DYNAMICAL APPROACH TO THE $R^{1/4}$ LAW OF ELLIPTICALS

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ABSTRACT. The $R^{1/4}$ luminosity law has often been used as an observational constraint in the construction of self-consistent models of ellipticals. In contrast, in this and in the following paper, developing some ideas proposed in the past (BS84, SB85), we investigate certain theoretical arguments that may lead to a dynamical justification of this important empirical law.

Phase space properties of elliptical galaxies suggested by numerical experiments of dissipationless galaxy formation are used as a dynamical constraint (essentially to provide a boundary condition at large radii) in the construction of self-consistent equilibrium models. In particular, from a discussion of the quasi-spherical case, the following simple selection criterion is proposed: the combination of integrals of the motion in the relevant distribution function should guarantee that asympotically at large radii (i) the pressure anisotropy parameter $\alpha = 2 - \langle v_T^2 \rangle / \langle v_r^2 \rangle$ tends to 2 and (ii) the mass density decreases as r^{-4}.

We have already used this working hypothesis to construct a simple family of distribution functions which was found to possess the $R^{1/4}$ law as a built-in property (BS84,SB85). Detailed comparison with photometric and kinematical data has been made by Saglia (1986) for eight ellipticals. Given the interest of these results we now test the generality of our approach in several ways. First we test structural stability by constructing a number of new families of equilibrium distribution functions all following the above defined selection criterion. They are all found to be qualitively similar to our original family and, in particular, to be consistent with the $R^{1/4}$ law in the region of parameter space where the central potential well is sufficiently deep.

Then we address the problem of the dynamical stability of our equilibria by performing N-body experiments using van Albada's (1982) code. No significant instabilities are found in our survey.

All the results described above are restricted to the quasi-spherical case. We finally show that following the prescription

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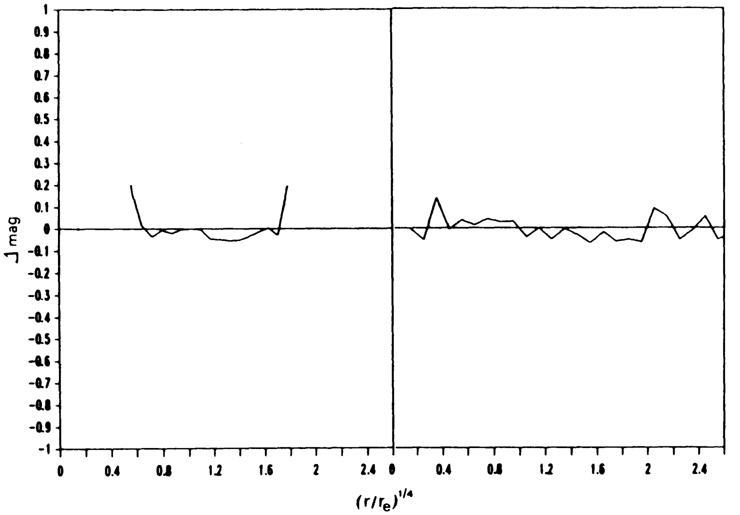
of our distribution function we can easily generate N-body equilibria with the qualitative features of our analytic models but with finite flattening, so that (oblate) E2/E3 galaxies are reasonably modelled. Again the agreement with the $R^{1/4}$ law is preserved.

Bertin, G. and Stiavelli, M., 1984. Astron. Astrophys. 137, 26 (BS84)
Saglia, R., 1986. Tesi di laurea, Università di Pisa
Stiavelli, M. and Bertin, G., 1985. Mon.Not.Roy.astron.Soc. 217, 735 (SB85)
van Albada, T.S., 1982. Mon.Not.Roy.astron.Soc. 201, 939.

Table Summary of the results of N-body simulations

run id	N_{par} ($\times 10^3$)	T_{fin} (t_{cr})	$\Delta\mathcal{E}/\mathcal{E}$ % per t_{cr}	$\Delta N/N$	e_f	Δe
P7.7C	40	14.2	0.20	0.00	0.01	0.00
P7.7E	20	40.3	0.02	0.00	0.25	0.01
P7.7G	60	17.1	0.10	0.00	0.18	0.02
P6A	40	28.0	0.07	0.00	0.00	0.01
P6C	40	14.0	0.00	0.00	0.19	0.01
P5.2A	40	51.0	0.00	0.00	0.02	0.01
P5.2B	60	51.1	0.01	0.00	0.00	0.00
P5.2C	20	50.5	0.09	0.00	0.03	0.01



Projected mass distribution for the final equilibrium of the N-body run labeled P7.7G, initialized with the $\Psi = 7.7$ distribution function of BS84. On the left, deviations from the $R^{1/4}$ law are plotted in the range ($.1\ r_e$) to ($10\ r_e$), corresponding to a range of ten magnitudes. On the right, deviations from the initial model are plotted. Note that this case could simulate an E2 galaxy. In the Table e_f denotes final ellipticity of the model, Δe is the ellipticity variation during the run. The rest of the notation is self-explanatory.

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