ONTARIO MATHEMATICAL MEETINGS

The fifth Ontario Mathematical Meeting was held Saturday, December 2, 1967 at Sidney Smith Building, University of Toronto. Research papers were presented in the morning, followed by lunch at Sir Daniel Wilson Residence, followed by an address by Professor J. Glimm (Massachusetts Institute of Technology) entitled: Gas Dynamics and Related Non-linear Hyperbolic Equations. Abstracts of research papers which were presented are as follows:

67.16 S.H. Smith (University of Toronto)
The Impulsive Motion of a Wedge in a Viscous Fluid

A wedge is situated at rest in a viscous liquid when it is given an impulsive velocity along its line of symmetry; this constant velocity is maintained for all subsequent time. In this work we consider the resultant motion in the boundary layer along the face of the wedge.

We first consider two different (linear) differential equations which approximate the non-linear equations of motion; one is more accurate at the edge of the boundary layer and the other at the face of the wedge. From these it is concluded that there is a certain time, depending on the position, during which the fluid is unaware of the presence of the leading edge. This effect is then initially felt at the edge of the boundary layer before proceeding to diffuse inwards through the layer to the face of the wedge. Estimates are given for the time required for this steady state to be virtually complete.

The full non-linear equations are then considered to further investigate the motion at three particular stages. This is achieved firstly soon after the impulsive start, and secondly when the effect of the leading edge first enters the expressions for the velocity, which it is shown to do with an essential singularity. Finally we notice that for large times the unsteady motion has an exponential decay.

67.17 Wolfgang Eichhorn (Universitaet Wuerzburg and (visiting) University of Waterloo)

Functional Equations in Vector Spaces, Composition Algebras, and Systems of Partial Differential Equations

The following problem is considered: Let X be a vector space over a field F of char. $\neq 2$. Find linear mappings $L=L(x)$ of X into the vector space Hom (X, X) of the linear transformations of X with the property that there exist other such mappings, say $M=M(x)$, such that the functional equation

$$(1) \quad M(x)L(x) = \mu(x)I \quad \begin{cases} I\text{: identity map } X \to X \\ \mu\text{: } X \to F, \ \mu \not\equiv 0, \text{ a quadratic form} \end{cases}$$

holds. Writing

$$\sum_{i=1}^{n} \alpha_{ijk} \frac{\partial}{\partial \xi_i} \quad \text{for } L(x) \quad \text{and} \quad \sum_{i=1}^{n} \beta_{ijk} \frac{\partial}{\partial \xi_i} \quad \text{for } M(x)$$

$(j, k = 1, 2, \ldots, n)$

one sees that (1) contains the problem of the existence of "generalized Cauchy-Riemann equations" which are connected with a single second order differential equation (analogous to the connection between the classical C.-R. equations and the Laplacian).

Results. 1) Let $\bar{L}, \bar{M}, \bar{\mu}$ be a solution of (1). Let the algebra $\bar{A}$ defined by $\bar{L}$ [by $xy := \bar{L}(x)y$; this is a bilinear mapping $X \times X \to X$, i.e., (X, xy) is an algebra] be isotopic [see Albert, Ann. Math. 43 (1942), p.696] to an algebra A with identity element e. Then A is an alternative quadratic algebra, i.e., a composition algebra in case $\bar{\mu}$ is nondegenerate. It is known that every composition algebra is one of the following 1, 2, 4, or 8 dimensional algebras: Fe, $F \oplus F$, extension fields of F of degree 2, quaternion algebras, Cayley algebras. 2) If $\bar{A}$ as defined in 1) is a division algebra, then it is isotopic to a composition algebra. 3) If the dimension of X is finite and even if μ is nondegenerate then there exist solutions of (1) whose corresponding algebras are not isotopic to composition algebras. However such solutions again only exist in dimensions 2, 4, 8.

The application of the above results to the problem of the existence of generalised C.-R. equations is immediate.

67.18 Benno Artmann (McMaster University)
<u>On the relations between Hjelmslev Planes and Modular Lattices</u>

A list of elements $a_1, \dots, a_n$ of a modular lattice L with greatest element U and least element N is said to be a homogeneous basis of L, if the a_i are independent, pairwise perspective and their union is U. We consider the case where n = 3 and the quotients $L(N, a_i)$ are chains. We define $A = \{p \in L/p$ is a complement of $a_1 \cup a_2$ or $a_1 \cup a_3$ or $a_2 \cup a_3\}$ and $B = \{g \in L/g$ is a complement of a_1 or a_2 or $a_3\}$ and call A the set of points, B the set of lines. An incidence relation $|$ is induced by the order relation of L. With further assumptions concerning the existence of certain relative complements, the system $H = (A, B, |)$ turns out to be a projective Hjelmslev plane in the sense of W. Klingenberg, Math. Zeitschr. <u>60</u>. If the chains $L(N, a_i)$ are of length 2, then H is uniform (defined as done for the affine case by H. Lüneburg, Math. Zeitschr. <u>79</u>) or a projective plane. Every uniform Hjelmslev plane can be obtained from a suitable lattice L in the way described above.

67.19 Walter Benz (Bochum and Waterloo, Ontario)
<u>A Characterisation of the Geometry of Circles in the Minkowskian Plane over a Field</u>

Given the ring L of pseudo-complex numbers over the commutative field K of characteristic $\neq 2$. The projective line K' over K can be considered as a subset of the projective line L' over L. Every

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subset of L', which is of the form $(K')^{\gamma}$ with $\gamma \in PGL(2, L)$ is called a pseudo-euclidean (or minkowskian) circle. By means of the two maximal ideals of L two parallel-relations on the set of points can be introduced. The results are an axiomatic characterisation of the above described geometrics and the determination of some automorphism groups connected with these geometrics.