	Brought forward,	385
Turnbull, Esq.; Dr John Charles Ogilvie Will; Dr Andrew Wilson, ...		18
Deduct Deceased.—Alexander J. Adie, Esq.; John Blackwood, Esq.; Dr Colledge; E. W. Dallas, Esq.; Dr J. G. Fleming; Edward J. Jackson, Esq.; Professor Kelland; Dr James M'Bain; Professor Clerk-Maxwell; Professor Nicol; Dr Montgomerie Robertson; J. F. Rodger, Esq.; Dr John Smith; Sir Walter C. Trevelyan; Arthur, Marquis of Tweeddale, ...	15	
Resigned.—Professor Fuller; O. G. Miller, Esq.; Professor John Young, ...	3	
	18	18
Total number of Ordinary Fellows at November 1879, .		385
Add Honorary Fellows, ...		54
Total number of Ordinary and Honorary Fellows at commencement of Session 1879–80, ...		439

During the last session the Keith Prize for the biennial period 1875–77 was awarded to Professor Heddle for his papers on the "Rhombohedral Carbonates" and on the "Felspars of Scotland," originally communicated to the Society, and containing important discoveries. The Makdougall-Brisbane Prize for the biennial period 1876–78 was awarded to Professor Geikie for his memoir "On the Old Red Sandstone of Western Europe," which has been published in the Society's Transactions, and forms one of an important series of contributions by Professor Geikie to the advancement of geological science.

OBITUARY NOTICES.

The Rev. Professor Kelland. By Professors Chrystal and Tait.

Professor Kelland was the son of the Rev. Philip Kelland, who, at the time of the birth of his son, was rector of the parish of Dunster, in Somersetshire. Afterwards it would appear that he removed to Landcross, in Devonshire. Though an Oxford man himself, he sent his son Philip to Queen's College, Cambridge, where he greatly distinguished himself among his contemporaries, and in 1834 stood at the head of the honour list as Senior Wrangler and First Smith's Prizeman. Mr Kelland, who had taken orders in the Church of England, became Tutor of Queen's College, and held the post for

https://doi.org/10.1017/S0370164600043911 Published online by Cambridge University Press

the next three years. In 1838 he was appointed to the Chair of Mathematics in the University of Edinburgh, as successor to Professor Wallace. He has been a Fellow of this Society ever since he came to Edinburgh; and only last year was elected to its highest office.

The loss which this Society suffered in the death of its President has already been characterised in fitting words by Sir Alexander Grant (*ante* p. 208). What we are now called upon to do, is to give a general account of his services alike to the University of Edinburgh and to science.

Kelland occupied the Mathematical Chair from the time of the resignation of Professor Wallace in 1838—a period including forty-one complete sessions. During six months of each year he gave at least thirteen lectures in all per week to his three classes; and for at least four sessions, during the illness of Professor Forbes, he conducted the Natural Philosophy course also. He was, besides, for long periods secretary of Senatus, and of the Board of Visitors of the Observatory, and was constantly employed in conducting examinations for various public bodies and institutions, *e.g.*, the Colleges of Physicians and Surgeons, the Dick Bequest, the Edinburgh High School, &c., and on several important occasions his services were engaged by one of the Scottish Insurance Offices with a view to the septennial investigation of its affairs from the actuarial point of view. In this connection he made a tour in Canada and the United States, of which he published an account in a charming little volume called "Transatlantic Sketches." If we add to this the labour entailed by his various published works and original scientific papers, as well as his constant contributions to educational publications, we can easily see what an active life he spent. To the very end of his career his activity never seemed to flag. His college duties grew from year to year, partly in consequence of the great increase in the number of students, but mainly because of the enormous increase of graduation. And he kept up, year after year, from 1869 at least, two Mathematical Classes for the Edinburgh Ladies' Educational Association. Yet his teaching was to the last as thorough as ever; and no better proof could be desired than the fact that three of the last four awards of the Ferguson Mathematical Scholarship, which is open to all the Scottish Universities, have been made in favour of Edinburgh students.

As Sir A. Grant has well said, he came to know the Scottish Universities better even than do Scotsmen themselves. To this we may add that he knew also, as few have ever known them, the characteristics and the wants of Scottish students. Our grief for his loss is at least tempered by the fact that he died at his post after an unusually extended term of active usefulness. He who has in person instructed, alike by clear precept and noble example, many thousands of the youth of a nation, cannot fail to have a happy and lasting influence on that nation's progress. Philip Kelland was, in the very highest sense, a benefactor to Scotland.

In spite of all his hard week-day work, he did not shrink from clerical duty on Sundays, very often reading the service, or preaching, in some of the Episcopal Churches in Edinburgh. In his sermons, as in his secular addresses, he was studiously quiet and simple, avoiding all mere popular arts of word-painting; but he was none the less effective in consequence. No one in the crowded Assembly Hall on the 22d April last, when he appeared for the last time before the public, can forget the profound impression produced on the whole audience by the few but earnest and loving words which he then addressed to the graduates. A fortnight later he was followed to his grave by the majority of those who had vociferously applauded his simple and touching eloquence.

His earlier mathematical work was very much influenced by his admiration for Fourier and Cauchy. The latter, indeed, was his personal friend. His *Theory of Heat*, and various elaborate papers in the "Camb. Phil. Trans." and the "Phil. Mag.," show how Kelland attempted to base the explanation of the phenomena of heat upon the mutual action of systems of particles exerting forces on one another at a distance. The analysis employed is of a nature very similar to Cauchy's; but we need not examine these attempts closely, for, though they show great mathematical ingenuity, they are now known to be based upon an unsound physical assumption.

He was much more successful when his physical assumptions were more accurate, as in his investigations of the motion of waves in canals, and in the calculation of the intensity of light which had passed through a grating. Another real service which he did to physical science consisted in his having edited and reprinted the very valuable "Lectures" of Thomas Young. Kelland's edition is now unfortunately, like its predecessor, entirely out of print.

But his forte unquestionably lay in pure mathematics, and in that department, and others closely allied to it, he has enriched our "Transactions" with several very excellent papers.

An idea of Professor Kelland's scientific activity will be obtained by looking at the list of papers under his name in the Royal Society's Catalogue of Scientific Memoirs.

Several of his memoirs deal with physical optics. Two of these are especially interesting. They deal with the question of the aggregate effect of interference. In the first ("Trans. Camb. Phil. Soc." vii.) he shows that when light falls on a lens, part of which is covered and part uncovered, the whole quantity of light on a screen placed in the focus is to that which falls on the lens as the area of the uncovered part of the glass is to the whole area of the glass. Hence he infers that the whole quantity of light is not diminished or increased by interference. In the second ("Trans. R.S.E." xv.), starting from the principle thus established, he treats a very interesting point which arises in the application of Huyghens' principle in the undulatory theory. In forming the expression for the vibration due to any element of an aperture on the surface of a lens, we multiply the maximum intensity of vibration by the area of the element, and to keep the dimension correct we must divide by a factor D whose dimension is the square of a line. Kelland investigates a variety of cases for different forms of aperture, and finds in each case that D must be $b\lambda$ where λ is the wave-length of the incident light, and b the distance from the lens of the screen placed in its focus. The question was afterwards discussed by Stokes ("Trans. R.S.E." xx.), who generalised Kelland's analysis, and showed that the result may be deduced for an aperture of any form.

In a memoir read before the Royal Society of Edinburgh in April 1839 ("Trans." xiv.) Kelland took up the subject of wave motion. He discusses the case of a canal of finite depth h, adopting the hypothesis of parallel sections. Assuming the motion to be undulatory, and taking

$$z = h + a \sin \frac{2\pi}{\lambda}(ct - x)$$

for the equation to the surface, he deduces the approximate formulæ, in which $\alpha = \frac{2\pi}{\lambda}$,

$$c^2 = \frac{g}{\alpha} \frac{e^{\alpha h} - e^{-\alpha h}}{e^{\alpha h} \quad {}^{-\alpha h}} . \frac{1}{1 - a^2 \alpha^2 (e^{\alpha h} - e^{-\alpha h})} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$a = \frac{2}{\alpha(e^{\alpha h} + e^{-\alpha h})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the first of which gives the velocity of transmission, the second the height of the wave.

In the latter part of the paper he applies his method to canals having a vertical section of any shape whatever, and deduces the following elegant formula—

$$c^2 = g \frac{\text{area of vertical section}}{\text{breadth at surface}}$$

for the velocity of propagation. This gives the result, for canals of triangular section, that the velocity of propagation is that in a rectangular canal of half the depth. This conclusion is tested by means of Scott-Russell's observations, and is found to be in close agreement with fact.

The same result was also arrived at independently by Green, who, in point of fact, anticipated Kelland in the matter, for he gives it in a note read before the Cambridge Philosophical Society on the 18th February 1839, whereas Kelland's paper was read on the 1st April of the same year. Scott-Russell's observations were the exciting cause of both investigations, which have little in common beyond this particular result.

In a memoir on General Differentiation ("Trans. R.S.E." xiv.), read December 1839, Professor Kelland deals with one of the most abstruse and difficult branches of analysis. The process by which we extend the meaning of the symbol x^m where m is integral to the case where m has any value whatever, is familiar enough, although it has its difficulties as every algebraist knows. General differentiation is a problem of a similar kind but of a much higher order of difficulty. Thus,

$$\frac{d}{dx}, \qquad \left(\frac{d}{dx}\right)^2, \qquad \left(\frac{d}{dx}\right)^3, \text{\&c.,}$$

are symbols which have for their effect to deduce in a particular

https://doi.org/10.1017/S0370164600043911 Published online by Cambridge University Press

way from any given function another set of functions, to which the name of first, second, third, &c., differential co-efficients are given. The corresponding problem here is to interpret the operating symbol,

$$\left(\frac{d}{dx}\right)^{\mu},$$

where μ may have any value whatever. This interpretation must be so made that it shall include the particular meanings already attached to the cases where μ is integral. This process of extension has been aptly called by De Morgan a case of the interpolation of forms, and there are difficulties in connection with it very like those that arise in the solution of functional equations or the inverse method of finite differences. The question was first raised by Leibnitz, and was treated successively by Euler, Laplace, Fourier, Liouville, Greatheed, Peacock, and Kelland.

The laws of operation to be conserved are—

$$D^n(u+v) = D^n u + D^n v,$$
$$D^m D^n u = D^{m+n} u.$$

The only question is:—What fundamental functions are we to select on which to base our calculus? It appears that different systems arise according as we select our fundamental functions. Peacock starts with x^m: Kelland, following Liouville and others, starts with e^{mx} as the ground function, and lays down the equation

$$\left(\frac{d}{dx}\right)^{\mu} e^{mx} = m^{\mu} e^{mx}$$

as the foundation of his system.

By means of a definite integral he then deduces the general formula

$$\left(\frac{d}{dx}\right)^{\mu} x^{-n} = \frac{(-1)^n \overline{/n+\mu}}{/n \, x^{n+\mu}},$$

where $/n$ is a function like the gamma function, satisfying the equation

$$\overline{/n+1} = n\,\overline{/n},$$

but unlike it not restricted to positive values of n.

This formula is, then, applied in a variety of particular cases, and is shown to be perfectly general provided certain conventions are adopted, and from it are derived working formulæ convenient in different cases.

The theory is applied to the logarithmic and circular functions, and at the end of Part I. are given some very ingenious applications to expansions in fractional powers of x.

In Part II. is given the following extremely elegant formula—

$$\int_0^z d\theta\phi(\theta+\alpha)(z-\theta)^p = (-1)^{p+1} \sqrt{p+1}\cos(p+1)\pi\left(\frac{d}{dz}\right)^{-(p+1)}\phi(z+\alpha),$$

which is applied to the solution of a variety of problems.

The whole of the mathematical work in this memoir is of great simplicity and elegance, and for that reason alone it is well worth the attention of students of the higher mathematics. It has, moreover, intrinsic value as an important contribution to the elucidation of a difficult branch of analysis. How great that importance may be it is impossible to estimate until the future of the method is more certain than it can at present be said to be; but, in any case, the work will remain a lasting monument to the skill and ingenuity of its author.

Closely connected with the paper just mentioned is another on a process in the differential calculus and its application to the solution of differential equations. Nothing farther need be said regarding it except that it is characterized by the same elegance and simplicity that mark the memoir on general differentiation.

Perhaps the most important of all Professor Kelland's scientific papers is his Memoir on the limits of our knowledge respecting the Theory of Parallels. He there deals with the subject now better known as absolute or non-Euclidean geometry. It would scarcely be possible to convey to those who have not busied themselves with pangeometry (or the geometry of pure reason as one might venture to call it, as opposed to the geometry of experience which is Euclid's) a full idea of the importance of this work of Kelland's, and of the evidence that it affords of his grasp of purely mathematical speculation. Suffice it to say that he reasons out correctly, and perhaps even more elegantly than is done in one of the last works on the subject,* the consequences of denying Euclid's "parallel axiom," or what is its equivalent, viz., the proposition that the sum of the three angles of any triangle is two right angles. It can be shewn by means of the properties of congruent figures, which, with all the consequences as

* Frischauf, "Elemente der Absoluten Geometrie."

to the nature of space that follow therefrom, are hereby assumed that—(1) The sum of the angles of any triangle can never exceed two right angles; (2) If the sum of the angles of any one triangle is two right angles, then the sum of the angles in every triangle is two right angles. But independently of the theory of parallels, this is in substance as far as we can go. If we assume that the sum of the angles of any triangle is less than two right angles, then we arrive at the conclusion that this sum depends on the area of the triangle, the defect from two right angles being less the less the area, and the same for all triangles of the same area, consequently therefore proportional to the area of the triangle. The effect of this assumption on the theory of parallels is very remarkable. Defining parallels as straight lines in the same plane that do not intersect (this is not the definition adopted in recent books, such as that of Frischauf, above named, but that is a mere question of words), we find that there are an infinite number of straight lines passing through the same point all parallel to a given straight line; that through one point on one of a pair of parallels only one straight can be drawn that makes the alternate angles equal; that parallel straight lines are not equidistant; that the locus of the points equidistant from a given straight line is not a straight but a curved line; that equal parallelograms on the same base cannot be between the same parallels, and so on. All this, and much more, is shewn by Kelland to form part of a system of geometry as logical as Euclid's.

As far as can be gathered from the memoir, and the form of the demonstrations, all but the fundamental propositions (the mere idea in fact) is Kelland's own work. It is characteristic of the man that he was in the habit of treating this subject in his class lectures.

Clearness in dealing with the fundamental principles of mathematical science was one of the virtues of Kelland's thinking and teaching. His text-book on *Algebra* is distinguished over other text-books in present use by its attempts to give a rational account of the first principles of the subject. The same readiness to grasp a new elementary idea, and trace its consequences, is exemplified by the fact that he took up Quaternions with his class in the University, and so late as 1873 published, in conjunction with Professor Tait, an excellent elementary treatise on this branch of mathematics. (See the *Preface* to that work.)

The minds of most men stiffen with age, and after a certain period the faculty of reception in most disappears. It was evidently not so with Professor Kelland.

ALEXANDER JAMES ADIE, Esq. By David Stevenson, M.I.C.E.

ALEXANDER JAMES ADIE, Civil Engineer, son of the late Alexander Adie, F.R.S.E., the eminent optician, was born in Edinburgh in 1808. A course of study at the High School, and afterwards at the University of Edinburgh, prepared him for entering on an apprenticeship under Mr James Jardine, Civil Engineer, with whom he was afterwards associated in carrying out various works.

In 1836 he became Resident Engineer of the Bolton, Chorley, and Preston Railway, and communicated some interesting papers to the Institution of Civil Engineers regarding that work, particularly one on Skew Bridges.

On leaving Lancashire he removed to Glasgow to take charge of some of the colliery railways there, and ultimately became engineer and manager of the Edinburgh and Glasgow Railway, which post he resigned about 1863.

Mr Adie made a series of important experiments on the expansion of stone by heat, which he communicated to the Society in his paper entitled "The Expansion of Different Kinds of Stone from an Increase of Temperature, with a Description of the Pyrometer used in making the Experiments," which is published in vol. xiii. of the Transactions.

Mr Adie was elected a Member of the Society in 1846. He latterly retired to reside at Rockville, near Linlithgow, where he had an opportunity of cultivating his taste for horticulture and the fine arts, and of receiving visits from many who esteemed his friendship, and valued his accomplishments.

JOHN BLACKWOOD, Esq. By Principal Sir Alex. Grant, Bart.

JOHN BLACKWOOD, who died on the 29th October last, was for a long period one of the most widely known and highly esteemed worthies of Scotland. As head of the last remaining of the great