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LACUNARITY ON NONABELIAN GROUPS AND SUMMING OPERATORS

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Abstract

In this note we investigate lacunarity or 'thin' subsets in the dual object of a compact group via different classes of summing operators between Banach spaces. In particular, we give characterisations of Sidon and $\Lambda(p)$ sets, $2 < p < \infty$.

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1. Introduction and notation

Lacunary sets have been studied in a wide variety of settings. Here we will consider lacunarity mainly for compact (not necessarily abelian) groups. In [1, 2, 3] Sidon and $\Lambda(p)$ sets for compact abelian groups were studied by investigating special operator ideals. Here we will generalise these results for the nonabelian case.

We start by introducing our main tool. These are the ideals of operators which are summing with respect to a given infinite orthonormal system. Henceforth, we consider (σ-finite) measure spaces (Ω, Σ, μ) such that $L_2(\mu)$ is infinite dimensional and $B \subset L_2(\mu)$ always denotes an infinite orthonormal system. Moreover, we use standard Banach space notation. In particular, for any Banach space X we denote by $\mathbb{B}_X$ its closed unit ball and by X^* its topological dual.

DEFINITION 1. An operator $T : X \to Y$ is said to belong to the *class* $\Pi_B(X, Y)$ *of* B-*summing operators* if there exists a constant $c > 0$ such that for all finite sequences

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$(b_i)_1^n$ in B and $(x_i)_1^n$ in X we have:

$$(1) \qquad \left(\int_\Omega \left\| \sum_{i=1}^n b_i T x_i \right\|^2 d\mu \right)^{1/2} \le c \sup_{x^* \in \mathbb{B}_{X^*}} \left(\sum_{i=1}^n |\langle x^*, x_i \rangle|^2 \right)^{1/2}.$$

We write $\pi_B(T)$ for the smallest constant c satisfying (1). Also we abbreviate the supremum on the right hand side of (1) by $\|(x_i)_1^n\|_{l_2^{\text{weak}}(X)}$.

The corresponding operator ideal of B-summing operators will be denoted by $[\Pi_B, \pi_B]$.

For further theory of operator ideals, we will follow Pietsch [8]. Given two Banach ideals $[\mathscr{A}, \alpha]$ and $[\mathscr{B}, \beta]$ we write $\mathscr{A} \subset \mathscr{B}$ if we have $\mathscr{A}(X, Y) \subset \mathscr{B}(X, Y)$ for all pairs of Banach spaces X and Y. If $\mathscr{A} \subset \mathscr{B}$, then there exists a constant $c > 0$ such that $\beta(T) \le c\alpha(T)$ for all Banach spaces X, Y and all $T \in \mathscr{A}(X, Y)$ (see [8, Theorem 6.1.6]). For the Banach ideals of bounded, Gaussian-summing and p-summing operators, $1 \le p < \infty$, we use the standard notations $[\mathscr{L}, \|\cdot\|]$, $[\Pi_\gamma, \pi_\gamma]$ and $[\Pi_p, \pi_p]$ respectively.

For the sake of completeness we recall the definitions of p-summing and Gaussian-summing operators.

A Banach space operator $T: X \to Y$ is called *p-summing*, $1 \le p < \infty$, if there exists a constant c such that, for any choice of finitely many $x_1, \dots, x_n$ in X, we have

$$(2) \qquad \left(\sum_{i=1}^n \|T x_i\|^p \right)^{1/p} \le c \sup_{x^* \in \mathbb{B}_{X^*}} \left(\sum_{i=1}^n |\langle x^*, x_i \rangle|^p \right)^{1/p}.$$

The infimum of all numbers c satisfying (2) is denoted by $\pi_p(T)$.

The Banach ideal of Gaussian-summing operators was introduced in 1974 by Linde and Pietsch [6]. Given an infinite set $\mathscr{G}$ of real valued, independent standard Gaussian variables on a suitable probability space (Ω, Σ, P) an operator $T: X \to Y$ is said to be *Gaussian-summing* if there exists a constant c such that for any choice of $g_1, \dots, g_n \in \mathscr{G}$ and $x_1, \dots, x_n \in X$ we have

$$(3) \qquad \left(\int_\Omega \left\| \sum_{i=1}^n g_i T x_i \right\|^2 dP \right)^{1/2} \le c \, \|(x_i)_1^n\|_{l_2^{\text{weak}}(X)}.$$

The least constant c for which (3) holds is denoted by $\pi_\gamma(T)$.

It is known (compare [1, 15]) that

$$\Pi_2 \subset \Pi_B \subset \Pi_\gamma$$

holds with continuous embeddings of norm 1 for all infinite orthonormal systems B.

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Throughout, we will use the following convenient notation. Let H be a finite dimensional (real or complex) Hilbert space of dimension d and let $\{e_j : 1 \le j \le d\}$ be an orthonormal basis of H. Given a matrix $\underline{x} = \{x_{j,k} : 1 \le j, k \le d\}$ with entries in X and an operator A in $\mathscr{L}(H)$ represented with respect to $\{e_j : 1 \le j \le d\}$ by the matrix $\{a_{j,k} : 1 \le j, k \le d\}$ we will denote by $\operatorname{tr} A\underline{x}$, or $\operatorname{tr} \underline{x}A$, the element of X defined by

$$\operatorname{tr} A\underline{x} = \operatorname{tr} \underline{x}A = \sum_{j,k=1}^{d} a_{j,k} x_{k,j} .$$

Given n matrices $\underline{x_i} = \{x^i_{j,k} : 1 \le j, k \le d_i\}$, $1 \le i \le n$, with entries in X we write

$$\|(\underline{x_i})_1^n\|_{l_2^{\text{weak}}(X)} = \sup_{x^* \in \mathbb{B}_{X^*}} \left(\sum_{i=1}^{n} \sum_{j,k=1}^{d_i} |\langle x^*, x^i_{j,k}\rangle|^2 \right)^{1/2} .$$

Moreover, G will always denote a compact group, m the normalised Haar measure on G and Σ the dual object of G, that is, the set of all equivalence classes of continuous irreducible unitary representations of G. For $\sigma \in \Sigma$, U^σ is a representative of the class σ with degree d_σ and trace χ_σ. For each $\sigma \in \Sigma$, let $\{u^\sigma_{j,k} : 1 \le j, k \le d_\sigma\}$ be a set of coordinate functions for $U^\sigma \in \Sigma$ and a fixed basis $\{e_i : i = 1, \dots, d_\sigma\}$ in the representation space H_σ of U^σ. Recall that the set of all $d_\sigma^{1/2} u^\sigma_{j,k}$, $j, k \in \{1, \dots, d_\sigma\}$ and $\sigma \in \Sigma$, forms an orthonormal basis in $L_2(G)$ and $\{u^\sigma_{j,k}(g) : 1 \le j, k \le d_\sigma\}$, $g \in G$, is the matrix representing U^σ_g with respect to $\{e_i : i = 1, \dots, d_\sigma\}$.

The Fourier series of a function $f \in L_1(G)$ is given by

$$f(g) \sim \sum_{\sigma \in \Sigma} d_\sigma \operatorname{tr}(A^f_\sigma U^\sigma_g) \qquad (g \in G),$$

where A^f_σ is

$$A^f_\sigma = \int_G f(g^{-1}) U^\sigma_g \, dm(g) .$$

A subset E of Σ is called a *Sidon set* with *Sidon constant* c if

$$\|f\|_A = \sum_{\sigma \in \Sigma} d_\sigma \operatorname{tr}(|A^f_\sigma|) \le c \, \|f\|_\infty \tag{4}$$

for every continuous E-function (f is an E-function if $A^f_\sigma = 0$ for all σ not in E).

E is called a $\Lambda(p)$ *set*, $1 \le p < \infty$, if there is a constant Φ such that for some $q < p$ we have

$$\|f\|_p \le \Phi \, \|f\|_q \tag{5}$$

for every E-function f in $L_p(G)$. The least constant Φ satisfying (5) is denoted by $\Phi(B, p, q)$ and is called the $\Lambda(p, q)$-constant of E. From Hölder's inequality we infer that if (5) is satisfied for some $\hat{q} < p$, then the same is true for any $q < p$, albeit with different constants.

E is called a *central Sidon set* (respectively *central* $\Lambda(p)$ *set*) if (4) (respectively (5)) holds for all central E-functions, that means for those E-functions belonging to the center of the algebra $L_1(G)$. Functions in the center are those with Fourier series of the form

$$f(g) \sim \sum_{\sigma \in \Sigma} \hat{f}(\sigma) \chi_\sigma(g),$$

where

$$\hat{f}(\sigma) = \int_G f(g) \chi_\sigma(g^{-1}) \, dm(g).$$

Given an infinite subset E of Σ we write Π_{χ_E} for $\Pi_{\{\chi_\sigma : \sigma \in E\}}$ and Π_E for

$$\Pi_{\left\{d_\sigma^{1/2} u_{j,k}^\sigma : \sigma \in E, 1 \leq j, k \leq d_\sigma\right\}}.$$

The second notation is justified since a routine calculation shows that Π_E does not depend on the choice of the representations U^σ of the σ's. It follows from [2, Theorem 7] that Π_E (respectively Π_{χ_E}) coincides with Π_2 isometrically if and only if E fails to be a $\Lambda(2)$ system (respectively a central $\Lambda(2)$ system). Since it is known that the special unitary group $\mathcal{SU}(n)$, $n > 1$, does not have any infinite $\Lambda(2)$ sets (see [10, 14]) this yields the following proposition.

PROPOSITION 2. *For all infinite subsets E of the dual object of $\mathcal{SU}(n)$, $2 \leq n$, we have $\Pi_E = \Pi_2$ with equal norms.*

2. Main results

Henceforth, we will make repeated use of the following easy observations.

PROPOSITION 3. (i) $\Pi_E \subset \Pi_{\chi_E}$ *for all* $E \subset \Sigma$.
(ii) *If $E \subset \Sigma$ is of uniformly bounded degree* ($\sup_{\sigma \in E} d_\sigma < \infty$), *then* Π_{χ_E} *coincides with* Π_E.

We will see later that the reverse implication in (ii) does not hold. It has been shown in [3] that in case where G is a commutative compact group, an infinite subset E of the dual group is a Sidon set if and only if $\Pi_E = \Pi_\gamma$ whereas E is a $\Lambda(p)$-set, $2 < p < \infty$, if and only if $\Pi_p \subset \Pi_E$ holds. In the following we will generalise these two results to arbitrary compact groups.

THEOREM 4. *Let E be an infinite subset of Σ, $2 < p < \infty$ and $c > 0$ be given. Then E is a $\Lambda(p)$ set with $\Lambda(p, 2)$-constant c if and only if $\Pi_p \subset \Pi_E$ holds with $\pi_E \leq c\, \pi_p$.*

PROOF. Fix an E-polynomial $f = \sum_{i=1}^n d_{\sigma_i} \operatorname{tr}(A_{\sigma_i} U^{\sigma_i})$, $n \in \mathbb{N}$. Using the properties of the normalised Haar measure we have, for all $h \in G$,

$$\|f\|_p = \left(\int_G \left| \sum_{i=1}^n d_{\sigma_i} \operatorname{tr}(A_{\sigma_i} U^{\sigma_i}_{gh}) \right|^p dm(g) \right)^{1/p} .$$

Hence, assuming that $\Pi_p \subset \Pi_E$ holds with $\pi_E \leq c\, \pi_p$ and taking into account that the embedding $i_p : \mathscr{C}(G) \to L_p(G)$ is p-summing with $\pi_p(i_p) = 1$ (see for instance [5, Example 2.9]) we get

$$\|f\|_p = \left(\int_G \left\| \sum_{i=1}^n d_{\sigma_i} \operatorname{tr}(A_{\sigma_i} U^{\sigma_i} U^{\sigma_i}_h) \right\|_p^2 dm(h) \right)^{1/2} \leq c \left\| (d_{\sigma_i}^{1/2} A_{\sigma_i} U^{\sigma_i})_1^n \right\|_{l_2^{\text{weak}}(\mathscr{C}(G))} .$$

Since point masses form a norming subset of $C(G)^*$ and $U_g^{\sigma_i}$ is a unitary matrix for all $g \in G$, $1 \leq i \leq n$, we have

$$\begin{aligned} \left\| (d_{\sigma_i}^{1/2} A_{\sigma_i} U^{\sigma_i})_1^n \right\|_{l_2^{\text{weak}}(\mathscr{C}(G))} &= \sup_{g \in G} \left(\sum_{i=1}^n \sum_{j,k=1}^{d_{\sigma_i}} \left| \sum_{r=1}^{d_{\sigma_i}} (f \,|\, d_{\sigma_i}^{1/2} u^{\sigma_i}_{r,j}) u^{\sigma_i}_{r,k}(g) \right|^2 \right)^{1/2} \\ &= \left(\sum_{i=1}^n \sum_{j,r=1}^{d_{\sigma_i}} |(f \,|\, d_{\sigma_i}^{1/2} u^{\sigma_i}_{r,j})|^2 \right)^{1/2} = \|f\|_2 . \end{aligned}$$

Hence, E is a $\Lambda(p)$ set with $\Lambda(p, 2) \leq c$. Since the reverse implication holds even for general orthonormal systems (see [1, Corollary 7.9]) we are done. □

Making use of a description of orthonormal systems B for which the inclusion $\Pi_p \subset \Pi_B$, $2 < p < \infty$, holds (compare [1, Theorem 7.7]) we get the following characterisation of $\Lambda(p)$-sets.

COROLLARY 5. *Let $2 < p < \infty$, $c > 0$ and $E \subset \Sigma$ be given. Then E is a $\Lambda(p)$ set with $\Lambda(p, 2) \leq c$ if and only if the embedding $I : L_2^E(G) \hookrightarrow L_2(G)$ is p-convex such that the p-convexity constant of I is smaller or equal to c. That means that*

$$\left\| \left(\sum_{i=1}^n |f_i|^p \right)^{1/p} \right\|_2 \leq c \left(\sum_{i=1}^n \|f_i\|_2^p \right)^{1/p}$$

holds for every choice of finitely many $f_1, \dots, f_n$ in $L_2^E(G)$. Here $L_2^E(G)$ denotes the space of all E-functions in $L_2(G)$.

Let $\{g^{\sigma}_{j,k} : 1 \le j,k \le d_\sigma, \sigma \in \Sigma\}$ be a collection of independent real valued Gaussian random variables with mean 0 and variance 1. We denote by G_σ the random operator on H_σ which admits $(d_\sigma^{-1/2}\, g^{\sigma}_{j,k})_{j,k=1}^{d_\sigma}$ as its representative matrix with respect to the basis $\{e_j : 1 \le j \le d_\sigma\}$ in H_σ. Obviously, an operator $T \in \mathscr{L}(X,Y)$ is Gaussian-summing if and only if we can find a constant c such that

$$\left(\int_\Omega \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2}\, T(\operatorname{tr}(\underline{x}_i\, G_{\sigma_i}))\right\|^2 dP\right)^{1/2} \le c\, \|(\underline{x}_i)_1^n\|_{l_2^{\text{weak}}(X)}$$

for all choices of finitely many σ_i's and $\underline{x}_i$'s.

The following statement is a generalisation of a result by Pisier [9, Theorem 2.1] about Sidon sets in the abelian case. Pisier's proof can be extended to establish the analogue result in the setting of compact nonabelian groups.

THEOREM 6. *Let $E \subset \Sigma$ be a Sidon set with Sidon constant c. Then there exists a constant C such that for all $n \in \mathbb{N}$, $\sigma_i \in \Sigma$ and all $d_{\sigma_i} \times d_{\sigma_i}$ matrices $\underline{x}_i$ with values in a Banach space X, $1 \le i \le n$, we have*

$$\left(\int_G \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, U^{\sigma_i})\right\|^2 dm\right)^{1/2} \le C \left(\int_\Omega \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, G_{\sigma_i})\right\|^2 dP\right)^{1/2}.$$

PROOF. Given $n \in \mathbb{N}$, $\sigma_1, \dots, \sigma_n \in E$ and $V_i \in \mathscr{U}(H_{\sigma_i})$ represented with respect to $\{e_i : i = 1, \dots, d_{\sigma_i}\}$ by the unitary matrix $(v_{j,k})_{j,k=1}^{d_{\sigma_i}}$, $i = 1, \dots, n$, we define the linear map

$$\xi : \mathscr{C}^E(G) \to \mathbb{C}, \; f \mapsto \sum_{i=1}^{n} d_{\sigma_i} \operatorname{tr}(A^f_{\sigma_i} V_i),$$

where $\mathscr{C}^E(G)$ denotes the space of all continuous E-functions. Since E is a Sidon set with Sidon constant c the functional ξ is continuous with $\|\xi\| \le c$. By the Hahn-Banach Theorem, we find a measure $\mu \in \mathscr{M}(G)$ such that $\|\mu\| \le c$, where $\|\mu\|$ denotes the total variation of μ, and $\int_G f\, d\mu = \xi(f)$ for all $f \in \mathscr{C}^E(G)$. In particular, we have $\xi(u^{\sigma_i}_{j,k}) = v^i_{j,k}$. Given $\underline{x}_i = (x^i_{j,k})_{j,k=1}^{d_{\sigma_i}}$, $1 \le i \le n$, we put $Z(g) = \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, U^{\sigma_i}_g)$, $g \in G$. By ν we denote the image measure of μ under $g \mapsto g^{-1}$. Then

$$\begin{aligned}(\nu * Z)(g) &= \sum_{i=1}^{n} \sum_{j,k=1}^{d_{\sigma_i}} x^i_{j,k} \int_G u^{\sigma_i}_{k,j}(h^{-1}g)\, d\nu(h) \\ &= \sum_{i=1}^{n} \sum_{j,k,s=1}^{d_{\sigma_i}} x^i_{j,k} u^{\sigma_i}_{s,j}(g) \int_G u^{\sigma_i}_{k,s}(h^{-1})\, d\nu(h)\end{aligned}$$

$$= \sum_{i=1}^{n} \sum_{j,k,s=1}^{d_{\sigma_i}} x^i_{j,k}\, v^i_{k,s}\, u^{\sigma_i}_{s,j}(g) = \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, V_i\, U^{\sigma_i}_g)\,.$$

Moreover, by the convexity of $t \mapsto t^2$,

$$\|(\nu * Z)(g)\|^2 \le \|\nu\| \int_G \|Z(h^{-1}g)\|^2\, d|\nu|(h)$$

and so we get, by integrating,

$$\begin{aligned}\int_G \|(\nu * Z)(g)\|^2\, dm(g) &= \int_G \left\| \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, V_i\, U^{\sigma_i}_g) \right\|^2 dm(g) \\ &\le \|\nu\| \int_G \int_G \|Z(h^{-1}g)\|^2\, dm(g)\, d|\nu|(h) \\ &= \|\nu\|^2 \int_G \|Z(g)\|^2\, dm(g) \\ &\le c^2 \int_G \left\| \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, U^{\sigma_i}_g) \right\|^2 dm(g).\end{aligned}$$

Replacing in the later $\underline{x}_i$ by $\underline{x}_i\, V_i$ and V_i by V_i^{-1} reveals

$$\int_G \left\| \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, U^{\sigma_i}_g) \right\|^2 dm(g) \le c^2 \int_G \left\| \sum_{i=1}^{n} \operatorname{tr}(\underline{x}_i\, V_i\, U^{\sigma_i}_g) \right\|^2 dm(g).$$

Let $\tilde{G}$ denote the compact group $\prod_{\sigma \in \Sigma} \mathscr{U}(H_\sigma)$ and $m_{\tilde{G}}$ the normalised Haar measure on $\tilde{G}$. Integrating over $\tilde{G}$ and applying [7, Proposition 5.2.1], [7, Corollary 5.2.4] and [7, Proposition 5.2.6] we infer the existence of a constant C depending only on c such that

$$\begin{aligned}\int_G \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, U^{\sigma_i}) \right\|^2 dm &\le c^2 \int_{\tilde{G}} \int_G \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, V_i\, U^{d_{\sigma_i}}) \right\|^2 dm\, dm_{\tilde{G}} \\ &\le c^2 \int_{\tilde{G}} \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, V_i) \right\|^2 dm_{\tilde{G}} \\ &\le C^2 \int_\Omega \left\| \sum_{i=1}^{n} d_{\sigma_i}^{1/2} \operatorname{tr}(\underline{x}_i\, G_{\sigma_i}) \right\|^2 dP,\end{aligned}$$

and the proof is complete. □

Since $\Pi_B \subset \Pi_\gamma$ holds for all infinite orthonormal systems B it is immediate from Theorem 6 that $\Pi_E = \Pi_\gamma$ holds for all Sidon sets E in Σ. As in the abelian case the reverse implication holds as well.

THEOREM 7. *Let $E \subset \Sigma$ be given. Then E is a Sidon set if and only if $\Pi_\gamma = \Pi_E$ holds.*

PROOF. If $\Pi_E = \Pi_\gamma$, we find a constant $c > 0$ such that

$$1/c\, \pi_E \leq \pi_\gamma \leq B_p\, \pi_p \quad \text{for all} \quad 2 < p < \infty,$$

where B_p denotes the constant from the Khinchin Inequality (compare [6, Theorem 6]). Since $B_p \leq \sqrt{p}$ (see for instance [4, page 96 and page 100]) we have $\pi_E \leq c\sqrt{p}\,\pi_p$ and so, by Theorem 4, $\|f\|_p \leq c\sqrt{p}\,\|f\|_2$ for all E-functions $f \in L_2(G)$ and all $2 < p < \infty$. Now apply [7, Theorem 6.2.3] and [7, Remark 6.2.6] to see that E is a Sidon set. □

REMARK 8. (a) By Proposition 3 and Theorem 7 we have $\Pi_E = \Pi_{\chi_E}$ for all Sidon sets E. Since a Sidon set does not have to be of uniformly bounded degree the equality $\Pi_E = \Pi_{\chi_E}$ does not imply that $\sup_{\sigma \in E} d_\sigma < \infty$ holds.

(b) It follows from Theorem 7 that the union of two Sidon sets is a Sidon set. Since it is known that this does not hold for central Sidon sets (compare [11, Example 8]), an equivalent of Theorem 7 cannot hold for central Sidon sets. More precisely, there exist central Sidon sets E which are not central $\Lambda(p)$ sets for any $p > 0$ (compare [11, Example 1]) and so $\Pi_{\chi_E} = \Pi_2$ by [2, Theorem 7]. Hence, E being central Sidon does not imply that Π_{χ_E} coincides with Π_γ.

(c) An extension of Theorem 7 for the case where the compact group G is replaced by a compact (abelian) hypergroup does not hold either since central Sidonicity on a compact group is known to be equivalent to Sidonicity on the compact abelian hypergroup consisting of the group's conjugacy classes (compare [16]).

If E is of uniformly bounded degree things look better since in this case a central Sidon set is a Sidon set (see [17, Remark 7.2]) and a central $\Lambda(p)$ set is a $\Lambda(p)$ set. The latter can be seen by combining for example [1, Corollary 7.9], Proposition 3 (ii) and Theorem 4.

COROLLARY 9. *Let E be an infinite subset of Σ of uniformly bounded degree and $2 < p < \infty$. Then the following statements are equivalent.*

(i) *E is a Sidon [respectively $\Lambda(p)$] set.*
(ii) *E is a central Sidon [respectively $\Lambda(p)$] set.*
(iii) *$\Pi_E = \Pi_\gamma$ [respectively $\Pi_p \subset \Pi_E$].*
(iv) *$\Pi_{\chi_E} = \Pi_\gamma$ [respectively $\Pi_p \subset \Pi_{\chi_E}$].*

We conclude with the following remark.

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REMARK 10. If $E \subset \Sigma$ fails to be of uniformly bounded degree, the inclusion $\Pi_p \subset \Pi_{\chi_E}$ does not necessarily imply that $\Pi_p \subset \Pi_E$ holds. For example we know by Proposition 2 that $\Pi_2 = \Pi_E$, for all infinite subsets E of the dual of $\mathscr{SU}(n)$, $n \geq 2$. On the other hand, every infinite set E in the dual of $\mathscr{SU}(n)$ contains an infinite subset which is central $\Lambda(p)$ for all $p < 2 + 2/n$ (see [12]) and so, by [1, Corollary 7.9], $\Pi_p \subset \Pi_{\chi_E}$ for those p.

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