THE MASSES OF THE PRINCIPAL PLANETS

W. J. KLEPCZYNSKI, P. K. SEIDELMANN, and R. L. DUNCOMBE
U. S. Naval Observatory, Washington, U.S.A.

(Presented at IAU Colloquium No. 9, 'The IAU System of Astronomical Constants', Heidelberg, Germany, August 12–14, 1970.)

Abstract. A set of masses for the principal planets is derived systematically from all available fundamental and independent determinations. In deriving these values an attempt has been made to treat independently those determinations based on differing observational types or analytical methods.

1. Introduction

The planetary masses currently incorporated in the national ephemerides are essentially those deduced by Newcomb (1898a, 1898b). More recently, compilations have been made by Kozlovskaya (1963), Clemence (1964) and Kulikov (1965). Brouwer and Clemence (1961) review past determinations, and also discuss the several methods by which planetary masses can be deduced.

The determination of the mass of a planet, whether derived from its periodic perturbations of an adjacent body, from its effect on the secular motions of other planets, or from measures of the motion of its satellites, may be affected by systematic errors the extent of which are not reflected in the formal mean error given by the investigator. Attempts to evaluate the systematic errors present in the data were nonconclusive, so, while the presence of systematic effects is suspected, there is no known method of correcting for the unknown effect.

In the present study, the various independent determinations of the planetary masses are segregated by observation type and analytical method to form group means. The group means are examined to see if they form an accordant or discordant data set. The group means are then combined to form the final value.

In Tables I through IX are assembled in chronological order all of the published determinations of the masses of the principal planets known to us. Those results which in our judgment are fundamental and independent have been analyzed in groups depending on observation type and analytical method; that is those determinations depending principally on radar measures have been placed together; those resulting from satellite measures are grouped, etc. For each planet the determinations that have been superseded by later investigations utilizing the same data and the same method were omitted from consideration. If previously used data were analyzed again by a significantly different method, the determination was considered independent.

2. Formation of Group Means and Their Errors

The group means were formed by a weighted mean of the closely related but not redundant determinations, the weighting factors being determined usually by the

Celestial Mechanics **4** (1971) 253–272.

mean error associated with each value. Where a multitude of measures are available a method of elimination similar to Taylor *et al.* (1969) can be used, where any datum with an error greater than three times that of a similar datum is discarded. In the case of the planetary masses, there are not sufficient data.

There are two methods to determine the mean error of a weighted mean. The first method bases the mean error (e) of the weighted mean on the mutual discordances of the individual items with respect to the mean:

$$e = \sqrt{\frac{\sum w_i r_i^2}{(n-1)\sum w_i}} \tag{1}$$

where w_i is the weight of the ith item, r_i is the residual of the ith item with respect to the mean and n is the number of individual items entering the mean.

The second method considers the weighted mean to be a simple linear combination of several items and consequently considers the mean error of the mean to be a linear combination of the corresponding mean errors of the individual items. The resulting expression for the mean error of the weighted mean is

$$e' = \sqrt{\frac{e_0^2}{\sum w_i}} \tag{2}$$

where w_i is the weight determined from $w_i = e_0^2/e_i^2$, e_i is the mean error of an individual determination, and e_0 is an arbitrary, but consistent, quantity. In each case e_0 was selected as the largest mean error entering the group mean, so the smallest weight was unity.

Both methods suffer from the fact that they do not always give a true indication of the mean error of the mean. For the first method (e), consider the case when a mean value is formed from two items which are very close together, but each having large mean errors associated with them. Since the deviations from the mean are small, the resulting mean error will be small, but it will not reflect the large mean errors of the individual determinations.

In the second method (e'), examine the case when a mean value is formed from two items which are separated by a considerable amount, but each having small mean errors associated with them. Since the mean errors are small, the mean error of the mean value will be small, and the deviation of the values making up the mean will not be reflected in the resulting mean error.

In this investigation, due to the paucity of mass determinations, very few items are being combined to form a mean value. The character of these data is carefully examined before choosing the method to form the mean error of the weighted mean.

After forming a weighted group mean, it is frequently the practice in statistics to discard determinations exceeding three or more times the standard deviation of unit weight (σ) as determined by

$$\sigma = \sqrt{\frac{\sum w_i r_i^2}{(n-1)}}. \tag{3}$$

When a large variation of weights occurs, the value of sigma resulting is dependent on the system of weights used and is not a valid basis for rejecting independent determinations. In this investigation, no independent determinations have been rejected solely on the magnitude of their residuals.

3. Formation of Final Values

On inspecting the various group means for each planet, it was apparent that the values could be considered as fairly accordant sets of data. It was therefore decided to calculate the final mean value for each planet by forming a weighted mean of the group means. The final values are considered to comprise not only the mass of the planet, but its satellites and atmosphere, if any. As a check on the consistency of the results, a simple arithmetic mean of the group means was formed for each planet, which in all cases compared reasonably with the weighted mean.

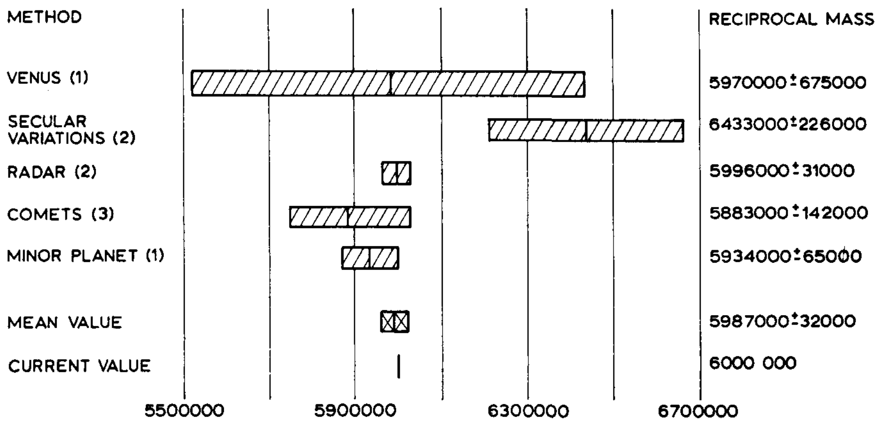

Fig. 1. Reciprocal mass of Mercury.

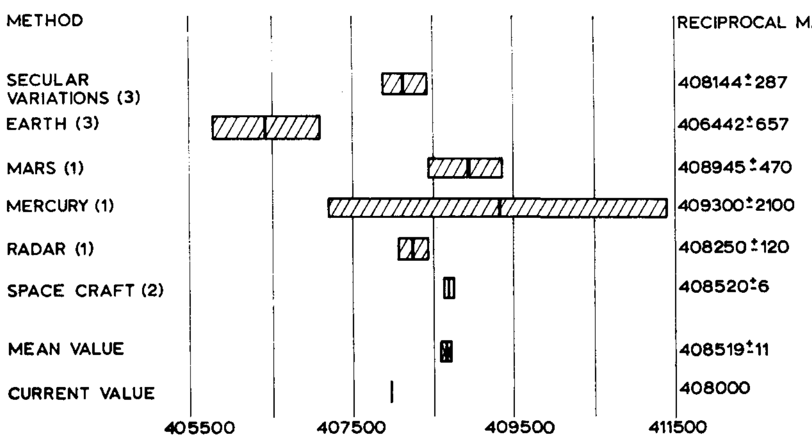

Fig. 2. Reciprocal mass of Venus.

https://doi.org/10.1017/S0252921100028372 Published online by Cambridge University Press

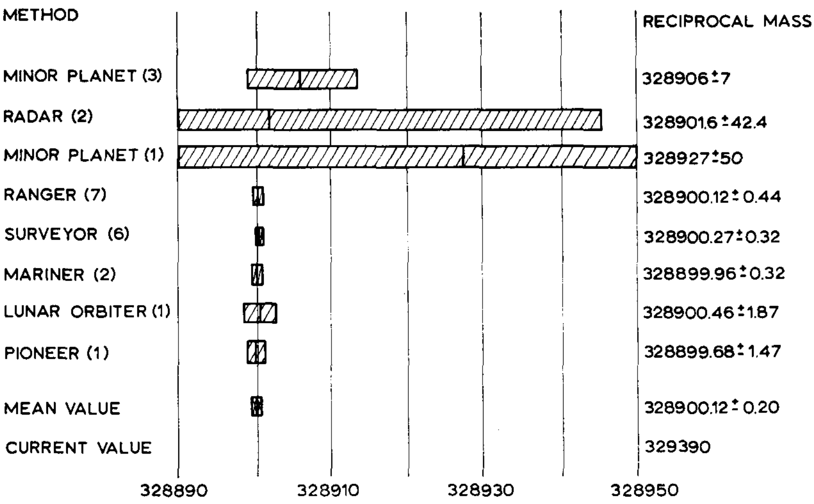

Fig. 3. Reciprocal mass of Earth–Moon.

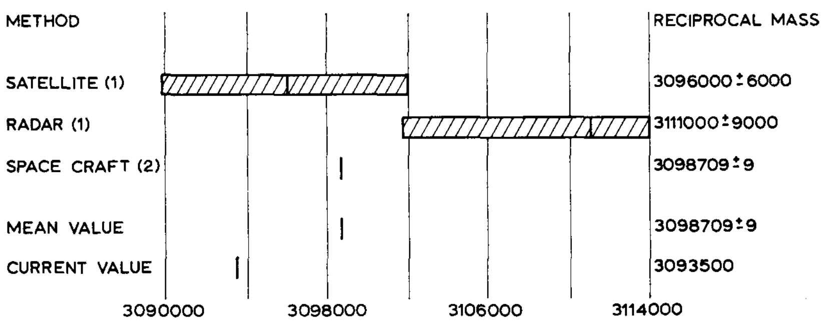

Fig. 4. Reciprocal mass of Mars.

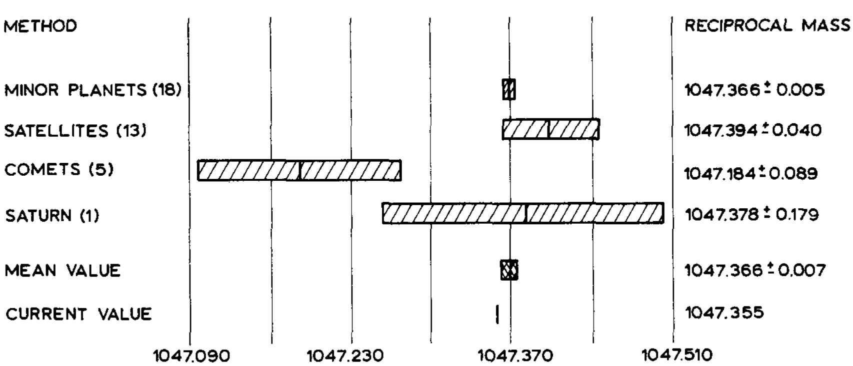

Fig. 5. Reciprocal mass of Jupiter.

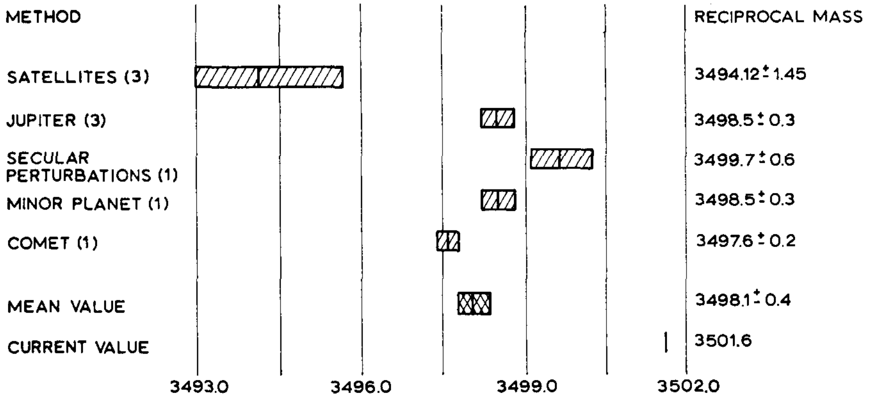

Fig. 6. Reciprocal mass of Saturn.

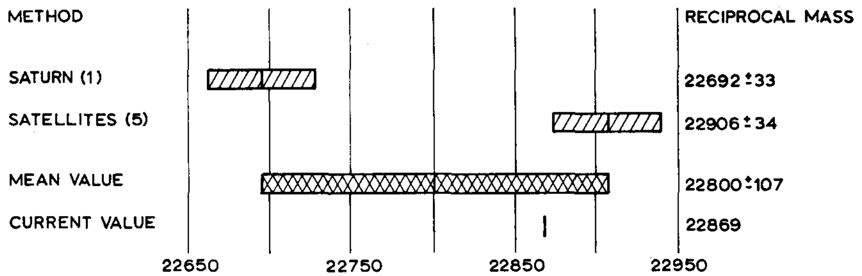

Fig. 7. Reciprocal mass of Uranus.

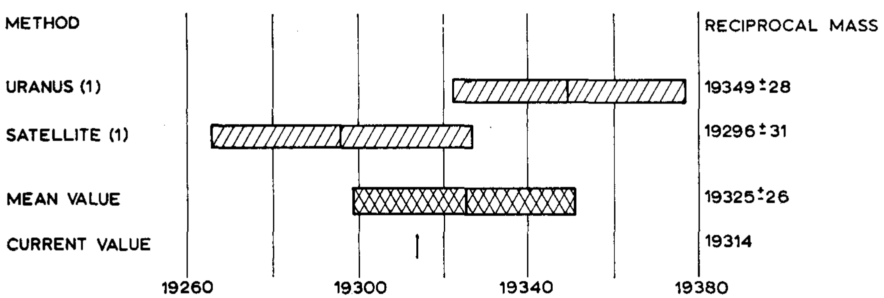

Fig. 8. Reciprocal mass of Neptune.

In Tables I to IX, determinations that enter the final weighted mean individually are marked by an asterisk (*) while those that are part of a group mean are marked by a cross (+). In addition, the value currently adopted by the IAU and our deduced weighted mean are listed at the bottom of each table. The associated mean errors are those resulting from the solution in all cases except Pluto.

Figures 1 to 8 present graphically the value and its mean error for the group means, the final mean value, and the currently adopted value for each planet. The number in parentheses after the description of the method is the number of independent determinations entering the group mean.

4. Mercury, Table I

(a) The reciprocal mass of Mercury from the periodic perturbations in the motion of Venus determined by Duncombe (1958) supersedes that of Newcomb (1895) and was adopted.

(b) From the analyses of the secular variations of the inner planets, von Haerdtl (1889) is superseded by Clemence (1949) and Brouwer (1950) and a weighted mean of 6433000 for the reciprocal mass of Mercury was determined. While these two values differ only slightly, they each have large mean errors, so the mean error of the group mean (±226000) was based on the given mean errors rather than on the discordance of the determinations.

(c) The values of the reciprocal mass of Mercury based principally on radar range measurements by Ash *et al.* (1967) and Melbourne (1968) were combined to give a group mean of 5996000±31000. This mean error is based on the mean error of the determinations rather than on their residuals from the mean.

(d) The results based on observations of the comets Winnecke by von Haerdtl (1889) and Encke by Backlund (1894) were rejected because of the admittedly poor quality of the observations involved. The determinations based on Encke's Comet by von

TABLE I
Investigations of the mass of Mercury

No.	Reciprocal mass	Mean error	Author	Year	Object
1	5012842	±1034637	von Haerdtl	1889	Winnecke's comet
2	5514700	± 148000	von Haerdtl	1889	Secular perturbations
3+	5648600	± 3000	von Haertdl	1889	Encke comet (1819–68)
4+	5669700	± 890000	von Haertdl	1889	Encke comet (1871–85)
5	9700000	±1000000	Backlund	1894	Encke comet
6	7210000	±2307000	Newcomb	1895b	Venus (1750–1892) α and δ
7	7943000	±2780000	Newcomb	1895b	Venus (1750–1892) α only
8+	6400000	± 300000	Clemence	1949	Motion of perihelion of Earth and Mercury
9	6120000	± 64000	Rabe	1950	Eros (1926–45)
10+	6480000	± 350000	Brouwer	1950	Secular perturbations
11	6280000	± 350000	Makover	1956	Encke comet (1937–54)
12*	5970000	± 675000	Duncombe	1958	Venus (1750-1949)
13+	5980000	± 170000	Makover and Bokhan	1961	Encke comet (1898–1954)
14+	6021000	± 53000	Ash *et al.*	1967	Radar data (1959–66) Optical data (1950–65)
15+	5983000	± 37000	Melbourne, O'Handley and Reed (see Melbourne *et al.*, 1968)	1968	Venus radar observations
16*	5934000	± 65000	Lieske and Null	1969	Icarus
	6000000		Currently adopted		
	5987000	± 32000	Weighted mean		

https://doi.org/10.1017/S0252921100028372 Published online by Cambridge University Press

Haerdtl (1889) from observations over the period 1819–1868, by von Haerdtl (1889) from observations 1871–1885, and by Makover and Bokhan (1961) from observations 1898–1954, which supersedes Makover (1956), are combined with relative weights of 15, 8 and 54 respectively. These weights are the product of the number of oppositions of the comet in each observational period, 15, 4 and 18 respectively, times a quality factor 1, 2 or 3, based on the epoch of the observations. This method of weighting was used because the published mean errors have a questionably large variation in magnitude. The resulting group mean for the reciprocal mass of Mercury was 5883000 ± ± 142000 where the mean error is derived from the published mean errors and assigned weights.

(e) The analysis of perturbations induced in the motion of minor planet Icarus by Lieske and Null (1969) was adopted. The mass derived from Eros by Rabe (1950) is omitted because subsequent analysis of the observations by Rabe (1967) indicates the value is poorly determined. The values of the mass of Venus, the Earth–Moon, and Mars resulting from the same investigation are also omitted.

In forming the weighted mean the group means were given weights of 1, 9, 474, 23, and 108 respectively. The resultant reciprocal mass of Mercury is 5987000 ± 32000 where the mean error is based on the residuals from the mean value.

5. Venus, Table II

(a) The reciprocal masses of Venus determined from secular perturbations by Fotheringham (1935), Clemence (1949) and Brouwer (1950) were combined to give a weighted group mean of 408 144 ± 287. The mean error is based on the mean errors of the independent determinations.

(b) While there is some overlapping of observational data, the determinations of the reciprocal mass of Venus derived from periodic perturbations in the motion of the Earth by Newcomb (1898), Jones (1926) and Morgan and Scott (1939) were combined in a group mean to give 406442 ± 657. The mean error quoted is based on the discordance among the determinations. The determination by Cowell (1906) is superseded by the other determinations.

(c) Based on the periodic perturbations induced in the motion of Mars, the reciprocal mass of Venus derived by Duncombe (1964), which supersedes Ross (1917) and Jones (1925 and 1927) is adopted.

(d) The reciprocal mass determined from periodic perturbations in the motion of Mercury by Clemence (1943), which supersedes Newcomb (1898) and Williams (1939), is adopted.

(e) The reciprocal mass of Venus derived by Ash *et al.* (1967) from a solution incorporating both radar and optical observations is adopted.

(f) The value of the reciprocal mass of Venus deduced from perturbations in the motion of the spacecraft Mariner 2 by Anderson (1967) and the spacecraft Mariner 5 by Anderson *et al.* (1968) were combined to give a weighted group mean of 408 520 ± 6. The mean error is determined from the discordance between the two values.

Several investigators have derived new values for the masses of Venus and Mercury on the hypothesis that errors in these masses are the sole cause of the difference between the observed and theoretical secular variation of the obliquity of the ecliptic. Morgan (1933) deduced a reciprocal mass of Venus of 405560 and Jones (1932) determined values of 403030 and 405800. Jones (1932) also derived a reciprocal mass of Mercury of 7000000. Since other independent information does not support this hypothesis, these mass values are omitted from the discussion.

In determining the final weighted mean, the group means received weights of 54, 10, 20, 1, 306, and 122500, respectively. The resulting value is 408519 ± 11 where the mean error is based on the discordance of the group means.

TABLE II
Investigations of the mass of Venus

No.	Reciprocal mass	Mean error	Author	Year	Object
1+	406650	±1400	Newcomb	1895b	Sun (1750–1889)
2	406770	±2070	Newcomb	1895b	Mercury (1765–1889)
3	399000		Cowell	1906	Sun (1864–1900)
4	403490	±3600	Ross	1917	Mars (1753–1912)
5+	412700	±2000	Jones	1925	Mars (1899–1924)
6	404700	±1200	Jones	1926	Sun (1836–1923)
7	411300	±1400	Jones	1927	Mars (1899–1924)
8+	408400	±2800	Fotheringham	1935	Perturbations of the obliquity of the Ecliptic, Sun (1884–1932)
9	406400		Williams	1939	Transits of Mercury (1799–1927)
10+	407000	± 700	Morgan and Scott	1939	Sun (1900–37)
11*	409300	±2100	Clemence	1943	Mercury (1765–1937)
12+	408150	± 296	Clemence	1949	From motion of perihelia of Earth and Mercury
13	408645	± 308	Rabe	1950	Eros (1926–45)
14+	408000	±1200	Brouwer	1950	Secular perturbations
15*	408945	± 470	Duncombe (see IAU Trans.)	1964	Mars (1750–1955)
16	408533.5	± 44	Anderson *et al.*	1964	Mariner 2, 1962
17	408532	± 37	Anderson	1964	Mariner 2
18+	408507.7	± 7	Anderson	1967	Mariner 2
19*	408250	± 120	Ash *et al.*	1967	Radar data (1959–66) Optical data (1950–65)
20+	408522	± 3	Anderson *et al.*	1968	Mariner 5
	408000		Currently adopted		
	408519	± 11	Weighted mean		

6. Earth–Moon, Table III

(a) Based on the perturbations of the motion of Eros, the determinations of the reciprocal mass of the Earth–Moon by Rabe and Francis (1967), Schubart and Zech (1967) and Lieske (1968) supersede the previous work. The seven parameter solution by Rabe and Francis was selected rather than the sixteen parameter solution because

TABLE III
Investigations of the mass of Earth–Moon

No.	Reciprocal mass	Mean error	Author	Year	Object
1	329097		von Haerdtl	1889b	Winnecke comet
2	328016		Newcomb	1895b	Solar parallax
3	328882	±1462	Witt	1905	Eros (1893–1903)
4	328659	± 123	Witt (see Noteboom,(1921)	1908	Eros (1893–1907)
5	327463	± 301	Hinks	1909	Eros (1900–01) Photo. α by Trig.
6	327575	± 446	Hinks	1910	Eros (1900–01) Micrometer α by Trig.
7	328370	± 102	Noteboom	1921	Eros (1893–1914)
8	327950	± 297	van den Bosch	1927	Combination of parallaxes and minor planets
9	328390	± 103	Witt	1933	Eros (1893–1931)
10	329367	± 169	Jones	1941	Solar parallax (1930–1)
11	328452	± 64	Rabe	1950	Eros
12	328446	± 64	Rabe	1954	Eros (1926–45)
13	328560	± 133	Sky and Telescope	1960	Pioneer 5
14	328910	± 74	See Kotelnikov	1962	Venus radar obs.
15+	328903.2		Muhleman	1965	Radar (1961–3)
16	328631	± 133	Rabe	1967	Eros (1926–45) 16 parameters
17	328875	± 31	Rabe	1967	Eros (1926–45) 7 parameters
18	328939	± 40	Rabe and Francis	1967	Eros (1926–65) 16 parameters
19+	328899	± 15	Rabe and Francis	1967	Eros (1926–65) 7 parameters
20+	328900	± 60	Ash *et al.*	1967	Radar data (1959–66) Optical data (1950–65)
21	328900	± 1.5	Sjogren *et al.*	1967	Ranger lunar probes
22+	328894	± 30	Schubart and Zech	1967	Eros (1926–65)
23*	328927	± 50	Schubart	1969	Amor (1932–64)
24+	328915	± 4	Lieske	1968	Eros (1893–1966)
25	328900.1	± 0.4	Anderson (see Melbourne *et al.*, 1968)	1968	Ranger data
26+	328899.86	± 2.04	Sjogren *et al.*	1966	Ranger 3 (Jan. 62)
27+	328899.67	± 10.85	Sjogren *et al.*	1966	Ranger 4 (April 62)
28+	328901.84	± 10.76	Sjogren *et al.*	1966	Ranger 5 (Oct. 62)
29+	328900.61	± 1.87	Sjogren *et al.*	1966	Ranger 6 (Jan. 64)
30+	328899.89	± 1.35	Sjogren *et al.*	1966	Ranger 7 (July 64)
31+	328900.27	± 0.67	Sjogren *et al.*	1966	Ranger 8 (Feb. 65)
32+	328899.96	± 0.73	Sjogren *et al.*	1966	Ranger 9 (Mar. 65)
33+	328900.15	± 0.85	Wong	1968	Surveyor 1 (May 66)
34+	328900.29	± 0.86	Wong	1968	Surveyor 3 (Apr. 67)
35+	328900.23	± 1.02	Wong	1968	Surveyor 4 (July 67)
36+	328900.30	± 0.68	Wong	1968	Surveyor 5 (Sept. 67)
37+	328900.30	± 0.60	Wong	1968	Surveyor 6 (Nov. 67)
38+	328900.30	± 0.84	Wong	1968	Surveyor 7 (Jan. 68)
39*	328900.46	± 1.87	Mottinger and Sjogren	1967	Lunar Orbiter 2 (Nov. 66)
40+	328899.70	± 1.14	Anderson	1967	Mariner 4 (Nov. 64)
41+	328899.98	± 0.33	Anderson and Hilt	1968	Mariner 5 (Jun. 67)
42*	328899.68	± 1.47	Anderson and Hilt	1968	Pioneer 7 (Aug. 66)
	329390		Currently adopted		
	328900.12	± 0.20	Weighted mean		

https://doi.org/10.1017/S0252921100028372 Published online by Cambridge University Press

it is believed to be more determinate. It is recognized that the three analyses contain some common observational data. Due to the greater extent of his observations, Lieske's determination was given a weight of two, while the other determinations received a weight of one. The resulting group mean was 328906 ± 7, where the mean error is based on the discordance among the determinations.

(b) The mass determined from radar range measurements by Muhleman (1965) and Ash *et al.* (1967), which supersede Kotelnikov *et al.* (1962), were combined with equal weight to give a group mean of 328901.6 ± 42.4. The mean error is based on the published mean errors of the independent determinations.

(c) Schubart (1969) derived the reciprocal mass of the Earth–Moon from the perturbations induced in the minor planet Amor and this value was adopted.

(d) Since there is a possibility of systematic effects in results derived from different types of spacecraft due to differing instrumentation and measuring techniques, spacecraft determinations have been considered separately. The values based on the orbits of the Ranger spacecraft give a weighted group mean of 328900.12 ± 0.44, where the mean error is based on the mean errors of the independent determinations.

(e) Based on the Surveyor spacecraft series, the weighted group mean is 328900.27 ± ± 0.32 where the mean error is based on the errors of the independent determinations.

(f) The values from the Mariner spacecraft data give a weighted group mean of 328899.96 ± 0.32 with the mean error based on the uncertainty of the determinations.

(g) The Lunar Orbiter result was adopted.

(h) The Pioneer 5 determination (1960) was discarded because of the presence of systematic errors, and the determination from Pioneer 7 (1968) was adopted.

The values based on solar parallax and comet determinations were omitted from the solution because of the suspected presence of systematic errors. The group means were weighted 51, 1.4, 1, 12913, 24414, 24414, 715, and 1157, respectively, giving the final mean value of 328900.12 ± 0.20. The mean error is based on the accuracies of the group means rather than their discordance.

7. Mars, Table IV

(a) For the reciprocal mass of Mars derived from the motion of the satellites, Wilkins' (1966) analyses based on both satellites was selected since it includes more data than the determination based on Deimos alone. This supersedes prior determinations.

(b) The values derived by Anderson *et al.* (1970) from doppler observations of Mariners 4 and 6 were combined to give a weighted group mean of 3098709 ± 9 where the mean error is based on the given mean error of the determinations. The determination by Anderson (Himmel, 1969) from Mariner 6 data is superseded by the later analysis of Anderson *et al.* (1970).

(c) For the mass derived from planetary perturbations deduced from radar range measurements of the planet the determination of Anderson *et al.* (1970) was not used

TABLE IV
Investigations of the mass of Mars

No.	Reciprocal mass	Mean error	Author	Year	Object
1	3093500	± 5000	Hall	1878	Satellites
2	3088000	± 7400	van den Bosch	1927	Satellites (1877–1909)
3	3110000	± 11400	Rabe	1950	Eros (1926–45)
4	3079000	± 8500	Urey	1952	Deimos
5*	3096000	± 6000	Wilkins	1966	Satellites (1877–1928)
6	3094000	± 4000	Wilkins	1966	Deimos (1877–1928)
7*	3111000	± 9000	Ash *et al.*	1967	Radar data (1959–66) Optical data (1950–65)
8	3098600	± 890	Null *et al.*	1967	Mariner 4
9	3098700	± 148	Null (see Melbourne *et al.*, 1968)	1968	Mariner 4
10	3098700	± 148	Anderson (see Himmel)	1969	Mariner 6
11	3098697	± 100	Anderson *et al.*	1970	Mariner 6 Radar data of Mars
12+	3098716	± 132	Anderson *et al.*	1970	Mariner 6
13+	3098709	± 9	Anderson *et al.*	1970	Mariner 4
	3093500		Currently adopted		
	3098709	± 9	Weighted mean		

since it is primarily dependent upon the Mariner 6 data. The determination by Ash *et al.* (1967) was adopted.

The group means were given weights of 2, 1000000, and 1, respectively in forming the weighted mean of 3098709±9 where the mean error is determined from the accuracy of the group means.

8. Jupiter, Table V

(a) In forming the group mean for the mass of Jupiter derived from its effects on the motion of minor planets, the determinations by Encke (1826), Powalky (1864), Hansen and Krueger (1873) and Newcomb (1894) were omitted from consideration since there are more recent determinations which incorporate the same observational material. A weighted mean of the 18 remaining determinations was formed. In those cases where no weight was given by the investigator a weight of unity was arbitrarily assigned. The resulting weighted mean of 1047.366±0.005 for the reciprocal mass of Jupiter is thought to be representative of the modern determinations. The mean error was determined by the discordances of the individual values from the weighted mean.

(b) The determinations of Jupiter's mass from satellite observations made by Newton (1686), Lagrange (1782), Laplace (1802) and Schur-Triesnecker (1882) are not considered since they rest solely on observations made before 1800 and therefore can not contribute significantly to the final result. The investigations of Santini (1835), Airy (1837), Bessel (1841), Vogel (1880) and Kulikov (1950) are superseded and have

TABLE V
Investigations of the mass of Jupiter

No.	Reciprocal mass	Mean error	Author	Year	Object
1+	1053.924		Nicolai	1826	(3) Juno
2	1053.36		Encke	1826	(4) Vesta
3+	1048.69		Encke	1837	(2) Pallas
4	1047.88		Powalky	1864	(48) Doris
5+	1048.23		Schubert	1866	(23) Thalia
6	1051.12	±1.2	Hansen (see Kempf, 1882)		(13) Egeria
7+	1047.37	±1.538	Becker	1872	(29) Amphitrite
8	1047.538	±0.285	Krueger	1873	(24) Themis
9+	1045.25	±0.68	Dubjago (see Kempf, 1882)		(78) Diana
10+	1048.42		Bryant	1889	(80) Sappho
11	1047.34	±0.044	Newcomb	1894	(33) Polyhymnia
12+	1045.63		Leveau	1896	(4) Vesta
13*	1047.378	±0.179	Hill	1898	Saturn
14+	1047.558	±0.40	Samter	1910	(13) Egeria (1850–1906)
15+	1047.57	±0.095	Osten	1928	(447) Valentine (1899–1918)
16+	1047.387	±0.004	O'Handley	1967	(65) Cybele
17+	1047.356	±0.004	Fiala	1968	(57) Mnemosyne (1859–1965)
18+	1047.340	±0.024	Zielenbach	1968	(48) Doris
19+	1047.351	±0.006	Klepczynski	1969	(10) Hygeia (1849–1966)
20+	1047.359	±0.010	Klepczynski	1969	(24) Themis (1853–1964)
21+	1047.372	±0.006	Klepczynski	1969	(31) Euphrosyne (1854–1964)
22+	1047.337	±0.027	Klepczynski	1969	(52) Europa (1858–1964)
23+	1047.341	±0.011	Janiczek	1969	(33) Polyhymnia (1854–1969)
24+	1047.340	±0.013	Doggett	1969	(49) Pales (1857–1968)
25	1067		Newton	1686	Satellite 4
26	1067.195		Lagrange	1782	Satellite 4
27	1067.09		Laplace	1802	Satellite 4
28	1050.2	±2.5	Santini	1835	Satellite 4
29	1046.77		Airy	1837	Satellites 1–4 (1833–37)
30	1047.879	±0.348	Bessel (see Schur, 1882)	1841	Satellites 1–4 (1834–39)
31	1047.760	±0.550	Vogel (see Kempf, 1882)	1880	Satellites 3, 4 (1868–70)
32+	1047.232	±0.365	Schur	1882	Satellites 1–4 (1874–80)
33	1048.55	±2.15	Schur-Triesnecker	1882	Satellites 1–4 (1794–95)
34+	1047.805	±1.11	Schur-Airy	1882	Satellites 1–4 (1833–37)
35+	1051.088	±2.242	Schur-Santini	1882	Satellite 4
36+	1047.905	±0.199	Schur-Bessel	1882	Satellites 1–4 (1834–39)
37+	1047.54		Schur-Jacob	1882	Satellites 3–4 (1860)
38+	1047.37	±1.20	Kempf-Jacob	1882	Satellites 3–4 (1860)
39+	1047.767	±0.460	Kempf-Vogel	1882	Satellites 3–4 (1868–70)
40+	1047.641	±0.724	Kempf-Airy	1882	Satellites 1–4 (1833–37)
41+	1047.439	±0.138	Cookson	1906	Satellites 1–4 (1901)
42+	1047.247	±0.087	Cookson	1906	Satellites 1–4 (1902)
43+	1047.498	±0.086	de Sitter	1915	Satellites 1–4 (1891)
44	1047.411	±0.6	Kulikov	1950	Satellite 8 (1930–46)
45+	1047.335	±0.077	Herget	1968	Satellite 8 (1908–65)
46+	1047.386	±0.061	Bec	1969	Satellite 9 (1914–68)

Table V (Continued)

No.	Reciprocal mass	Mean error	Author	Year	Object
47+	1047.788	±0.408	Möller	1872	Faye's comet
48+	1050.478		Von Asten (see von Haerdtl, 1889)		Encke's comet
49+	1047.175	±0.014	von Haerdtl	1889	Winnecke's comet (1858–86)
50+	1050.93	±0.33	Rasmussen	1967	Olber's comet (1815, 1887, 1955)
51+	1050.99	±0.98	Rasmussen	1967	Halley's comet (1759, 1835, 1910)
	1047.355		Currently adopted		
	1047.366	±0.007	Weighted mean		

been omitted. A weighted mean of the 13 remaining investigations gives 1047.394±±0.040 where the mean error is derived from the discordance of the individual values.

(c) The five determinations based on analyses of the motion of comets were combined to yield a weighted mean of 1047.184±0.089. This result is strongly influenced by the large weight given to the determination by von Haerdtl (1889) due to its unrealistic mean error. The mean error of the weighted mean was derived from the mutual discordances with respect to the weighted mean.

(d) Hill's determination of the mass of Jupiter from its influence on the motion of Saturn was adopted.

The resulting mass of Jupiter reflects the small mean error associated with the mass as determined from minor planet studies. In forming the mean value, relative weights of 1282, 20, 4, and 1 were assigned to the four group means. The resulting weighted mean along with its mean error determined by discordances with respect to the mean is 1047.366±0.007.

The authors feel that the true value probably lies within a range of five times the mean error on either side of the mean, i.e., between 1047.331 and 1047.401.

9. Saturn, Table VI

(a) Since van den Bosch (1927) includes the work of Bessel (1833) in his investigation, the latter is not considered further. The investigations of van den Bosch (1927), Woltjer (1928) and Jeffreys (1954) which utilized observations of the satellites to determine a value for the mass of Saturn were combined with weights 1.0, 1.2, and 7, respectively, to yield a weighted mean of 3494.12±1.45. Since the three values happen to be in close agreement, it was felt that determining the mean error by the discordances with respect to the mean would not truly reflect the accuracy of the result. Rather, the mean error of the mean was obtained by dividing the mean error of unit weight by the square root of the sum of the weights.

(b) The analysis of the mass of Saturn by Hill (1898) from its effect on the motion of

Jupiter is superseded by Gaillot (1913). Forming a weighted mean of the determination by Gaillot (1913), Hertz (1953) and Klepczynski *et al.* (1970), the value 3498.5±0.3 was obtained as the group mean. Here, the mean error was derived from the discordances with respect to the mean.

(c) The investigation by Clemence (1960), since it analyzed only the secular perturbations of Saturn on Jupiter, was considered independent of those investigations which utilized periodic perturbations and was adopted.

(d) Marsden's (1970) analysis using the motion of the minor planet Hidalgo is the only such determination and is adopted.

(e) Similarly, the use of the comet P/Schwassmann-Wachmann 1 by Herget (1970) is the only such analysis and it is adopted.

The five group means were combined with weights 1, 23, 6, 23, 53 and the resulting weighted mean is 3498.1±0.4. The mean error was determined by the mutual discordances with respect to the mean.

TABLE VI
Investigations of the mass of Saturn

No.	Reciprocal mass	Mean error	Author	Year	Object
1	3501.6	±1.15	Bessel	1833	Titan
2	3502.2	±0.79	Hill	1898	Jupiter (1750–1888)
3+	3499.8	±1.75	Gaillot	1913	Jupiter (1750–1907)
4+	3496	±4.4	van den Bosch	1927	Satellites (1830–1915)
5+	3493	±4	Woltjer	1928	Hyperion
6+	3497.64	±0.40	Hertz	1953	Jupiter (1884–1948)
7+	3494.04	±1.65	Jeffreys	1954	Satellites (1924–37)
8*	3499.7	±0.6	Clemence	1960	Secular perturbations Jupiter (1779–1941)
9*	3498.5	±0.30	Marsden	1970	Hidalgo (1920–64)
10*	3497.6	±0.20	Herget	1970	Comet P/Schwassmann-Wachmann 1 (1927–65)
11+	3498.7	±0.2	Klepczynski *et al.*	1970	Jupiter (1913–68)
	3501.6		Currently adopted		
	3498.1	±0.4	Weighted mean		

10. Uranus, Table VII

(a) There are two investigations which analyze the perturbations of Uranus on Saturn to derive its mass. The investigation by Hill (1898) depends on Hill's theory of Saturn which is known to have errors (Clemence, 1951). For this reason, Hill's value is eliminated from consideration, and the determination by Klepczynski *et al.* (1970) is adopted.

(b) The value of 22530 given by van den Bosch (1927), obtained from observations of the satellites, is dropped from consideration because it includes observations which are thought to be affected by systematic errors. These observations are not included

TABLE VII
Investigations of the mass of Uranus

No.	Reciprocal mass	Mean error	Author	Year	Object
1	23239	±132	Hill	1898	Saturn (1751–1888)
2+	22685	± 31	van den Bosch	1927	Oberon (1874–1911)
3+	22831	± 25	van den Bosch	1927	Titania (1874–1911)
4+	22919	±255	van den Bosch	1927	Umbriel (1874–1901)
5+	23073	±144	van den Bosch	1927	Ariel (1874–1901)
6	22530	± 74	van den Bosch	1927	Satellites (1874–1911)
7+	22934	± 9	Harris	1950	Satellites (photographic)
8*	22692	± 33	Klepczynski *et al.*	1970	Saturn (1913–68)
	22869		Currently adopted		
	22800	±107	Weighted mean		

in the determinations from individual satellites. The remaining five determinations of the mass of Uranus which used observations of the satellites were combined to yield a weighted mean of 22906±34. The mean error is based on the discordance among the values.

The mean value obtained by combining these two group means is 22800±107. The value and the mean error reflect the two discordant values which received equal weight in forming the mean.

11. Neptune, Table VIII

(a) The determination of the mass of Neptune by investigating its effects on the motion

TABLE VIII
Investigations of the mass of Neptune

No.	Reciprocal mass	Mean error	Author	Year	Object
1	19700	±200	Newcomb	1874	Uranus (1690–1872)
2	19314		Newcomb	1898b	Uranus (1781–1896) Triton
3	19094	±33	Gaillot	1910	Uranus (1690–1903)
4	19176	±37	Eichelberger and Newton	1926	Triton visual
5	19655	±53	Eichelberger and Newton	1926	Triton photographic
6	19331	±31	Eichelberger and Newton	1926	Triton combined
7	18889	±92	van Biesbroeck	1957	Nereid (1949–55)
8*	19296	±31	Gill and Gault	1968	Triton (1887–1958)
9*	19349	±28	Seidelmann *et al.*	1969	Uranus (1781–1968)
	19314		Currently adopted		
	19325	±26	Weighted mean		

of Uranus obtained by Seidelmann *et al.* (1969) supersedes the analyses of Newcomb (1874, 1898b) and Gaillot (1910) and is adopted.

(b) Of the determinations of the mass of Neptune from the motion of its satellites, we adopt the value of Gill and Gault (1968) since it supersedes the investigation of Eichelberger and Newton (1926). However, the mean error of Eichelberger and Newton is assigned to the determination of Gill and Gault, since 83% of the observational material is in common and the accuracy of the added observations appears comparable to that used earlier. Due to correlations existing among the unknowns, which may systematically affect the results, it was decided to not include the determination by by Van Biesbroeck (1957) in this discussion.

The two group means are in fairly good agreement and each contributes almost equally to the weighted mean of 19325±26. The mean error was determined from the mutual discordances with respect to the mean.

12. Pluto, Table IX

(a) The previous gravitational determinations are superseded by investigation 10. Item 2 is omitted since it is derived only from brightness measures. The value 1812000±50000 is therefore adopted as the provisional mass of Pluto; the error has been estimated rather than derived formally.

TABLE IX

Investigations of the mass of Pluto

No.	Reciprocal mass	Mean error	Author	Year	Object
1	> 330000		Jackson	1930	Neptune (1795–1928)
2	3000000		Bower	1931	Brightness
3	350000	± 90000	Nicholson and Mayall	1931	Neptune (1795–1930)
4	> 660000		Brown	1931	Uranus (1780–1900)
5	330000	± 46500	Wylie	1942	Neptune (1795–1938)
6	350000	± 50000	Kourganoff	1944	Neptune
7	930000	± 726000	Eckert *et al.*	1951	Uranus (1781–1938)
8	400000	± 59000	Brouwer	1955	Uranus and Neptune (1712–1941)
9	450000	± 118000	Brouwer	1955	Longitudes of Uranus and Neptune (1712–1941)
10*	1812000	± 50000	Duncombe *et al.*	1968	Neptune (1795–1968)
	360000		Currently adopted		
	1812000	± 50000	Recommended value		

13. Summary

The final set of masses is exhibited in Table X along with some previous determinations.

TABLE X

Discussions of the masses of the planets

	Newcomb (1898) (currently adopted)	Clemence (1964)	Kulikov (1965)	Klepczynski *et al.* (1970)
Mercury	6000000	6110000 ± 60000	6127000 ± 60000	5987000 ± 32000
Venus	408000	408539 ± 18	408120 ± 190	408519 ± 11
Earth–Moon	329390	328906 ± 9	328546 ± 37	328900.12 ± 0.20
Mars	3093500	3050000	3087000 ± 4200	3098709 ± 9
Jupiter	1047.355[a]	1047.41 ± 0.03	1047.394 ± 0.027	1047.366 ± 0.007
Saturn	3501.6	3499.6 ± 0.6	3498.85 ± 0.3	3498.1 ± 0.4
Uranus	22869	22930 ± 9	22929 ± 9	22800 ± 107
Neptune	19314	19070 ± 31		19325 ± 26
Pluto	360000[a]	400000 ± 60000		1812000 ± 50000

[a] adopted from Eckert *et al.* (1951)

https://doi.org/10.1017/S0252921100028372 Published online by Cambridge University Press

References

(Combined list of references to both this paper and J. Kovalevsky, 'Détermination des Masses des Planètes', pp. 213–223).

Airy, G. B.: 1833–37, *Mem. Roy. Astron. Soc.* **6**, 83; **8**, 33; **9**, 7; **10**, 43.
Anderson, J. D.: 1964, *AIAA Proceedings* (ed. by Szebehely).
Anderson, J. D.: 1967, Dissertation, UCLA.
Anderson, J. D.: 1967, *Jet Propul. Lab. Tech. Rep.* No. 32-816.
Anderson, J. D. and Efron, L.: 1969, *Bull. AAS* **1**, 231.
Anderson, J. D. and Hilt, D. E.: 1968, *Am. Astron. Soc. Pap.* 68–130.
Anderson, J. D., Efron, L., and Pease, G. E.: 1968, *Astron. J. Suppl.* **73**, S-162.
Anderson, J. D., Efron, L., and Wong, S. K.: 1970, *Science* **167**, 277.
Anderson, J. D., Null, G. W., and Thornton, C. T.: 1964, 'Celestial Mechanics and Astrodynamics', in *Progress in Aeronautics and Astronics Series* **14**, Academic Press, New York, p. 131.
Anderson, J. D., Esposito, P. B., Martin, W., and Muhleman, D. O.: 1970, *Communication à la 13e Assemblée Generale du COSPAR*, Léningrad, Mai 1970.
Ash, M. E., Shapiro, I. I., and Smith, W. B.: 1967, *Astron. J.* **72**, 338.
Backlund, M. O.: 1894, *Bull. Astron.* **11**, 473.
Bec, A.: 1969, *Astron. Astrophys.* **2**, 381.
Becker, E.: 1872, *Publ. Astron. Gesellsch.* Nr. 10.
Bessel, F. W.: 1833, *Astron. Nachr.* **11**, 17.
Böhme, S. and Fricke, W.: 1965, *Bull. Astron.* **25**, 280.
Bower, E. C.: 1931, *Lick Obs. Bull.* **XV**, No. 437.
Brouwer, D.: 1950, *Bull. Astron.* **15**, 171.
Brouwer, D.: 1955, *Monthly Notices Roy. Astron. Soc.* **115**, 221.
Brouwer, D. and Clemence, G.: 1961, 'Orbits and Masses of Planets and Satellites' in *The Solar System* (ed. by Kuiper and Middlehurst), **3**.
Brown, E. W.: 1931, *Monthly Notices Roy. Astron. Soc.* **92**, 80–101.
Bryant, R.: 1889, *Astron. Nachr.* **121**, 231.
Clemence, G. M.: 1943, *Astron. Papers Am. Ephem.* **11**, pt. 1.
Clemence, G. M.: 1949, *Proc. Am. Phil. Soc.* **93**, 532.
Clemence, G. M.: 1960, *Astron. J.* **65**, 21.
Clemence, G. M.: 1961, in *Planets and Satellites* (ed. by Kuiper and Middlehurst), University of Chicago Press, p. 61.
Clemence, G. M.: 1966, in 'Proceedings of the 12th General Assembly 1964', *IAU Transactions* **12B**, 610.
Cookson, B.: 1906, *Ann. Cape Obs.* **12**, pt. II.
Cowell, P. H.: 1906, *Monthly Notices Roy. Astron. Soc.* **66**, 302.
de Sitter, W.: 1915, *Ann. Cape Obs.* **12**, pt. I.
Doggett, L. E.: 1969, Thesis, Georgetown Univ.
Duncombe, R. L.: 1958, *Astron. Papers Am. Ephem.* **16**, pt. 1.
Duncombe, R. L.: 1965, *Bull. Astron.* **25**, 137.
Duncombe, R. L., Klepczynski, W. J., and Seidelmann, P. K.: 1968, *Astron. J.* **73**, 830.
Eckert, W. J., Brouwer, D., and Clemence, G. M.: 1951, *Astron. Papers Am. Ephem.* **12**.
Eichelberger, W. S. and Newton, A.: 1926, *Astron. Papers Am. Ephem.* **9**, pt. 3.
Encke, J. F.: 1826, *Abh. Berliner Akad. Wiss.*, 267.
Encke, J. F.: 1837, *Astron. Nachr.* **14**, 331.
Fiala, A. D.: 1969, *Bull. AAS* **1**, 342.
Fotheringham, J. K.: 1935, *Astron. Nachr.* **256**, 6121.
Gaillot, M. A.: 1910, *Ann. Obs. Paris* **28**, A 1.
Gaillot, M. A.: 1913, *Ann. Obs. Paris* **31**, 105.
Gill, J. R. and Gault, B. L.: 1968, *Astron. J.* **73**, S 95.
Hall, A.: 1878, *Observations and Orbits of the Satellites of Mars with Data for Ephemeris in 1879*, Washington, Government Printing Office.
Halliday, I., Hardie, R. H., Franz, O., and Priser, J. B.: 1965, *Astron. J.* **70**, 676.
Harris, D.: 1950, Dissertation, University of Chicago.

Herget, P.: 1968, *Astron. J.* **73**, 737.
Herget, P. and Carr, H. J.: 1970, *Bull. AAS* **2**, 245.
Hertz, H. G.: 1953, *Astron. Papers Am. Ephem.* **15**, pt. 2.
Hertz, H. G.: 1966, *Circul. Bur. Centr. Télegr. Astron.* No. 1983.
Hill, G. W.: 1898, *Astron. Papers Am. Ephem.* **7**, 160.
Himmel, N.: 1969, *Aviation Week* **91**, 95.
Hinks, A.: 1909, *Monthly Notices Roy. Astron. Soc.* **69**, 544.
Hinks, A.: 1910, *Monthly Notices Roy. Astron. Soc.* **70**, 588.
Jackson, J.: 1930, *Monthly Notices Roy. Astron. Soc.* **90**, 728–32.
Janiczek, P.: 1969, Dissertation, Georgetown Univ.
Jeffreys, H.: 1954, *Monthly Notices Roy. Astron. Soc.* **114**, 433.
Jones, H. S.: 1925, *Monthly Notices Roy. Astron. Soc.* **85**, 853.
Jones, H. S.: 1926, *Monthly Notices Roy. Astron. Soc.* **86**, 426.
Jones, H. S.: 1927, *Monthly Notices Roy. Astron. Soc.* **87**, 602.
Jones, H. S.: 1932, *Ann. Cape Obs.* **13**, pt. 3.
Jones, H. S.: 1941, *Mem. Roy. Astron. Soc.* **66**, pt. 2, 11.
Kempf, P.: 1882, *Publ. Astrophys. Obs. Potsdam* **3**, pt. II, No. 10.
Kempf, P.: 1883, *Astron. Nachr.* **105**, 325.
Klepczynski, W. J.: 1969, *Astron. J.* **74**, 774.
Klepczynski, W. J., Seidelmann, P. K., and Duncombe, R. L.: 1970, *Astron. J.* **75**, 739.
Kotelnikov, V. A. *et al.*: 1962, *Dokl. Akad. Sci. U.S.S.R.* **145**, 5.
Kourganoff, V.: 1944, *Ciel Terre* **60**.
Kovalevsky, J.: 1970, in *Surfaces and Interiors of Planets and Satellites* (ed. by Dollfus), Academic Press, p. I.
Kozlovskaya, S. V.: 1963, *Bull. Inst. Astron. Théor. Léningrad* **9**, 330.
Krueger, A.: 1873, *Astron. Nachr.* **81**, 331.
Kulikov, D. K.: 1950, *Bull. Inst. Astron. Théor. Léningrad* **4**, 311.
Kulikov, D. K.: 1965, *Bull. Astron.* **25**, 139–51.
Lagrange, J. L.: 1782, *Mem. Berlin*, p. 183.
Laplace, P. S.: 1802, *Traité Mécanique Céleste* **3**, 62.
Leveau, G.: 1896, *Ann. Obs. Paris* **22**, A 57.
Lieske, J. H.: 1968, *Astron. J.* **73**, 628.
Lieske, J. H. and Null, G. W.: 1969, *Astron. J.* **74**, 297.
Makover, S. G.: 1956, *Publ. Inst. Astron. Théor. Léningrad* No. 6, p. 67.
Makover, S. G. and Bokhan, N. A.: 1961, *Trudy Inst. Astron. Théor. Léningrad* **8**, 135.
Marsden, B. G.: 1970, *Astron. J.* **75**, 215.
Melbourne, W. J.: 1969, *COSPAR Symp. C*, Prague, Czechoslovakia.
Melbourne, W. J.: 1970, in *Dynamics of Satellites* (ed. by Morando), Springer Verlag, p. 63.
Melbourne, W. J. and O'Handley, D. A.: 1968, *Jet Propul. Lab. Space Program Summary*, 37–53, vol. III, p. 1.
Melbourne, W. J., Mulholland, J. D., Sjogren, W. L., and Sturm, F. M. Jr.: 1968, *Jet. Propul. Lab. Tech. Rep.* No. 32–1306.
Möller, A.: 1972, *Viertelj. Astron. Gesellsch.* **7**, 85.
Morgan, H. R.: 1933, *Astron. J.* **42**, 149.
Morgan, H. R. and Scott, F. P.: 1939, *Astron. J.* **47**, 193.
Mottinger, N. A. and Sjogren, S. L.: 1967, *JPL SPS* 37–46, III, 19.
Muhleman, D. O.: 1965, *Bull. Astron.* **25**, 153–175.
Mulholland, J. D.: 1967, *Jet. Propul. Lab. Space Program Summary*, 37–45, vol. IV, p. 17.
Mulholland, J. D.: 1968, *Astron. J.* **73**, S-28.
Newcomb, S.: 1874, *Smithsonian Contrib. Knowl.* **19**, 173–7.
Newcomb, S.: 1894, *Astron. Nachr.* **136**, 129.
Newcomb, S.: 1895a, *Astron. Papers Am. Ephem.* **5**, 381–449.
Newcomb, S.: 1895b, *Supplement to the American Ephemeris and Nautical Almanac for 1897*, p. 1–202.
Newcomb, S.: 1898a, *Astron. Papers Am. Ephem.* **6**.
Newcomb, S.: 1898b, *Astron. Papers Am. Ephem.* **7**.
Newton, I.: 1686, *Principia*, Book III, Prop. 8.
Nicholson, S. B. and Mayall, N. U.: 1931, *Astrophys. J.* **73**, 1–12.

Nicolai, F.: 1826, *Berliner Astron. Jahrbuch*, p. 227.
Noteboom, E.: 1921, *Astron. Nachr.* **214**, 154.
Null, G. W.: 1969, *Bull. AAS* **1**, 356.
Null, G. W. and Lieske, J. M.: 1969, *Bull. AAS* **1**, 198.
Null, G. W., Gordon, H. J., and Tito, A. D.: 1967, *JPL Tech. Rep.* 32–1108.
O'Handley, D. A.: 1967, Dissertation, Yale Univ.
O'Handley, D. A.: 1968, *Astron. J.* **73**, S-29.
Osten, M.: 1928, *Astron. Nachr.* **232**, 225.
Powalky, C.: 1864, *Astron. Nachr.* **62**, 321.
Rabe, E.: 1950, *Astron. J.* **55**, 112.
Rabe, E.: 1954, *Astron. J.* **59**, 409.
Rabe, E.: 1967, *Astron. J.* **72**, 852.
Rabe, E. and Francis, M. P.: 1967, *Astron. J.* **72**, 856.
Rasmussen, H. Q.: 1967, *Publ. Copenhagen Obs.* Nr. 194.
Ross, F. E.: 1917, *Astron. Papers Am. Ephem.* **9**, p. 261.
Samter, H.: 1910, *Astron. Abhandl. Erg. Z. Astron. Nachr.* **3**, No. 17, 9.
Santini, G.: 1835, *Astron. Nachr.* **12**, 285.
Scholl, H.: 1971, *Celes. Mech.* **4**, 250.
Schubart, J.: 1969, *Astron. Astrophys.* **2**, 173.
Schubart, J.: 1970, *Circul. Bur. Centr. Télegr. Astron.*, No. 2268.
Schubart, J. and Zech, G.: 1967, *Nature* **214**, 900.
Schubert, E.: 1866, *Astron. Nachr.* **66**, 25.
Schur, W.: 1882, *Astron. Nachr.* **104**, 81–6.
Seidelmann, P. K., Duncombe, R. L., and Klepczynski, W. J.: 1969, *Astron. J.* **74**, 776.
Sjogren, W. L., Trask, D. W., Vegos, C. J., and Wollenhaupt, W. R.: 1966, *JPL Tech. Rep.* 32–1057.
Taylor, B. N., Parker, W. H., and Langenberg, D. N.: 1969, *Rev. Mod. Phys.* **41**, 375.
Urey, H. C.: 1952, *The Planets*, New Haven.
Van Biesbroeck, G.: 1957, *Astron. J.* **62**, 272.
Van den Bosch, C. A.: 1927, *De Massa's van de Groote Planeten*, G. Bakker, Baarn.
von Haerdtl, F.: 1889a, *Denkschr. Kaiserl. Acad. Wiss.* **56**.
von Haerdtl, F.: 1889b, *Astron. Nachr.* **120**, 257.
Wilkins, G. A.: 1967, in *Mantles of the Earth and Terrestrial Planets* (ed. by Runcorn,), Interscience Publ., p. 77.
Williams, K. P.: 1939, *Publ. Kirkwood Obs. Indiana Univ.*, No. 1.
Witt, G.: 1905, Doctoral dissertation, Berlin.
Witt, G.: 1908, *Viertelj. Astron. Gesellsch.* **43**, 295.
Witt, G.: 1933, *Astron. Abhandl. Erg. Z. Astron. Nachr.* **9**, No. 1.
Woltjer, J.: 1928, *Ann. Leiden* **16**, 3.
Wong, S. K.: 1968, *JPL SPS* 37–52, II, p. 12.
Wylie, L. R.: 1942, *Publ. U.S. Naval Obs.*, 2nd ser., **15**, pt. 1.
Zech, G.: *Veröffentl. Astron. Rechen-Inst. Heidelberg* No. 21.
Zielenbach, J. W.: 1960, *Sky and Telescope* **20**, 6, 337.
Zielenbach, J. W.: 1962, *Astron. J.* **67**, 1299.
Zielenbach, J. W.: 1964, *Transactions of IAU*, **12A**, 16.
Zielenbach, J. W.: 1968, Dissertation, Georgetown Univ.
Zielenbach, J. W.: 1969, *Astron. J.* **74**, 567.