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Towards Estimating the Solar Meridional Flow and Predicting the 11-yr Cycle Using Advanced Variational Data Assimilation Techniques

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Abstract. We present in this work the development of a solar data assimilation method based on an axisymmetric mean field dynamo model and magnetic surface data. Our mid-term goal is to predict the solar quasi cyclic activity. We focus on the ability of our variational data assimilation algorithm to constrain the deep meridional circulation of the Sun based on solar magnetic observations. Within a given assimilation window, the assimilation procedure minimizes the differences between data and the forecast from the model, by finding an optimal meridional circulation in the convection zone, and an optimal initial magnetic field, via a quasi-Newton algorithm. We demonstrate the capability of the technique to estimate the meridional flow by a closed-loop experiment involving 40 years of synthetic, solar-like data. We show that the method is robust in estimating a (stochastic) time-varying flow fluctuating 30% about the average, and that the horizon of predictability of the method is ~ 1 cycle length.

Keywords. Dynamo, meridional circulation, data assimilation, numerical

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1. Introduction

Solar activity impacts space-weather and influences our modern technology-based society significantly, so it is important to obtain good solar predictions. The most common index to quantify solar activity is the sunspot number (SSN) (Clette *et al.* 2014; Vaquero *et al.* 2016), and Wolf reconstructed the past SSN from 1749. In addition, the surface magnetic field of the Sun is also an important observable. Observations of solar magnetic field can at least be traced back to 1908 (Hale 1908). Solar activity is quasiperiodic with sunspot cycle of 11 ± 3 years (Clette & Lefèvre 2012). Those cycles, however, vary in both their period and amplitude (Svalgaard, Cagnotti & Cortesi 2015), which raises questions regarding their predictability (Ossendrijver *et al.* 2002; Tobias *et al.* 1998). Data assimilation is an emerging technique in solar prediction, which incorporates observations in numerical models (Brun 2007). For examples of application in studying solar activity, see Dikpati, Anderson & Mitra (2014), Dikpati, Anderson & Mitra (2016) and Jouve, Brun & Talagrand (2011). In this work, we develop a variational data assimilation technique to analyze magnetic observations of the Sun. By controlling selected parameters of the physical model, an optimal fit of data is obtained over the time window when observations are available. We adopt the 4D-Var method, which minimizes the misfit or objective function, by using the adjoint model (Fournier *et al.* 2010; Talagrand 2010). This is based on the Babcock-Leighton flux transport mean field dynamo model (Babcock 1961; Leighton 1969; Jouve & Brun 2007) and its adjoint. The initial magnetic field and the meridional flow are the control parameters, as they determine the phase and the cycle length of the magnetic field. While recent estimate of meridional flow from helioseismology below the solar surface suggests the possibility of more complex structures other than a unicellular one (Zhao *et al.* 2013; Schad, Timmer & Roth 2013; Haber *et al.* 2002; Kholikov, Serebryanskiy & Jackiewicz 2014), there is no unique conclusion on the meridional structure, which raises the interest of estimating the meridional flow with a dynamo model. We show that we get a satisfactory estimate of the true time varying profile of the meridional circulation, and reconstruct the magnetic trajectory.

2. Data Assimilation Framework

We develop a data assimilation technique to estimate the initial magnetic field and the flow of the Sun on the meridional plane. We aim to minimize an objective function $\mathcal{J}$, defined in terms of the differences between the observations and the model trajectory,

$$\mathcal{J} = \sum_{\alpha} \sum_{i=1}^{N^o_{\alpha,t}} \sum_{j=1}^{N^o_{\alpha,\theta}} \frac{[y_\alpha(\theta_j, t_i) - y^o_\alpha((\theta_j, t_i)]^2}{\sigma^2_\alpha(\theta_j, t_i)}, \tag{2.1}$$

where α denotes the type of magnetic proxy y to be compared. The proxies with the superscript ^{o} stand for observations, and without for the forecast values, and σ_α stands for the errors. For each type α we sum over the observation times and latitudes, $N^o_{\alpha,t}$ and $N^o_{\alpha,\theta}$, respectively. In the assimilation setup, the magnetic observations are compared with the forecast values from the flux transport dynamo model. The latter is controlled by the state vector $\mathbf{x}$, which consists of the initial magnetic field and the flow of the model on the meridional plane. We adopt a quasi-Newton algorithm in the minimization of the objective function (Gilbert & Lemaréchal 1989), and the gradient of the function with respect to $\mathbf{x}$ is obtained by the adjoint model (Hung *et al.* 2015, 2017).

3. Closed-loop experiment and results

We verify and study our assimilation framework with a closed-loop numerical experiment. We generate synthetic observations from the same model used in the assimilation procedure with a known set of control parameters, contribute to the truth state $\mathbf{x}^t$. Then, by ingesting the synthetic observations, we show that we are able to estimate the truth state. In our example shown, we create an meridional circulation as a linear combination of 2 time varying expansion coefficients of the basis functions on the meridional plane. These 2 basis functions are of opposite parity about the equator, so that the flow and and corresponding field can be equatorially asymmetric. The scale of the stochastic time variability ($\sim$ 3 years) is chosen to be of the same order as the observed flow (Ulrich 2010). Furthermore, the amplitude of the fluctuation (30% of the average) is adjusted so that the corresponding fluctuation of the cycle length in the model is close to the observed 11 ± 3 years. We show the time-latitude plot of the surface flow in Figure 1. The synthetic magnetic proxy we used in this experiment is the surface magnetic field at the line of sight (B_{los}), and the toroidal field energy at the tachocline in both hemispheres (which mimic the hemispheric SSNs), and the data is noised artificially with 10% of its root mean square. We show the magnetic butterfly diagrams of the B_{los} and the toroidal field at the tachocline ($r = 0.7R_\odot$) in Figure 2. We assimilate the synthetic data yearly

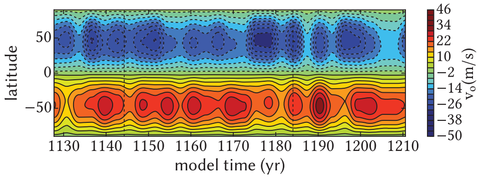

Figure 1. Latitudinal component of the flow at the surface as a function on arbitrary time unit. The assimilation period in the numerical experiment which follows is indicated by the dashed vertical lines. The sign convention is positive for a flow due south.

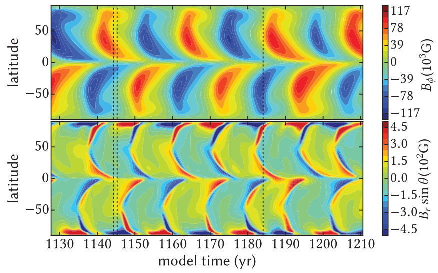

Figure 2. Top: time-latitude plot of the toroidal field at the tachocline. Bottom: Same for B_{los} at the surface.

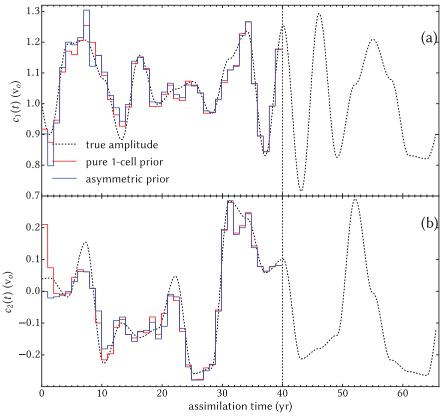

Figure 3. (a) Time series of the coeff. of the unicellular component of the meridional circulation. The reference time series is in broken line. The piecewise constant red (blue) curve is the result of assimilating synthetic observations from a unicellular (asymmetric) prior information. (b) Same for the antisymmetric component.

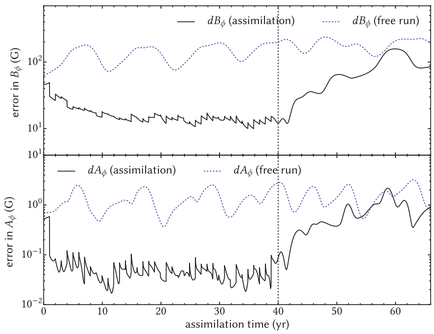

Figure 4. Top: Absolute difference between various estimates of the toroidal magnetic field and the true magnetic field versus time. Blue broken: free run of the dynamo model (unconstrained by data). Black: data assimilation estimate, with data consisting of B_{los} and pseudo sunspot number (toroidal field energy at the tachocline). Bottom: Same for the poloidal magnetic field.

for 40 consecutive years. Figure 3 shows the result of the estimate of the expansion coefficients of the meridional circulation. We approximate the true, smooth, time varying profiles with piecewise constant functions, by ingesting the synthetic observations yearly and assuming the flow is constant within each year. This estimate reasonably captures the time variability of the flow. Figure 4 shows the total error of the estimated poloidal field potential (A_ϕ) and the toroidal field (B_ϕ) (black curves), defined by integration of the differences between the truth and the estimate over the meridional plane. The errors are compared with the counterparts of a free dynamo run without assimilation (blue broken curves). When the synthetic observations are available, the errors in the estimate are 5% or less than the counterparts of the free run. After year 40, the errors start to grow and reach the free run case after $\sim$ 15 years without input of the data, which indicates the predicative capability in this example.

4. Summary

Our numerical experiment shows the capability of the data assimilation framework in estimating the deep meridional circulation of the Sun using magnetic proxies. We

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construct synthetic magnetic proxies with artificial random noise from the model, like B_{los} and the sunspot number, by relating them to the surface poloidal field potential and the toroidal field in the tachocline computed from the flux transport dynamo model. We then show that, by ingesting the synthetic monthly observations using a 4D-Var assimilation method over one-year long time intervals, we are able to reconstruct the time varying flow for 40 years. The method can also account for the equatorial asymmetry of the observed magnetic field as well as an asymmetric meridional flow, hence it is not impaired by symmetry of any sort. By comparing the time evolution of the error of the estimated magnetic field with that of a free dynamo run where no data assimilation is done, we conclude that in this experiment, the predictive capability of the method is about 15 years. More details and analysis can be found in Hung *et al.* (2017). Our future development include (i) analyzing magnetic proxies of the real Sun with the data assimilation method, (ii) improving the representation of the meridional circulation in the assimilation framework, such as adding the acceleration of the fluid to the control vector, and (iii) including physical constraints in the objective function.

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