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Cycles in Cubic Graphs

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In a communication network it is assumed that every node is able to communicate (send or receive messages via direct links) with three other nodes of the network. When a set of nodes is specified, it is asked whether or not a node in the set is able to communicate with all other nodes of the set in two ways. Sometimes an unreliable link is to be avoided and sometimes a specified link is to be utilised. Situations occur in which a given link is unavoidable regardless of whether it is reliable or not. There are also situations in which a favourable link cannot be utilised at all.

The work in this thesis is a consideration of this type of question in a mathematical setting. A communication network can be adequately represented by a graph whose vertices and edges represent nodes and links of the network.

We consider cycles in regular graphs. The purpose of this work is to study conditions under which a 3-connected cubic graph has cycles through a specified set of elements. Cycles in graphs have been of intensive research interest for many years. The present work is a continuation of research in a few topics of the field and our method of research is classical.

We were able to obtain a characterisation of cyclable sets of nine vertices and an edge in 3-connected cubic graphs. We also show that every 3-connected cubic planar graph has a cycle containing any chosen set of fourteen vertices and an edge. This result is best possible as there are 3-connected cubic planar graphs in which there are some fifteen vertices and an edge not contained in any cycle. We classify cyclable sets of twelve vertices in 3-connected cubic graphs under a contraction.

The search of nonhamiltonian 3-connected cubic planar graphs was motivated by an historic attempt to prove the *four colour theorem*. We have shown that every cyclically 4-connected cubic planar graph of order at most 40 is hamiltonian. Three nonhamiltonian cyclically 4-connected cubic planar graphs of order 42 were known. Hence the

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nonhamiltonian cyclically 4-connected cubic planar graphs of the smallest order have 42 vertices.

We were able to show that every 3-connected cubic planar graph has a cycle containing any given set of 21 vertices.

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