

## Eigenfunctions of axisymmetric p-modes in the presence of a dipole magnetic field: Observational consequences

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**Abstract.** We calculate the effects of a 3.2-kG dipolar magnetic field on the eigenfunctions of high-order p-mode pulsations and discuss the consequences for pulsation amplitude modulation with rotation phase and for line profile variations.

### 1. Introduction

Rapidly oscillating Ap (roAp) stars pulsate in high order p-modes which are affected by a strong magnetic field. In addition to shifting the oscillation frequency and stabilizing the pulsation, the magnetic field modifies the angular dependence of pulsation amplitude on the stellar surface such that the disturbances become more strongly concentrated to the magnetic axis. This may have important observational consequences (e.g., Cunha & Gough 2000; Bigot et al. 2000; Saio & Gautschy 2003a,b). In this paper we discuss the magnetic effects on observable quantities such as amplitude modulation with rotation phase of the star and line-profile variations.

We present results obtained from the eigenfunction of a high-order axisymmetric p-mode with  $(\ell_m, k) = (1, 32)$  of a ZAMS model of  $1.7 M_{\odot}$  that is permeated by a dipole magnetic field of  $B_p = 3.2$  kG;  $B_p$  stands for the field strength at a pole on the stellar surface. At this field strength, the mode suffers only from a very weak magnetic damping (Saio & Gautschy 2003a,b). The damping is expected to be overcompensated by the driving due to the  $\kappa$ -mechanism in the H-ionization zone. The angular dependence of the eigenfunction is represented by a sum of terms proportional to Legendre functions  $P_{\ell}(\cos \theta)$  with  $\ell = 1, 3, \dots, 23$ . Although the kinetic energy of the mode is well concentrated in the  $\ell = 1$  component, the surface amplitude of the  $\ell = 3$  component is about 70% of that of the  $\ell = 1$  component, indicating a considerable magnetically induced effect on the pulsation amplitude on the surface.

### 2. Amplitude modulation

The light variation of a roAp star modulates as a function of the stellar rotation phase. The amplitude modulation (AM) is interpreted as an axisymmetric mode whose axis aligns with the magnetic axis rotating about the rotation axis (oblique pulsator model; Kurtz 1982). The curve of the AM depends on the

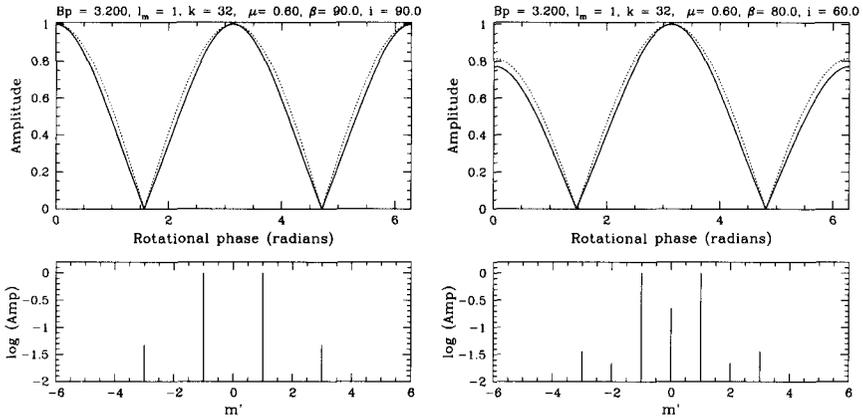


Figure 1. Amplitude modulation curves (upper panels) and corresponding amplitude spectra (lower panels). Dotted lines show the amplitude modulation curves obtained by assuming that the eigenfunction is proportional to  $P_1(\cos \theta)$ . The vertical axis in the lower panels measures the *logarithm* of the amplitude; the horizontal axis shows the azimuthal degree with respect to the rotation axis. The left and right panels are for two different sets of inclination angles  $i$  and  $\beta$ .

temperature perturbation across the stellar surface. In the upper panels of Fig. 1, we compare AM curves obtained from eigenfunctions with (solid lines) and without (dotted lines) the magnetic effect. Although the magnetic effect on the amplitude distribution on the stellar surface is considerable, the effect on the AM curve is very small. This comes from the fact that the surface integral of a brightness perturbation proportional to  $P_\ell(\cos \theta)$  over a uniform brightness disk cancels out for odd values of  $\ell$ , except for  $\ell = 1$ . Therefore, the effect of the deviation from the  $P_1(\cos \theta)$  dependence appears on the AM curve only through coupling with the limb darkening of the stellar disk.

The lower panels of Fig. 1 show the corresponding (logarithmic) amplitude spectra, where the abscissa is azimuthal degree  $m'$  with respect to the rotation axis. A single oblique pulsation mode shows multiple frequencies in the observer's frame. Our amplitude spectrum is symmetric with respect to the sign of  $m'$  because we have neglected the effect of rotation on the eigenfunctions. The Coriolis force causes a small asymmetry as it was shown by Shibahashi & Takata (1993) and Bigot & Dziembowski (2002). If  $i = 90^\circ$  or  $\beta = 90^\circ$  (where  $i$  is the angle between the rotation axis and the line of sight, and  $\beta$  the angle between the rotation and magnetic axes), the heights of the two peaks in the AM produced by a full stellar rotation are identical and the amplitudes of the even  $|m'|$  components vanish. The amplitude of the  $m' = 0$  component varies sensitively with  $i$  and  $\beta$ . The right panels of Fig. 1 show a case with an AM curve that is comparable to the observed one for HR 3831 (Kurtz et al. 1997). Although the amplitude of the  $m' = 0$  component relative to that of the  $m' = \pm 1$  components

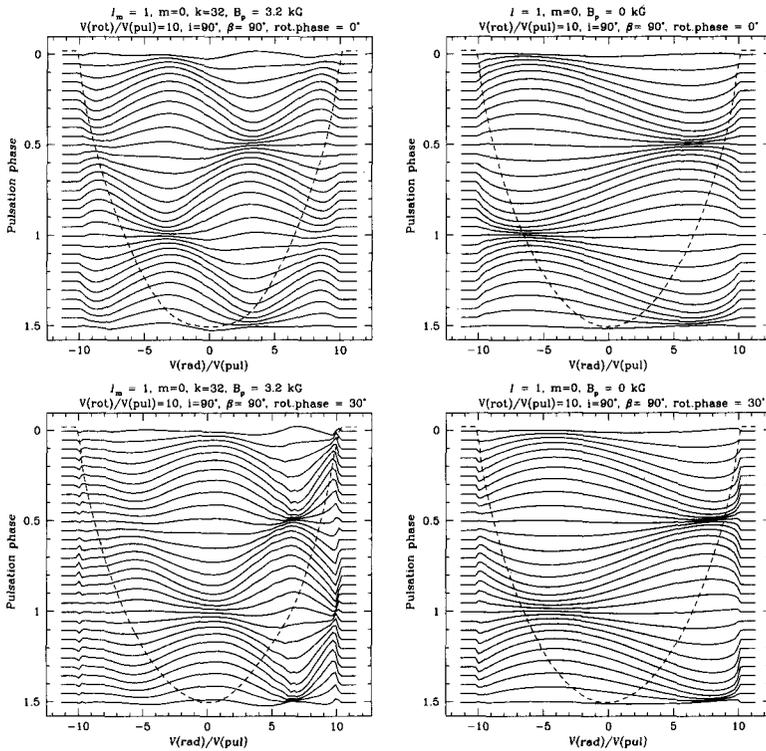


Figure 2. Line profile variations due to pulsation are shown as residuals (solid lines) from the mean profile (dashed lines). The velocity amplitude is assumed to be 0.1 times the rotation speed. The magnetic effect is included in the results on the left panels but not in the right ones. The horizontal axis measures the radial velocity in units of the pulsation amplitude.

is similar to the observed one, the amplitudes of  $|m'| = 2$  and 3 components are smaller than the observed ones.

### 3. Line-profile variations

Line-profile variations (LPVs) have been detected in some roAp stars (Kochukhov & Ryabchikova 2001a,b; Balona 2002). Fig. 2 shows theoretical LPVs derived from the pulsation velocity in the photosphere, neglecting the effect of temperature variations. The velocity amplitude was assumed to be 10% of the equatorial rotation speed. The top panels show the case of the magnetic axis pointing towards us, while bottom panels shows the situation after the star rotated by  $30^\circ$ . The left and right panels are for eigenfunctions with and without magnetic ef-

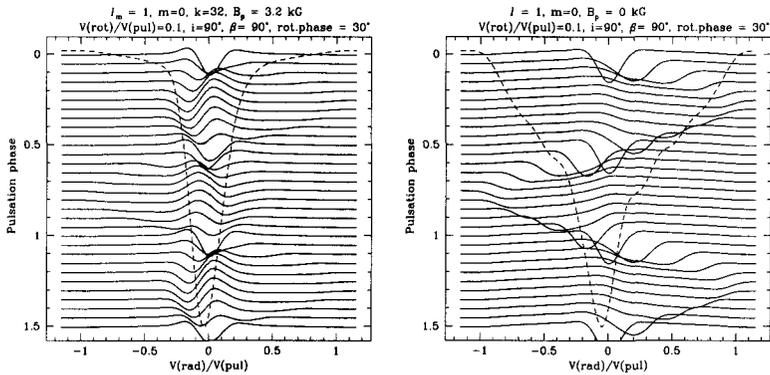


Figure 3. The same as the bottom panels of Fig. 2 but the velocity amplitude is assumed to be 10 times the rotation speed.

fects, respectively. Each solid line shows the *residual* from the mean line profile (dashed line) at each pulsation phase. The intrinsic line width is assumed to be 0.1 times pulsation amplitude. Fig. 3 shows LPVs that are expected if the pulsation velocity amplitude is 10 times the equatorial rotation speed.

Although the line-profile model used here is too primitive to allow a serious comparison with observed ones, it is evident that the effect of a magnetic field shortens the ‘wavelength’ of the LPVs. Therefore, to identify pulsation modes of roAp stars via LPVs, it is necessary to consider the effect of the magnetic field on the eigenfunctions.

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