viz., this will be so if we have the *two* relations

$$a = \frac{\pi}{2} - \beta \text{ ; and } b^3 = -\frac{4}{3}a^3\cos^2\beta.$$

I make (see fig. 84) Milner's lamp, with a circular section, β arbitrary, but a segment AM ($\angle$ SAM $= \beta$) made solid. G in the line SG at right angles to AM is the C.G. of the lamp, and G′ the C.G. of the oil.

And this seems to be the *only* form—for the pole of r must, it seems to me, be *on* the bounding circle—viz., in the equation $r^2 - 2ar\cos\theta = C$, we must have $C = 0$.

An Exercise on Logarithmic Tables.

By Professor TAIT.

In reducing some experiments, I noticed that the logarithm of 237 is about 2.37 ... . Hence it occurred to me to find in what cases the figures of a number and of its common logarithm are identical :—*i.e.*, to solve the equation

$$\log_{10}x = x/10^m,$$

where m is any positive integer.

It is easy to see that, in all cases, there are two solutions ; one greater than, the other less than, ϵ. This follows at once from the position of the maximum ordinate of the curve

$$y = (\log x)/x.$$

The smaller root is, for

$$m = 1,\ x = 1{\cdot}371288 \quad \ldots \quad \ldots$$

$$m = 2,\ x = 1{\cdot}023855 \quad \ldots \quad \ldots$$

For higher values of m, it differs but little from 1, and the excess may be calculated approximately from

$$y - y^2/2 + \quad \ldots \quad = (1 + y)\log_\epsilon 10/10^m.$$

Ultimately, therefore, the value of the smaller root is

$$1.00 \quad \ldots \quad \ldots \quad 0230258 \quad \ldots \quad \ldots$$

where the number of cyphers following the decimal point is $m - 1$.

The greater root must have $m + p$ places of figures before the decimal point ; p being unit till $m = 9$, thenceforth 2 till $m = 98$, 3 till $m = 997$, &c. Thus, for example, if $m > 8 < 98$ we may assume

$$x = (m + 1)10^m + y$$

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so that

$$\log_{10}\frac{m+1}{10}+\log_{10}\left(1+\frac{y}{(m+1)10^m}\right)=\frac{y}{10^m}$$

which is easily solved by successive approximations.

But it is simpler, and forms a capital exercise, to find, say to six places, the greater root, by mere inspection of a good Table of Logarithms.

Thus we find, for instance,

m	x
17	$182{,}615.10^{13}$
18	$192{,}852.10^{14}$
96	$979{,}911.10^{92}$
97	$989{,}956.10^{93}$

Geometrical Proof of the Tangency of the Inscribed and Nine-Point Circles.

By William Harvey, B.A.

S (fig. 85) is the circumscribed centre, and O the orthocentre of the triangle ABC; AX the perpendicular from A on BC, and P the middle point of BC.

SP produced bisects the arc BC in V, and I, the centre of the inscribed circle, lies on AV, and is so situated that $AI.IV = 2Rr$. (*See Note*). Also the angle XAV = angle AVS = angle SAV.

N, the centre of the nine-point circle, bisects the distance OS, and the circumference passes through P, X and L, the middle point of AO. Hence N bisects both LP and OS, and

$$SP = OL = AL;$$

therefore LP is parallel to AS.

NHM is a radius of the nine-point circle, bisecting the chord XP in H, and the arc XP in M; ID is a radius of the inscribed circle.

Since the chord XMP is bisected at M,

$$\text{the angle XPM} = \frac{1}{2}\text{angle XLP},$$
$$= \frac{1}{2}\text{ angle XAS},$$
$$= \text{angle XAV or AVS}.$$

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