, as discovered by numerical integrations of the comets' orbits backwards in time, allowing for the perturbation by the non-uniform field of the Galaxy.

In this paper, we are concerned only with the evidence under (b). We have repeated van Flandern's calculations, using a different integration technique. We have also carried out integrations for some sets of fictitious comets, whose orbital elements were chosen at random. We wish to decide in what respects, if any, the real long-period comets differ, in their orbital characteristics, from the fictitious (random) sets.

The present orbital elements of 60 long-period comets, corrected for planetary perturbations during the apparition of observation to give pre-encounter values, have been listed by van Flandern (4). We adopted the same

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V. Szebehely (ed.), Dynamics of Planets and Satellites and Theories of Their Motion, 101-107. All Rights Reserved. Copyright © 1978 by D. Reidel Publishing Company, Dordrecht, Holland. orbital elements for our "Real" comets. Our "Random" sets of elements were generated using a pseudo-random-number generating program. The only restriction placed upon the elements was that the perihelion distance, l, should lie between 0.5 a.u. and 4.5 a.u. These limits correspond roughly to the range found in van Flandern's sample, since he rejected all real comets whose pre-encounter value of l was less than 0.5 a.u. on the grounds that for such comets non-gravitational perturbations would be significant. In this paper, for reasons of economy, we display the results of only one of our random sets.

Integrations (backwards in time) were performed using a library program for the solution of the circular restricted three-body problem. The program uses adjusted time-steps, and integrations were found to be reversible over a time of 10^6 years with a precision better than 0° 1 in angle and 0.001 in \mathcal{I} . The mass and distance of the galactic center were taken to be 2.32×10^{11} solar masses and 11.7 kpc respectively. With these values, in the restricted problem Galaxy-Sun-Comet, the correct galactocentric solar angular velocity $\tilde{\omega} = 25 \text{ km s}^{-1}\text{kpc}^{-1}(6)$ and the correct gradient $\delta \tilde{\omega}/\delta R = -3.2 \text{ km s}^{-1}\text{kpc}^{-2}$ (7) are obtained.

If the hypothetical planetary disruption occurred T years ago,the periods of those long-period comets actually observed within the last few centuries must be very close to T/n, where n is an integer, since $T \sim 10^7$ years ago (2; *vide* 3). On the grounds that the pre-encounter aphelion distances of long-period comets tend to cluster about 50,000 a.u.,whereas the post-encounter aphelion distances do not, van Flandern [following Marsden and Sekaninan (8)] argues that for the selected comets n=1. It is therefore of interest to note that the period of any comet observable near perihelion, with an aphelion distance of 50,000 a.u., is $\sim 10^7$ years. However, the actual aphelion distances are poorly determined for such longperiod comets. van Flandern therefore integrates his orbits backwards for various assumed values of T, taking n = 1 for each value of T.

In the plots of our integrations, given in Figure 1, the upper left-hand number is the value of T in years, and the upper right-hand number the value of n. All the plots show the comets' heliocentric coordinates on Mollweide's Equal Area Projection [which is not the same projection as that used by van Flandern (4)]. The zero of ecliptic longitude is taken to be the direction of the vernal equinox at the present time, taken to be fixed in an inertial frame. The zero of galactic longitude is taken to be the direction of the center of the Galaxy, as seen from the Sun, at the time T years ago, allowing for a constant angular velocity of the Sun around the center of the Galaxy, in an inertial frame, of &=25 km s⁻¹kpc⁻¹.

When the integration backwards of a comet, for a time T, is begun, it is assumed that the osculating period of the comet at the present epoch is T. This will not be precisely true, so that if the times of integration were taken to be exactly T for all comets, the various comets would, at that time, be in different phases of their orbits. Since the orbits are nearly rectilinear, this means that the orbital elements of the different comets would not be strictly comparable. In a few cases, we actually found

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the sidereal period T' of the comet, and then adjusted the initial assumed osculating period by trial and error until T' = T. However, we found that it was quite sufficiently accurate to carry out our integrations for a time slightly in excess of T, but to evaluate the orbital elements at*perihelion passage*. The plots are produced from these perihelion orbital elements, and are restricted to those phases of the comets' motions for which the heliocentric distance is less than 30 a.u.

In each figure, the left-hand diagrams show the results for the real comets - i.e. the comets in van Flandern's list. The right-hand diagrams show the corresponding results for the fictitious random set of elements.

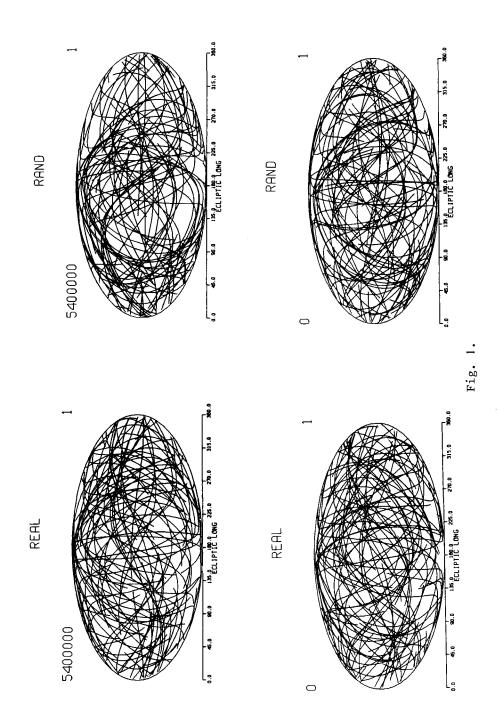
Figure 1 shows that the present pre-encounter orbital elements of van Flandern's list show no more preference for clustering than does the random set. Figure 2 shows that, for $T \sim 10^7$ years, n = 1, both the real set and the random set show extreme clustering. The plots in terms of galactic longitude show that the clustering is strongly related to the direction to the galactic center. This clustering is called by van Flandern "galactic polarization". It arises from the fact that for a comet to have had l < 30 a.u. one period $T \sim 10^7$ years ago, and *still retain* a value of l < 5 a.u., it must have had an original orbit that was insensitive to galactic perturbation. This "galactic polarization" has the consequence that if the hypothetical explosive event had occurred more than 10^7 years ago, the evidence for it in the comets' orbits would have been lost.

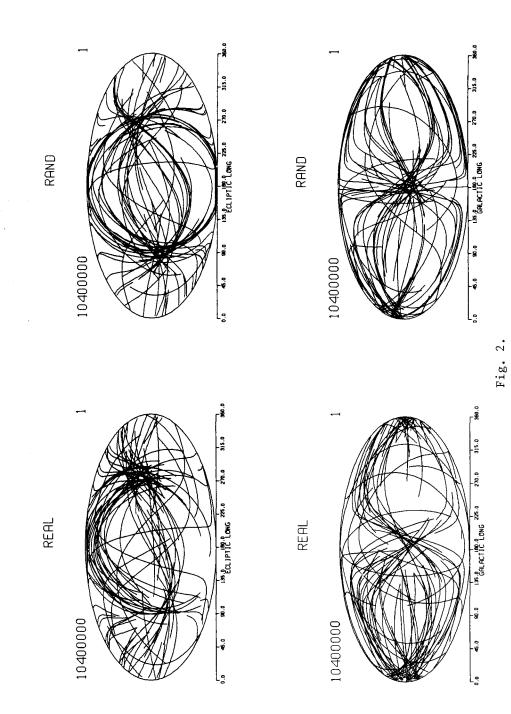
Figure 1 also shows that for n = 1, $T \simeq 5 \times 10^6$ years, the clustering of orbits is more marked for the "real" set than for the "random" set. Of course, it might be argued that if the clustering is purely accidental, the value of T has no significance, and that we could have found as strong a clustering of the fictitious set for some other value of T. This seems not to be the case for the random sets which we have investigated, with T restricted to past time. We find that the strongest clustering for the real set occurs for $T^{25} \times 10^6$ years, as compared with 6×10^6 years found by van Flandern. The difference is probably not significant. In these direction-coordinate plots, two centers of clustering, in opposite directions, are to be expected simply as a result of the fact that any orbit has two nodes on any given plane.

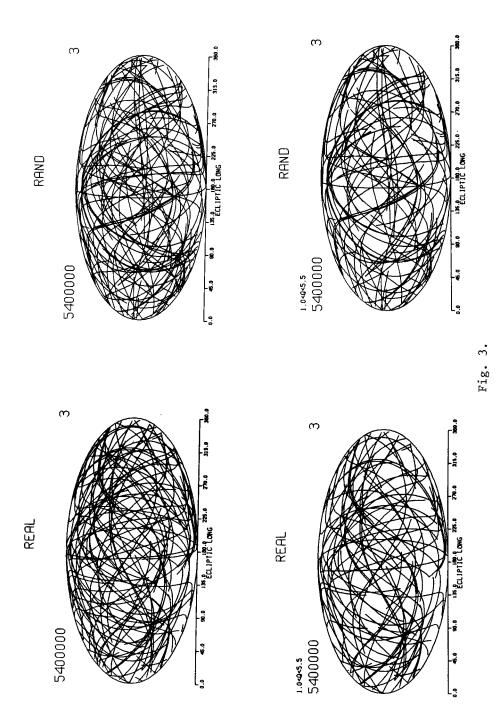
van Flandern did not investigate the consequences of assuming values of n different from 1. Figure 3 shows an important distinction between the real comets and the fictitious comets. The plots show the results of integrations assuming $T \simeq 5 \times 10^6$ years, but n = 3. The clustering is markedly enhanced for the real comets, whereas this is clearly not true for the random set.

The plots of heliocentric directions give no indication of whether the clustering orbits also intersect in space, in the region of the belt of asteroids. van Flandern argues that the effects of stellar encounters will have a greater effect on l than on the direction elements. This is undoubtedly true. Nevertheless we might hope to find some residual evid-

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ence for the explosive event also in the clustering of Q, the heliocentric distance at the node of the comet's orbit on the ecliptic. The bottom plots in Figure 3 show the same calculations as the upper plots, except that they have been restricted to those orbits for which 1.0 < Q < 5.5 a.u. A clear distinction now emerges between the real and the random sets. There is a correlation between those real orbits which contribute to the clustering, and those for which Q lies in the restricted range about the asteroidal distance. No such correlation can be seen for the residual clustering in the random set.

We conclude that the long-period comets of van Flandern's list do show a significant clustering in space in the region of the asteroid belt some 5×10^6 years ago (when allowance is made for the perturbing field of the Galaxy) as compared with randomly-chosen sets of "fictitious" comets. However, the clustering is more marked if it is assumed that the real comets have made more than one revolution about the Sun since the epoch of greatest clustering.

The time of 5×10^6 years is uncomfortably short for the dynamical arguments for a missing planet (3). Since "galactic polarization" would preclude discovery of an "event" prior to 10^7 years ago, and since the total cometary mass involved is much less than that of a planet, we suggest that the "event" discovered by van Flandern, and confirmed by our work, is not the catastrophic disruption of a major planet, but the latest in a series of minor catastrophies, such as (for example) the collisional break-up of two large asteroids.

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