ROTATION OF THE RIGID EARTH

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Abstract. We present the results of a solution of the Earth's rotation built with analytical solutions of the planets and of the Moon's motion. We take into account the influence of the Moon, the Sun and all the planets on the potential of the Earth for the zonal harmonics $C_{j,0}$ for j from 2 to 5, and also for the tesseral harmonics $C_{2,2}$, $S_{2,2}$, $C_{3,k}$, $S_{3,k}$ for k from 1 to 3 and $C_{4,1}$, $S_{4,1}$. We determine three Euler angles ψ, ω, and φ by calculating the components of the torque of the external forces with respect to the geocenter in the case of the rigid Earth. The analytical solution of the precession-nutation has been compared to a numerical integration over the time span 1900–2050. The differences do not exceed 16 μas for ψ and 8 μas for ω whereas the contribution of the tesseral harmonics reaches 150 μas in the time domain.

1. Introduction

The precision of the VLBI data allows one to measure the nutations for a non-rigid Earth at the level of a few $\times 10^{-5}$ arcseconds. It is thus necessary to build a solution for a rigid Earth with an accuracy of 1 μas. Furthermore, the accuracy of theoretical rigid Earth nutation series must be tested by comparing the results in the time domain with a numerical integration solution computed with the same model as the analytical solution. Now, we have compared the obtained analytical solution to a numerical integration using the same development of the terrestrial potential but in the analytical solution we used a solution for the lunar motion truncated to 0.″001.

2. Analytical Solution and Numerical Integration

We have determined the secular and periodic variations of the three Euler angles ψ, ω, and φ describing the motion of the true equator with respect

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TABLE 1. Difference between the analytical solution and the numerical integration. Unit is μas.

Time span	ψ	ω	ϕ
50 days	0.4	0.1	0.3
150 years	16	8	15

to the ecliptic and equinox J2000 as well as the precession-nutation in longitude $\mathcal{P}$, in obliquity ε and the angle χ (= arc $R\gamma_D$) where γ_D is the equinox of date and R the intersection of the equator of date and the ecliptic J2000. Finally, the series of the sidereal time is computed by

$$\text{sidereal time} = \varphi - \chi.$$

We computed the torques on the oblate Earth by using analytical theories for the motion of the solar system bodies including completely the relevant perturbations. In accordance with the observed value of the constant of precession $p = 50''\!.2877$ per year, we determined the dynamical ellipticity H_d :

$$H_d = \frac{2C - A - B}{2C} = 0.003\,273\,7671,$$

the value which agrees well with the values of Williams (1994) and Souchay and Kinoshita (1996). For the zonal harmonics, we take into account the coupling perturbations of the Moon with $C_{2,0}$, $C_{3,0}$, $C_{4,0}$, $C_{5,0}$, of the Sun with $C_{2,0}$, $C_{3,0}$, and of the planets with $C_{2,0}$. The tesseral harmonics empolyed are given in the next section. The same development of the terrestrial potential is used in the analytical solution and the numerical integration.

The motion of the Sun and of the planets is represented by the VSOP87A solution (Bretagnon and Francou, 1988) and that of the Moon by the ELP solution (Chapront-Touzé and Chapront, 1983). In the numerical integration the terrestrial equator is perturbed by the full ELP solution of the lunar motion but, in the analytical solution, we used an ELP solution truncated to $0''\!.001$ (8 000 terms solution instead of 40 000 terms) that ensures an accuracy of only $0''\!.09$ for the position of the Moon.

The comparison of the analytical and numerical solutions are illustrated in Bretagnon *et al.* (1996). In order to test the accuracy of the diurnal terms, we performed a numerical integration over 50 days. We have also performed a numerical integration over 150 years and the most important differences with the analytical solution are given in Table 1 for the three angles ψ, ω, and φ.

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TABLE 2. Most important perturbations of the diurnal nutations. Amplitudes are in μas, periods in days.

Origin		Argument	Amplitude of ψ	Amplitude of ω	Period
Moon	$C_{2,2}$, $S_{2,2}$	$2\lambda_3 + 2D - 2\varphi$	29.44	11.71	0.52
	$C_{3,1}$, $S_{3,1}$	$\lambda_3 + D + \varphi$	38.44	15.25	0.96
	$C_{3,2}$, $S_{3,2}$	$\lambda_3 + D - 2\varphi$	0.39	0.14	0.51
	$C_{3,3}$, $S_{3,3}$	$3\lambda_3 + 3D - 3\varphi$	0.14	0.05	0.35
	$C_{4,1}$, $S_{4,1}$	φ	1.68	0.67	1.00
Sun	$C_{2,2}$, $S_{2,2}$	$2\lambda_3 - 2\varphi$	12.32	4.90	0.50
	$C_{3,1}$, $S_{3,1}$	$\lambda_3 + \varphi$	2.79	1.11	1.00

3. Tesseral Harmonics

Table 2 gives the tesseral harmonics taken into account and the most important contribution for each harmonic. One may note the amplitude of the one day period terms coming from the resonance with the Euler period term. All the diurnal terms greater than 1 μas are given in Table 3. The last column contains the periods of the non-diurnal part of the arguments. The values of the components of the arguments are taken from Simon *et al.* (1994) :

$$\begin{aligned}
\lambda_3 &= 1.753\,470\,459\,50 + & 6\,283.075\,849\,9914\,t \\
D &= 5.198\,466\,741\,03 + & 77\,713.771\,468\,1205\,t \\
F &= 1.627\,905\,233\,38 + & 84\,334.661\,581\,3083\,t \\
l &= 2.355\,555\,898\,30 + & 83\,286.914\,269\,5536\,t \\
\varphi &= 4.894\,961\,212\,82 + & 2\,301\,216.753\,651\,535\,t
\end{aligned}$$

where t is the time measured in thousand Julian years from J2000.

To evaluate the importance of the diurnal terms, we have substituted the time in the terms of Table 3. Figure 1 gives the variations of the diurnal terms over 50 days. It shows up contributions to the full solution reaching 100 μas in the time domain whereas the most important term has an amplitude of only 38 μas and various beatings, for instance, of 13.66 day period.

We also performed a substitution of time in the series of Table 3 over 150 years. Taken into account the very high frequencies of these terms, Figure 2 shows only the upper and lower envelopes of these curves. The diurnal perturbations reach 153 μas for ψ, 61 μas for ω, and 141 μas for φ. The most important beatings have periods of 4.4 years and 18.6 years.

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TABLE 3. Diurnal terms of ψ, ω, and φ. Unit is μas, non-diurnal part periods in days.

Argument	ψ(sin)	ψ(cos)	ω(sin)	ω(cos)	φ(sin)	φ(cos)	Period
$\lambda_3 + D + \varphi$	−38.13	−4.69	−1.86	15.13	35.09	4.32	27.32
2φ	−31.85	18.28	7.27	12.67	31.97	−18.35	
$2\lambda_3 + 2D - 2\varphi$	−25.36	−14.56	5.79	−10.09	−0.13	−0.07	13.66
$\lambda_3 + D - \varphi$	34.82	−4.27	1.64	13.37	−32.29	3.96	27.32
$\lambda_3 + D - l + \varphi$	23.93	2.99	1.19	−9.52	−21.95	−2.75	3232.61
$\lambda_3 + D - l - \varphi$	19.85	−2.49	0.99	7.87	−18.23	2.29	3232.61
$2\lambda_3 - 2\varphi$	−10.63	−6.10	2.43	−4.23	−0.40	−0.24	182.63
$2\lambda_3 + 2D + l - 2\varphi$	−5.15	−2.96	1.18	−2.05	0.07	0.04	9.13
$\lambda_3 - D + l + \varphi$	−7.08	−0.88	−0.35	2.82	6.50	0.81	193.56
$\lambda_3 + D + F - 2\varphi$	4.78	2.75	−1.09	1.90	−5.26	−3.02	13.63
$\lambda_3 + D - F - 2\varphi$	−4.32	−2.48	0.99	−1.72	4.78	2.74	6793.48
$F + \varphi$	6.01	0.74	0.29	−2.37	−5.56	−0.68	27.21
$F - \varphi$	−5.34	0.66	−0.26	−2.15	4.85	−0.60	27.21
$F - l + \varphi$	−4.02	−0.50	−0.20	1.60	3.68	0.46	2190.35
$\lambda_3 + D + l - \varphi$	3.15	−0.39	0.14	1.17	−2.95	0.36	13.72
$F - l - \varphi$	−3.04	0.38	−0.15	−1.21	2.78	−0.35	2190.35
$\lambda_3 + D + l + \varphi$	−2.89	−0.36	−0.14	1.14	2.66	0.33	13.72
$l - 2\varphi$	1.88	1.08	−0.43	0.75	−1.89	−1.08	27.55
$l + 2\varphi$	−1.69	0.97	0.39	0.67	1.70	−0.97	27.55
$\lambda_3 + \varphi$	2.18	0.27	0.11	−0.87	−2.00	−0.25	365.26
φ	1.12	−1.22	−0.48	−0.45	−1.02	1.11	
$2\lambda_3 + 2D - F + \varphi$	−1.98	−0.24	−0.10	0.79	1.82	0.22	27.43
$2\lambda_3 + 2D - F - \varphi$	1.66	−0.20	0.09	0.75	−1.57	0.19	27.43
$\lambda_3 - D + l - \varphi$	1.60	−0.19	0.07	0.63	−1.47	0.17	193.56
$\lambda_3 + D + F + l - 2\varphi$	0.97	0.56	−0.22	0.39	−1.06	−0.61	9.12
$2\lambda_3 + 4D - l - 2\varphi$	−0.97	−0.56	0.22	−0.39	0.01	0.01	9.56
$2\lambda_3 + D - l + \varphi$	−0.13	−1.27	−0.50	0.05	0.12	1.16	328.18
$2\lambda_3 + 2D + 2\varphi$	−0.88	0.50	0.20	0.35	0.84	−0.48	13.66
$2\lambda_3 + 2D - F - l + \varphi$	1.18	0.15	0.06	−0.47	−1.08	−0.14	6167.21
$2\lambda_3 + 2D - F - l - \varphi$	1.06	−0.13	0.05	0.43	−0.98	0.12	6167.21

4. Comparison with the Kinoshita-Souchay Solutions

We have compared our solution with the Kinoshita-Souchay (1990) solution and with the Souchay-Kinoshita (1996) solution. The first comparison (Bretagnon, 1996) shows several terms with differences of a few 0.1 mas. The most important difference is 0.9 mas for the 122 day period term. With respect to the Souchay-Kinoshita (1996) solution (SK96) the most import-

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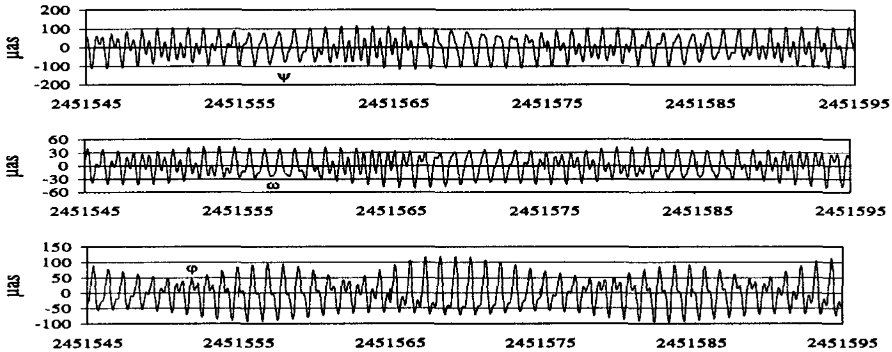

Figure 1. Diurnal terms of ψ, ω, and φ over 50 days.

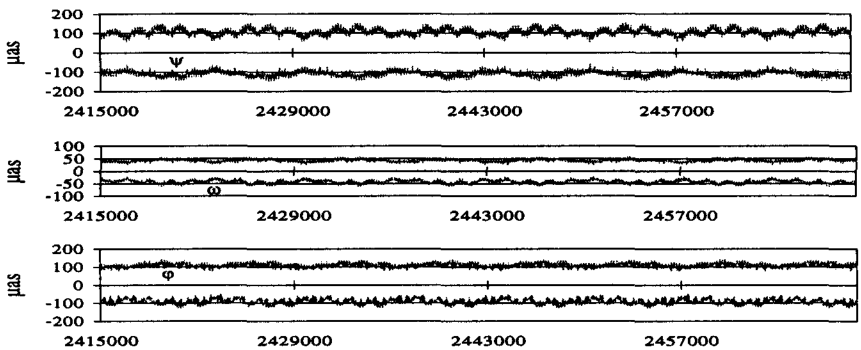

Figure 2. Envelopes of the diurnal terms of ψ, ω, and φ over 1900-2050.

ant difference concerns the 18.6 year term. The SK96 solution gives for the nutation in longitude :

$$\begin{aligned} \mathcal{P}_{SK96} = & - 1''\!.280\,585 \sin(\Omega_D) + 0''\!.000\,135 \cos(\Omega_D) \\ & - 0''\!.000\,128 \sin(-2l' + 2F - 2D + \Omega_D). \end{aligned} \tag{1}$$

In our solution, the longitude Ω_D of the node of the Moon reckoned in the ecliptic and equinox of date is represented by :

$$\Omega_D = \lambda_3 + D - F - 180° + pt$$

where p is the constant of precession. The mean anomaly l' of the Sun is :

$$l' = \lambda_3 - \varpi_3 = \lambda_3 - 1.796\,595\,647\,27 - 0.056\,298\,2756\,t$$

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where ϖ_3 is the longitude of the Earth's perihelion. Then, the two terms (1) become, for the periodic part :

$$\mathcal{P}_{SK96} = +17\overset{''}{.}280\,700 \sin(\lambda_3 + D - F) - 0\overset{''}{.}000\,190 \cos(\lambda_3 + D - F).$$

We find for the nutation in longitude :

$$\mathcal{P} = +17\overset{''}{.}280\,765 \sin(\lambda_3 + D - F) - 0\overset{''}{.}000\,440 \cos(\lambda_3 + D - F)$$

and thus

$$\mathcal{P}_{SK96} - \mathcal{P} = -65\,\mu\text{as} \sin(\lambda_3 + D - F) + 250\,\mu\text{as} \cos(\lambda_3 + D - F).$$

It must be noted that we find an out-of-phase term (440 μas) without dissipation mechanism.

For the nutation in obliquity we obtain

$$\varepsilon_{SK96} - \varepsilon = -5\,\mu\text{as} \sin(\lambda_3 + D - F) + 38\,\mu\text{as} \cos(\lambda_3 + D - F).$$

5. Conclusion

The comparison with numerical integrations shows that our present analytical solution has, for the chosen model, an accuracy of 16 μas. The amplitudes of the nutation series agree well with Souchay-Kinoshita's solution but we have a difference of 250 μas in the 18.6 year term. The diurnal terms reach 150 μas in the time domain and it is necessary to take into account the contribution of the tesseral harmonics with a high precision. We are going now to compute our solution with a greater precision and to introduce the full ELP solution in the determination of the rigid Earth rotation problem.

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