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ABSTRACTS OF PAPERS TO BE PRESENTED AT THE DISTRICT MEETING OF THE PSYCHOMETRIC SOCIETY, THE UNIVERSITY OF CHICAGO, ON SATURDAY, APRIL 3, 1937

Studies in the Learning Function, L. E. AND A. WILEY, Ohio Wesleyan University, Delaware, Ohio.

To the maze records of animals previously subjected to cerebral cortical injury we fitted Thurstone's theoretical learning curve

$$u = \frac{\sqrt{m}}{aK} - \frac{\sqrt{m}}{K} \cdot \frac{u}{R} \ . \tag{1}$$

In our analysis the curve appears in the form

$$u = A + \frac{BR}{C + R} \ . \tag{2}$$

A linear relationship is shown between the horizontal asymptote of the theoretical curve and the criterion which was formerly used, total-errors-minus-errors-on-the-first-trials.

Proof is given that the curve is an equilateral hyperbola which makes it possible to use the length of the semi-major axis to represent the learning situation. From this a solution for the learning constant of the animals and for the difficulty of the maze has been developed.

The properties of the vertex of the curve are discussed. At this point, all of the animals can be equated because of the fact that the first derivative is unity.

The relationship of the length of the semi-major axis to the limits of learning is discussed and a geometrical definition of insight is given. (15 minutes)

The Progress of General College Students in Mathematics, MARY L. ELVEBACK, University of Minnesota, Minneapolis, Minnesota.

An algebra test was given to a class on the first day of both the fall and winter quarters. The mean high school percentile rank was 30 and the mean C.A.T.P.R. 27: nevertheless, the mean score increased from 52 to 84 out of a possible 113 points.

Success in the course is as accurately predicted by this half-hour test as by the high school record.

Analysis of the variance reveals that students in different quarters of first score differ significantly in various measures of gain. Students of low initial status not only gain more points than those of higher initial status but their second scores are larger proportions of their first scores. In fact, the regression of the relative gain on the first score, p, is well fit by the rectangular hyperbola

$$\frac{p_2 - p_1}{p_1} = \frac{54 - .5\,p_1}{p_1 - 2.4} \;.$$

Thus, as the first score increases, the relative gain decreases with a diminishing rate of decrease so that it falls off very rapidly at first but becomes almost constant for large first scores. However, the effect of initial status on the ratio of gain to possible gain is negligible except within the upper quarter of first scores. This is easily established by correlating the gain with its "most likely" value. (15 minutes)

A Rational Theory of Discrimination Learning, HAROLD GULLIKSEN AND DAEL L. WOLFLE, University of Chicago, Chicago, Illinois.

A rational theory has been developed which describes the course of discrimination learning and predicts the nature and accuracy of transfer of the discrimination to new stimuli.

The theory is sufficiently general to handle any discrimination problem, but in this paper the mathematical development will be confined to the usual type of discrimination problem in which the subject is presented with two stimuli which differ in some respects (brightness, form, size, etc.). In half of the trials these stimuli are presented spatially in an AB order, and in half of the trials in a BA order. Each order constitutes a different stimulus configuration.

We postulate the existence of two response tendencies for each stimulus configuration—to go to the right and to go to the left. The relation between the strengths of the two response tendencies for a given configuration, i, is assumed to be a function of two factors.

1. *Initial Strength*: Before any practice has been given the strength may vary due to position preferences, brightness preferences, etc.
2. *Practice*: Each reward increases the strength of a response tendency and each punishment decreases it. Practice of a response to any configuration, j, affects the strength of the same response tendency to configuration i according to a diminishing function of the distance between i and j. One such function is:

$$C_i = e^{-d^2_{ij}} \cdot k \;.$$

C_i = change of strength of the response tendency to configuration i, due to one practice with configuration j.

d_{ij} = distance between configurations i and j. (d_{ij} can be measured by scaling techniques.)

k = the learning constant.

If configurations i and j are identical, $d_{ij} = 0$ and $C_i = k$.

The asymptotes of the learning curve derived from these postulates depend upon the distance separating the stimulus configurations to be discriminated. The rate at which an individual curve will approach the asymptotes depends upon the subject's learning ability.

Some of the deductions regarding the ease or difficulty of learning different discrimination problems, and regarding the nature and extent of transfer to new stimuli, which have been checked against the experimental literature are:

1. If training is given under the usual conditions, learning difficulty increases as the distance between the two configurations decreases.
2. If four configurations are used in training, either an absolute or a relative discrimination can be established. The latter is easier.
3. Variation of the stimuli during training retards but does not prevent learning.
4. If subjects have been trained to make a relative discrimination, transfer to new configurations will be made on a relative basis.
5. If subjects have been trained to make an absolute discrimination, transfer to new configurations will be made on an absolute basis.
6. The accuracy of transfer will decrease as the distance between training and test configurations increases.

The derivation of the learning equation and the reasoning leading to some of the theoretical deductions will be presented. (30 minutes)

The Role of Statistical Selection of Items in Test Construction, DOROTHY C. ADKINS, The University of Chicago, Chicago, Illinois.

Statistical selection of items is distinguished from rational analysis, which may be pre-statistical or post-statistical. Those using the term "item analysis" usually mean item selection, which does not of necessity imply any rational analysis.

The application of a time-consuming selection technique, involving the interrelationships of items, to 150 items with 800 subjects is reported. Such a selection technique offered but slight improvement in predictive value over simpler methods. The low average of the 11,175 intercorrelation coefficients (.09) is consistent with this result. Data are presented to show the decrease in predictive value of a selected composite when it is tested on a new population.

That more complicated statistical techniques may be of greater value in a relatively untried field is still a possibility. Even in such a field, however, the same items cannot be repeated indefinitely. Statistical selection can be justified, therefore, only when accompanied by post-statistical analysis which provides more or better clues for future construction of items than can be attained by simpler means. That such clues do result remains to be demonstrated.

It is thus argued that pre-statistical analysis may obviate the need for post-statistical analysis and that results to date scarcely justify the application of complicated selection techniques. At the present stage, we should determine whether *any* method of item selection is of value rather than attempt to devise additional short-cuts and approximations to available statistical methods. (15 minutes)

Simple Graphic Aids for Harassed Psychometricians, HAROLD D. GRIFFIN, Nebraska State Teachers College, Wayne, Nebraska.

It is the writer's contention that we might employ certain easily constructed graphic devices more than we do, and thereby effect a saving in time and energy without reducing accuracy.

For a number of years he has used a graphic method for coding variables in multiple regression problems, generally reducing scores in all variates to a range of 20 points with 0 as low score, which is equivalent to grouping into 21 classes. This is speedier than other coding devices, and the resulting uniform range is helpful for preliminary inspection of relationships among the variables, besides facilitating rapid calculation of the various zero-order coefficients.

The writer has also found that the construction of simple graphic prediction charts aids in applying the multiple regression equation, and that these prediction charts may quite readily be adjusted by means of graphic methods to care for curvilinear tendencies in any of the original variates without employing other than the Pearson product-moment formula in obtaining zero-order correlations. (15 minutes)

The Theory of the Estimation of Test Reliability, G. F. KUDER AND M. W. RICHARDSON, University of Chicago, Chicago, Illinois.

Beginning with a precise definition of the equivalence of test forms, the reliability coefficient is expressed in its theoretically best form. A series of special assumptions are made for cases in which item analyses of varying degrees of completeness are available. The resulting formulas represent various degrees of approximation to the theoretical coefficient. The last of these,

$$r_{tt} = \frac{n^2\sigma^2{}_t + M^2 - Mn}{n(n-1)\sigma^2{}_t},$$

gives a rapid method of estimating the reliability coefficient from the commonly computed parameters of the test score distribution.

An application of several of the formulations is made to test data, in order to illustrate the degree of approximation involved in each method. (30 minutes)

The Estimation of Factors: A-Five Methods Applied to a Bi-Factor Problem, FRANCES SWINEFORD, University of Chicago, Chicago, Illinois.

The ultimate goal of the factor analyst is the measurement of factors for individuals. In case the factors have been described in terms of a large number of tests, the labor of calculating regression estimates and applying them to individual scores is so great as to be out of the question in practice.

It is our purpose to outline a number of short-cuts, and to apply them to actual data. A small set of twelve tests is employed in order that the complete regression estimate may be used as a standard with which to compare the short-cut estimates.

The comparisons are made on the basis of the multiple correlation coefficients, and of the intercorrelations among the estimates. (15 minutes)

The Estimation of Factors: B—Systems of Regression Equations for the Estimation of Factors, HARRY H. HARMAN, University of Chicago, Chicago, Illinois.

In this paper we set up five methods of estimating an individual's factor abilities from a given factor solution. The regression equations for the estimation of each factor are of several types: those involving every test, those involving only some of the tests, and those involving groups of tests considered as unit tests.

These methods are used in the estimation of factors obtained by four different factor analyses of an hypothetical problem. We compare the multiple correlation coefficients of the different factor estimates for a given set of factors to obtain the most practical and statistically sound method. Then we compare the accepted estimates of factors obtained from different factorial solutions to see how nearly the estimates of factors follow the same laws of comparison as the actual factors. (15 minutes)

A Method of Factor Analysis by Means of which all Co-ordinates of the Factor Matrix are given simultaneously, PAUL HORST, The Proctor and Gamble Company, Cincinnati, Ohio.

In general, the methods of factor analysis developed during the past five years are based on the reduction of the correlational matrix by successive steps. The first factor loadings are determined and eliminated from the correlational matrix, giving a residual matrix. This process is continued for successive factor loadings until the elements of the last obtained residual matrix may be regarded as due to chance.

The method outlined in this paper does not involve the progressive disintegration of the correlational matrix. Letting r be the postulated number of factors and n the order of the correlational matrix, we calculate $r + 1$ vectors, where the $(i+1)$'th vector is equal to the i'th vector premultiplied by the correlational matrix. From these vectors certain vector equations are derived which involve only the roots of the correlational matrix. The factor matrix is then expressed as a function of the $(r+1)$ vectors and the r roots. The solution for the roots is not linear, but once these are obtained the solution for the factor matrix is linear.

It is possible to solve for the basic vectors directly from the score matrix without first calculating the correlational matrix. The method is well adapted to the use of Hollerith equipment. If the variables are dichotomous, the calculation of the basic vectors reduces to a simple sorting and tabulating routine, and several hundred variables or more may be easily manipulated.

The mathematical theory underlying the method is developed, and illustrative material is presented. (30 minutes)

Matrix Approximation Criteria, GALE YOUNG, University of Chicago, Chicago, Illinois.

Linear factor analysis requires approximating given data matrices in such way as to materially reduce the rank without introducing too serious error. Some criteria for estimating the seriousness of this error are considered:

(*A*) *Size of error*: Let $\alpha(A,B)$ be a scalar measure of the error in replacing A by B; and, for a given A, let $\phi(r)$ be the minimum value of α for B of rank r. ϕ is then the minimum error possible in reducing the rank to r; and consideration of ϕ against r shows if there is a value $\overline{r}$ for r significantly indicated. This is an empirical matter; the matrix A may or may not be susceptible to such rank reduction.

Some forms for α are listed. For some of these the minimizing problem is solved, for some it is not. One of the forms is given in connection with a precise definition for "communalities".

(*B*) *Form of error*: Besides being small, it is desirable that the residuals exhibit some degree of randomness. A characterization of an ideally random matrix is suggested, although the probability significance of departure therefrom is not yet worked out. (15 minutes)

Current Misuse of the Factorial Methods, L. L. THURSTONE, University of Chicago, Chicago, Illinois.

Those psychologists who have devoted themselves recently to the development of factor analysis should be encouraged by the very general interest in these new methods. The number of papers in the psychological journals that involve factor analysis is increasing rapidly. It is therefore a serious matter that the large majority of these papers involve misinterpretation of the factorial methods. Factor theory is still imperfect and there are some very challenging theoretical problems for the mathematicians to solve in making these analytical tools even more powerful than they now are. But if the misapplications of factor methods continue at the present rate, we shall soon find general disappointment with the results because they are usually meaningless as fas as psychological interpretation is concerned.

With the hope of assisting in the best use of these new methods in their present state of development we may summarize the most common reasons for the meaningless results in most of the current factorial studies of psychological problems.

1) The number of psychologically basic factors in a battery of tests must be considerably smaller than the number of tests in the battery.

2) The diagonals must be regarded as unknown communalities unless one can be certain that specific factors are absent.

3) No matter how the correlational matrix is factored, the axes must be rotated into a simple configuration before any psychological interpretation can be made. The frequent attempts to find psychological interpretation for the centroid axes without rotation and for the principal components without rotation are examples of this frequent error.

4) No meaningful component can be identified unless each factor is overdetermined with three or four or more tests. Most of the current studies are made on batteries that are far too short.

5) In order that meaningful factors shall emerge it is essential that the individual tests be as simple as possible as regards factorial composition. If all of the tests involve many psychological factors, then the identification of the factors becomes very difficult. Composite tests like the Stanford Binet and composite group tests of intelligence are so complex in factorial composition that they are not useful in the identification of basic factors unless they are analyzed by separate items.

While the factorial methods are not yet perfected, they are sufficiently developed so that they can be used in the solution of many fundamentally important psychological problems. The factorial methods must be so formulated that they satisfy not only the requirements of mathematical rigor but also the restrictions of the psychological problems which they are intended to solve. Some of the conditions here listed are not generally accepted but the writer is quite certain that psychologically meaningful results will not be obtained unless we satisfy these conditions. The practical answer to any controversy about the conditions here listed will be in terms of psychological results. These conditions refer to the present state of knowledge of factor theory. (30 minutes)