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Partitions into large unequal parts

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Let $u = (u_j)_1^\infty$ be a strictly increasing sequence of positive integers and for $x \geqslant 1$ let $U(x)$ be the number of terms of u which do not exceed x. For integers m and n such that $0 \leqslant m < n/2$ define $q_u(m,n)$ to be the number of partitions of n into distinct parts coming from the sequence u and exceeding m.

In the special case when u is the sequence of positive integers, the classical function $q(n) = q_u(0,n)$ and, more recently, the function $q(m,n) = q_u(m,n)$ have been investigated by several authors. Freiman and Pitman [**1**] have recently given asymptotic estimates for $q(m,n)$ as $n \to \infty$.

In the general case the function $q_u(m,n)$ has also been studied, mainly for $m = 0$. In particular, Roth and Szekeres [**2**] have given an asymptotic formula for $q_u(0,n)$ which is widely applicable.

This thesis studies the asymptotic behaviour of $q_u(m,n)$ as $n \to \infty$ for sequences such that $U(x) \sim C_0 x^s (\log x)^{-t}$ as $x \to \infty$, where $C_0 > 0$, $s > 0$ and $t \geqslant 0$ are constants. Chapter 1 introduces the problem and provides historical background and Chapter 2 gives auxiliary results.

Chapter 3 presents the main theorem. For u as above satisfying a suitable further condition, and for given small positive δ, this gives an asymptotic estimate for $q_u(m,n)$ which is valid uniformly in m such that $0 \leqslant m \leqslant n^{1-\delta}$ as $n \to \infty$. The result is motivated by probabilistic considerations similar to those of [**1**] and the proof uses the circle method as in [**1**].

The next two chapters cover applications of the main theorem. The first part of Chapter 4 shows that the theorem applies to three wide classes of sequences which together include all the specific examples in [**2**]. The remainder of the chapter shows that under the conditions of the main theorem, for relatively small m, we have, as $n \to \infty$

$$q_u(m,n) \sim 2^{-U(m)} q_u(0,n).$$

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Chapter 5 uses the main theorem to obtain precise results about $q_u(m,n)$ in the case when u is the sequence of k-th powers.

Chapters 6 and 7 are devoted to more detailed study of the case when u is the sequence of positive integers. This work extends the results of [**1**].

References

[**1**] G.A. Freiman and J. Pitman, 'Partitions into distinct large parts', *J. Austral. Math. Soc. Ser. A* **57** (1994), 386–416.

[**2**] K.F. Roth and G. Szekeres, 'Some asymptotic formulae in the theory of partitions', *Quart. J. Math. Oxford Ser. (2)* **5** (1954), 241–259.

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