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2. D. M. Cvetković, M. Doob, I. Gutman and A. Torgašev, *Recent results in the theory of graph spectra* (North-Holland, Amsterdam, 1988).

Ziegler, G. M. *Lectures on polytopes* (Graduate Texts in Mathematics, Vol. 152, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong 1995), ix+370 pp., softcover: 3 540 94365 X, £21, hardcover: 3 540 94329 3, £47.

Since Branko Grünbaum's monumental book *Convex polytopes* (Wiley-Interscience) appeared in 1967, several texts have been written on or around the same subject. However, few of them deserve more than a tepid recommendation (which is why I shall not mention their authors). Further, the exceptions to this stricture have usually been volumes of collected articles, which were not devoted solely to polytopes. Here, though, we do have a book which can be welcomed with considerable enthusiasm. While it does not attempt to emulate the (then) comprehensiveness of Grünbaum's work, it provides more than adequate compensation with its coverage of theories which have developed over the last nearly thirty years.

Let us recall that a *convex polytope* is simultaneously definable as the convex hull of some finite point-set in a euclidean space or as a bounded intersection of finitely many closed half-spaces. A polytope P (we shall usually drop the qualification 'convex' in what follows) has *faces* of each dimension up to $\dim P$, which are its intersections with supporting hyperplanes. The family $\mathscr{F}(P)$ of these faces (including $\emptyset$ and P itself) forms a lattice, partially ordered by inclusion. Two polytopes P and Q are *isomorphic* (or *combinatorially equivalent*) if their face-lattices $\mathscr{F}(P)$ and $\mathscr{F}(Q)$ are isomorphic (as lattices).

Like Grünbaum's book Ziegler's is largely concerned with the combinatorics of polytopes. Central to this area is the *Steinitz problem*, which asks which lattices are isomorphic to face-lattices of popytopes. Up till now complete answers are only available for d-polytopes (that is d-dimensional polytopes) with d at most 3 or with at most $d+3$ vertices (or facets). These polytopes are also 'nice' in that there always exist isomorphic polytopes with rational vertices and the spaces of realizations of isomorphic polytopes (factored out by affinities) are contractible. It has recently become clear that for all other polytopes the situation is completely different: all subfields of the algebraic numbers may be needed for realizations and the realization spaces may be arbitrarily complicated. However, one aspect of this new work is only mentioned in the book and the other is too new even for that. It should be emphasized, though, that polytopes occur in many different contexts; the connexions with algebraic geometry appear to be particularly deep and extensive.

Ziegler's book provides a fine introduction to these newly developing areas. The treatment of the equivalence of the two definitions of polytopes (given above) in Lecture 1 is non-standard, using the computational technique of Fourier–Motzkin elimination. This sets a tone, that of doing things in often slightly unusual ways. The basic results about face-lattices are established in Lecture 2. Graphs of polytopes (formed by their vertices and edges) are dealt with in Lecture 3; here we see recent important theorems such as Kalai's pseudopolynomial bound on the edge-diameter of a d-polytope with n facets and his proof of the fact that the graph of a simple d-polytope (one with just d facets through each vertex) determines its combinatorial type.

The next two lectures concern Steinitz-type problems. Lecture 4 treats 3-polytopes, giving the positive answer mentioned above. Lecture 5 looks at 4-polytopes, giving examples which show that objects which look as if they ought to be projections (in some sense) of 4-polytopes in fact need not be.

A Gale diagram of a d-polytope with n vertices is a set of n points in $(n-d-1)$-dimensional space and encodes its combinatorial properties. Gale diagrams formed a very new topic in

Grünbaum's book, but in Lecture 6 they and their variants, generalizations and applications receive a full treatment. One interesting topic here concerns whether the shape of a given face of a polytope can be specified (in some isomorphic polytope); examples are given to show that this is generally not possible. (This is in contrast to the case of dimension 3.) Diagrams also occur in Lecture 7, which covers *zonotopes*, or vector sums of line segments. These have close connexions with arrangements of hyperplanes and oriented matroids, a subject of independent interest.

If one cannot characterize face-lattices of polytopes, one might at least ask less detailed questions such as which sequences $(f_0, \ldots, f_{d-1})$ of numbers are *f-vectors*, with $f_j = f_j(P)$ the number of j-faces of some d-polytope P. Even this question cannot yet be answered if $d \geqq 4$, although it can when P is simple. This problem is only mentioned here, but the upper bound theorem (giving the maximum of each f_j when f_0 is fixed) is proved using the technique of shelling (established for polytopes after 1967), a favourable ordering of the facets of a polytope.

The final Lecture 9 deals with fibre polytopes, another recent development. These are concerned with subdivisions of one polytope arising from it as a projection of another. The lecture ends with a look towards the future.

In addition, each of the lectures concludes with (historical) notes and with problems and exercises; some of the last are unsolved. This is in keeping with the philosophy of the book as an introductory text which can be used by students to learn the subject by themselves. Finally, there is an extensive bibliography, enlivened by the provision of the authors' full names wherever known. My judgment is that the aim of the book, to give just such an introduction, is most successfully accomplished. But students would not be the only ones who could benefit from so sympathetic a treatment and I firmly recommend the volume to anyone who might wish to find out about a subject which is intrinsically fascinating and, in spite of having about the oldest pedigree in mathematics, is yet a lively area of reasearch and has increasingly many connexions with other branches.

P. McMULLEN

GLENDINNING, P. *Stability, instability and chaos* (Cambridge University Press, Cambridge 1994), xiii + 388 pp., hardcover: 0 521 41553 5, £45, paperback: 0 521 42566 2, £17.95.

This book adds to a fast growing library of undergraduate introductions to dynamical systems theory (where it joins titles by Perko, Drazin, Verhulst, to name only a few). It is clearly a reincarnation of a course lecture notes. These origins are reflected both in its (very many) strong points and in (the non-negligible number of) its shortcomings.

First, the good news. I enjoyed the light conversational style of this book (see Fig. 1.1 and the discussion of the Glendinning integral). I hope in this respect Dr. Glendinning's book sets a precedent. It is also very nicely typeset. I was also impressed with some of the material covered, in particular, the very coherent and readable discussion of resonances, a very modern exposition of invariant manifold theory, as well as computation of direction of Hopf bifurcation, discussions of Arnol'd tongues and of global bifurcation phenomena (chapter 12, based on the work by the author). Much of this material will find its way into the honours course on nonlinear ODEs which I am teaching.

I would say, however, that there is scope for improvement. There is no need for French spelling of Lyapunov's name; omission of Grobman from the theorem that usually bears his name as well requires an explanation. The style sometimes borders on the inadmissibly loose: for example, 'the solution can go all over the place before tending to the point' (p. 28) can easily be improved, as can the caption to Fig. 1.4. Fig. 5.8 is incomprehensible altogether. The motivation for defining stability for an arbitrary point in phase space (and not, say, for an invariant set) is not clear. The ordering of the material is also suspect. Centre manifolds belong by right in a discussion of invariant manifolds. Discrete dynamical systems are an interesting object on their own account and should not be hidden in a chapter on periodic orbits (Chapter 6). The section on canards