

Abstracts of Australasian PhD theses

Root-theory of involutive Banach-Lie algebras

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The structure and classification theory of finite dimensional Lie algebras has been extended to a class of complex involutive Banach-Lie algebras of infinite dimension.

A complex Banach-Lie algebra E with involution $*$ is called a symmetric Lie algebra if for all self adjoint $x \in E$, $|\exp(it \operatorname{ad} x)| = 1$ for all $t \in \mathbb{R}$, and is called an S -algebra if, in addition, it has no proper abelian ideals nor abelian quotients. A pair (E, M) consisting of an S -algebra E and a self adjoint maximal abelian sub-algebra $M \subset E$ is called chromatic if the orbits in E under the action of the group $G = \{\exp(i \operatorname{ad} h) : h \in M, h = h^*\}$ are relatively compact. An S -algebra E is spanned by the root spaces with respect to M if and only if (E, M) is a chromatic pair. All semisimple complex finite dimensional Lie algebras equipped with a compact real form, all semisimple L^* -algebras, and all completions of

$\mathfrak{sl}(H) = \{T \in B(H) : \operatorname{rank} T < \infty, \operatorname{trace} T = 0, H \text{ a Hilbert space}\}$
in uniform cross-norms are chromatic S -algebras.

The root theory for chromatic pairs is completely analogous to that for semisimple complex finite dimensional Lie algebras. A chromatic pair (E, M) is algebraically determined by its root system, and (E, M) is simple if and only if its root system is indecomposable. The simple

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chromatic pairs have been classified, and have been found to fall into the four big Dynkin classes A, B, C, D .