

## DARK MATTER: OBSERVATIONAL ASPECTS

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**ABSTRACT.** A review of observational work on dark matter in USSR is given. Dynamically the dark matter can be located (i) in the galactic disk and/or in dwarf galaxies, (ii) in coronas of galaxies and in clusters of galaxies, and (iii) distributed smoothly in voids. The possible amount of matter in all three forms is discussed. Physically dark matter can be baryonic or non-baryonic, in the latter case either hot, warm or cold. Available information on the nature of dark matter is indirect, coming from theories of the formation of structure in the Universe. Two constraints to the formation scenarios are discussed, the galaxian correlation function and their morphology.

### 1. INTRODUCTION

There are two kinds of matter, the visible or luminous matter, and the dark matter. If one believes in recent inflationary models of the Universe, the total density of matter equals the closure density of the Universe. From dynamical considerations dark matter can be divided into the local one in galactic disks and/or in dwarf galaxies, the halo (or coronal) dark matter around giant galaxies and in clusters of galaxies, and the smoothly distributed background in voids.

Direct dynamical data give us little information on the physical nature of the dark matter. However, indirect data on the distribution of galaxies in space, characterized by the galaxian correlation function and other statistics, as well as data on the microwave background, chemical composition of matter etc. can be used to draw conclusions on the nature of the dark matter. These conclusions are indirect and enter into the calculations through various scenarios of the formation of structure in the Universe. The scenarios depend on the nature of the dark matter, whether it is baryonic, hot (neutrinos), or cold (axions) or is it simulated by the  $\lambda$ -term. Observational data mentioned can be used to test the formation scenarios and respective dark matter models. The situation is illustrated in Fig. 1.

In the following we give a review of recent observational work in the USSR on both the dynamical and physical aspects of the problem.

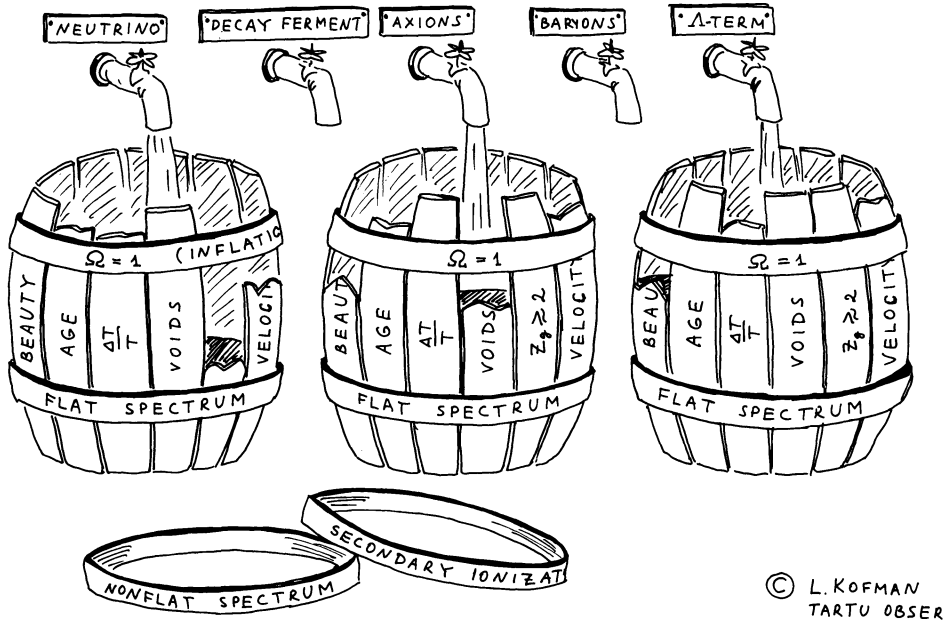


Figure 1. The barrel diagram - principal models of the formation of the structure in the Universe using different candidates of dark matter are shown as barrels. There are three main candidates of the dark matter: neutrino, axions (and other cold particles), and the cosmological constant. The barrels are hooped together by two principal assumptions,  $\Omega = 1$ , and a flat spectrum of initial perturbations. If these assumptions do not work, there are some hoops in reserve: a nonflat spectrum and secondary ionization. Various observational tests are expressed as staves. The height of a staff indicates the degree of accordance of the model with this particular test. One test is the beauty or internal harmony of the model. The level of the liquid in the barrel is equal to the height of the shortest staff, which determines the degree of acceptance of the model. If necessary, a cocktail from several liquids can be made, or some ferment as neutrino decay added.

Idea and artwork by L. Kofman.

## 2. LOCAL DARK MATTER IN GALACTIC DISK

The local mass density problem was introduced by Oort (1960). He demonstrated that dynamical mass density in the solar neighborhood,  $\rho_{\text{dyn}}$ , exceeds the mass density of known objects,  $\rho_{\text{vis}}$ , and explained this discrepancy by the gravitational attraction of an unknown population of invisible objects.

The dynamical density can be determined from the Poisson equation which in cylindrical coordinates has the form

$$4\pi G\rho_{\text{dyn}} = -dK_z/dz - dK_R/dR - K_R/R, \quad (1)$$

where  $G$  is the gravitational constant, and  $K_R$  and  $K_z$  are the components of the gravitational acceleration in the radial and vertical direction, respectively. For the solar vicinity the radial gravitational acceleration can be expressed through the Oort dynamical constants,  $A$  and  $B$ . Similarly, the vertical acceleration can be expressed as follows (Kuzmin 1952):

$$-dK_z/dz = C^2. \quad (2)$$

Using the Oort-Kuzmin constants one has instead of (1)

$$4\pi G\rho_{\text{dyn}} = C^2 - 2(A^2 - B^2). \quad (3)$$

In this expression the vertical acceleration term  $C^2$  is dominating, thus the dynamical density is determined essentially by the gradient of the vertical acceleration.

Oort (1960) derived the dynamical density by calculating the vertical gravitational acceleration  $K_z$  for a fairly large  $z$  interval and by determining its vertical gradient near the galactic plane. The dynamical density was also determined by Kuzmin (1952, 1955). Instead of calculating the acceleration in a large  $z$  interval Kuzmin concentrated from the very beginning on its gradient. Near the galactic plane  $K_z$  is proportional to  $z$ . For populations located entirely in this small  $z$  interval, the vertical velocity dispersion  $\sigma_z$  is independent of  $z$ , and the constant  $C$  can be expressed through  $\sigma_z$  and the dispersion of  $z$ -coordinates of stars of the same population,  $\sigma_z$  (Kuzmin 1952):

$$C = \sigma_z/\sigma_z. \quad (4)$$

This expression is valid for relatively young stars. Older populations have higher  $z$ -velocities and their stars move outside the linear  $K_z$  regime. To determine  $C$  stars are to be chosen from a narrow belt around the galactic equator. In his pioneering study Kuzmin used  $A$  and  $gK$  stars. The  $z$ -coordinates were calculated from the galactic latitudes, the  $z$ -velocities from proper motions perpendicular to the galactic plane, and distances were estimated from apparent magnitudes. This approach has the advantage that systematic errors in adopted distances of stars change both dispersions in the same manner, thus distance errors cancel out. A similar method was used later by a number of authors from Tartu whose results are summarized in Table 1. For cepheids instead of proper motions radial velocities were used, and the vertical component of the velocity dispersion was calculated using the standard relations between the dispersions  $\sigma_R$ ,  $\sigma_\theta$ , and  $\sigma_z$ .

As seen from Table 1, all determinations made in the Tartu Observatory yield dynamical densities about  $0.10 M_{\text{Opc}}^{-3}$  in good agreement with direct determinations of the mass density,  $\rho_{\text{vis}} = 0.092 M_{\text{Opc}}^{-3}$  (Joeveer, Einasto 1976). Eelsalu (1961) and Joeveer and Einasto (1976) have studied the reason for the discrepancy between the results obtained in these studies and in the Leiden studies (Oort 1960). The main reason lies in

the inhomogeneity of statistical samples selected on the basis of the HD spectral classification. This classification does not separate the metal deficient old disk and halo stars. The inclusion of a small number of old stars may significantly increase the velocity dispersion whereas the spatial dispersion remains practically the same.

TABLE 1. KUZMIN CONSTANT C AND LOCAL DYNAMICAL DENSITY

Stars	N	Method	C km/s/kpc	$\rho_{\text{dyn}}$ $M_{\odot}/\text{pc}^3$	Reference
Bright stars		$z, \mu_b, D(z)$	68	0.081	Kuzmin <sup>1</sup> 1955
Bright stars		$z, \Phi(M)$	66	0.076	Elsalu 1958
B8-B9	17	$\tau, z, \dot{z}$	70	0.086	Joeveer 1972
Cepheids	179	$\tau, z$	65	0.074	Joeveer 1974a
Cepheids	100	$v, D(z)$	80	0.114	Joeveer 1974b
gK			75	0.10	Balakirev 1976
B8-A5	278	$\mu_b, D(z)$	89	0.142	Joeveer 1985a
gK	253	$\mu_b, D(z)$	66	0.076	Joeveer 1985a

<sup>1</sup>) A rediscussion of earlier determinations of C by Oort 1932, Kuzmin 1952, Safronov 1952, and Parenago 1954

In such a confused case it is reasonable to use independent observational information to get a more reliable answer. Joeveer (1968, 1972, 1974a) noticed that very young populations are not in a stationary state, but oscillate in the  $z$ -direction:

$$z = z_0 \cos Ct, \quad (5)$$

$$V_z = -Cz_0 \sin Ct. \quad (6)$$

Here  $z_0$  is the maximum distance from the galactic plane for an individual star. The oscillation of a population as a whole can be easily explained if stars form outside the galactic plane in large complexes and "fall" together toward the plane (Dixon 1967a, b). Formula (5) was used by Joeveer (1974a) for a sample of cepheids, for which individual ages can be determined. Stars were grouped according to the age,  $t$ , and dispersions of the  $z$ -coordinates,  $\sigma_z$  were calculated. The  $\sigma_z$  versus  $t$  plot (Fig. 2) demonstrates the presence of a sinusoid as expected. The value of C can be estimated from two minima and one maximum.

If for a group of stars ages and the  $z$ -coordinates as well as the  $z$ -velocities are available, then each individual star yields an equation to determine C. C can be determined by minimizing the sums (Joeveer 1972)

$$f(C) = \sum [(z_i)_{\text{obs}}^2 - (z_i)_{\text{calc}}^2] \quad (7)$$

and

$$g(C) = \sum [(v_{zi})_{\text{obs}}^2 - (v_z)_{\text{calc}}^2]. \quad (8)$$

Accurate data are available for 17 B8-B9 stars. The plot of  $f$  and  $g$  versus C is given in Fig. 3. We see a pronounced minimum at  $C=70$  km/s/kpc.

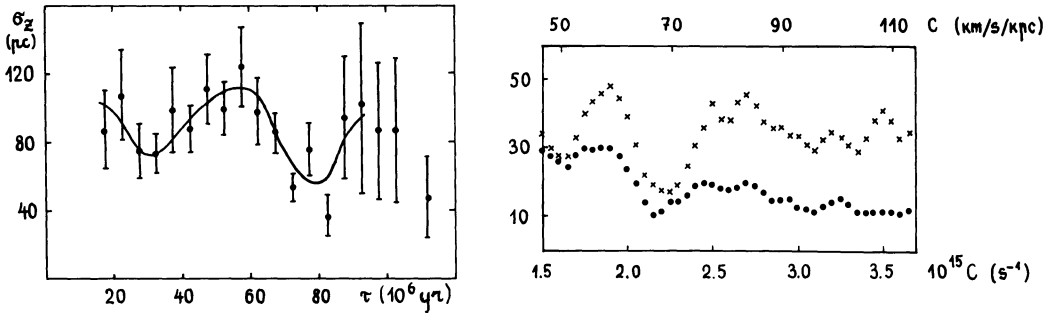


Figure 2.(left). Relation between the coordinate dispersion in the z-direction  $\sigma_z$  and the age parameter  $\tau = [\rho(R)/\rho(10)]^{-1/2}t$  in case of classical cepheids (Joeveer 1974).

Figure 3.(right). Dependence of the sums  $10^{-15}f(C_j)[\text{km}]$  and  $g(C_j)[\text{km/s}]$ , (dots and crosses, respectively) on the Kuzmin constant  $C$ , for nearby late B-type stars calculated on the basis of their evolutionary ages (Joeveer 1972).

This result as well as recent redeterminations of  $C$  using the classical method demonstrates that one can adopt a round value  $C = 70 \text{ km/s/kpc}$  which leads to the dynamical density  $\rho_{\text{dyn}} = 0.09 \text{ M}_{\text{opc}}^{-3}$  in good agreement with the direct density estimation. Thus there is no room left for the local dark matter. In a series of papers Bahcall (1984a, b) has obtained a considerably larger value. The reason for the discrepancy is not clear.

### 3. DARK MATTER IN DWARF SPHEROIDAL GALAXIES

According to Aaronson (1983) and Faber and Lin (1983) dwarf spheroidal galaxies may contain appreciable amounts of nonluminous matter. The preliminary evidence for this comes from unexpectedly large velocity dispersions in the Draco (Aaronson, 1983) and Carina (Cook, Schechter, Aaronson, 1984) dwarf spheroidal galaxies and from large tidal masses of the Sculptor, Draco, UMi and Carina galaxies (Faber, Lin, 1983). Both the virial as well as tidal mass-to-light ratios in these galaxies are an order of magnitude larger than in typical globular clusters.

The basic assumption to derive the virial and tidal masses is the dynamical equilibrium of gravitational systems under study. An alternative explanation to the rather large velocity dispersions in the Draco and Carina galaxies could be that these galaxies are just now being tidally disrupted, as already noted by Aaronson (1983). To decide between the two possibilities, the presence of dark matter or tidal disruption, a comparative analysis of morphological properties of dwarf spheroidal galaxies was performed by Joeveer (1985b). Several arguments in favour of the tidal disruption scenario were found.

A strong evidence for this comes from the tidal radii  $r_t$  versus absolute blue luminosities  $L_B$  diagram (Fig. 4). Here and in the following the data about the Virgo cluster galaxies are from Binggeli et al. (1984) and about dwarf spheroidal satellites of our Galaxy from Hodge (1966), and Faber and Lin (1983). As one can see the fit of three distant satellites of our Galaxy to the regression line determined by the Virgo cluster galaxies is excellent, but all nearby satellites are 2-5 times larger than can be expected on the basis of their luminosities. The deviations from the regression line are correlated with the tidal mass-to-luminosity relations by Faber and Lin. The simplest interpretation of this is that the nearest dwarf spheroidal satellites of the Galaxy are actually not tidally limited but tidally expanded. If one treats the diameters of these expanded galaxies as normal, he gets the fictitious abnormally large tidal masses and mass-to-luminosity ratios.

The data about the satellite  $r_t/r_c$  ratios and ellipticities  $e. = 1 - b/a$  are also in accordance with the tidal disruption scenario. The outer regions of satellite galaxies are more strongly influenced by tidal forces and must expand more intensively in comparison with inner regions, which define the core radius  $r_c$ . Indeed, on Fig. 5 such an effect is clearly seen. The probable expanded satellites of the Galaxy have larger  $r_t/r_c$  ratios in comparison with satellites outside the strong tidal field.

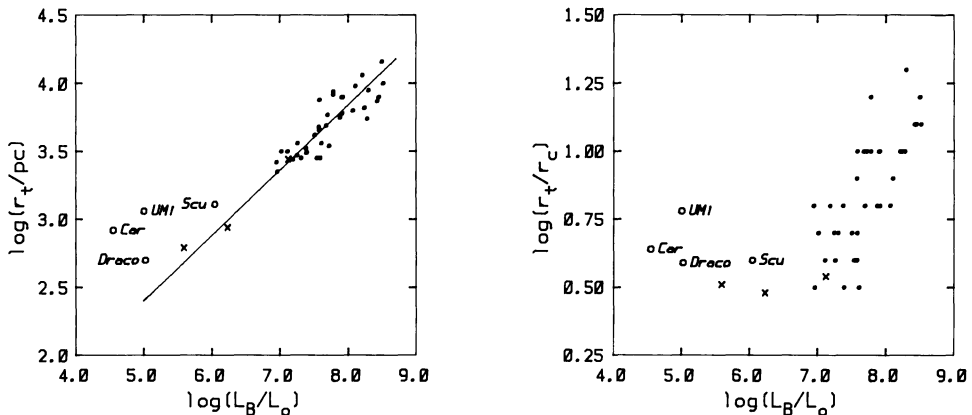


Figure 4.(left). Correlation of the tidal radius  $r_t$  with the absolute blue luminosity  $L_B$ . Dots stand for the Virgo cluster dwarf spheroidal galaxies, crosses and open circles represent resolved dwarf spheroidal satellites of the Galaxy. Open circles are nearby ( $R < 100$  kpc) satellites with high tidal mass-to-luminosity ratios ( $M/L_V > 6.8 M_\odot/L_\odot$ ), crosses are more distant satellites with  $M/L_V < 1.0 M_\odot/L_\odot$  from Faber and Lin (1983). The straight line is a mean regression for Virgo dwarfs.

Figure 5.(right). The ratio of the tidal radius  $r_t$  to the core radius  $r_c$  as a function of the absolute blue luminosity  $L_B$ . Coding is the same as in Fig. 4.

As the tidal expansion takes place mainly along the orbital plane, it is expected that tidally expanded galaxies have obtained additional ellipticities. So another support to the tidal disruption scenario comes from the mean ellipticities, which are  $\langle e \rangle = 0.22 \pm 0.09$  in case of three distant probably undisturbed satellites and  $0.38 \pm 0.05$  in case of four nearby satellites (Joeveer 1985b).

Summarizing, it is highly probable that the four nearest dwarf spheroidal satellites of our Galaxy (Sculptor, Draco, UMi and Carina) are strongly disturbed by tidal forces and the estimation of virial and tidal masses of these galaxies is impossible. It seems that only outer satellites can give us more or less reliable information about the masses of dwarf spheroidal galaxies. Of course, the result for the Fornax galaxy by Cohen (1983) leaves little hope to find a significant amount of dark matter in galaxies with masses  $< 10^8 M_{\odot}$ .

#### 4. DECOMPOSITION OF GALAXIES INTO VISIBLE AND DARK POPULATIONS

In a series of papers Einasto (1974 and references therein) developed a method to decompose galaxies into visible and dark populations. The method is based on the assumptions that physically homogeneous galactic populations can be described by ellipsoidal models and that mass-to-luminosity ratios within the populations are constant. The decomposition can be made by comparing photometric and dynamical data (velocity dispersion, rotation). Photometric data describe visible components, whereas dynamical data depend on both components (Haud 1985).

Models are available for a number of nearby galaxies. Table 2 lists the galaxies and calculated mass-to-luminosity ratios for visible populations. We see that for all galaxies studied so far, this ratio is rather small, between 2 and 7 in blue light. As a mean value we can adopt  $(M/L_B)_{gal} = 4.3$ .

TABLE 2. VISIBLE POPULATIONS IN GALAXIES

Name	Type	$M_B$	Distance (Mpc)	$M/L_B$	Reference
M 32	E2	-16.0	0.69	1.95	Einasto, Tenjes, Traat, 1980
M 87	EO-1	-22.2	20.	5.2	Tenjes, Einasto, Oleak, 1985
NGC 3115	SO	-20.0	10.	3.0	Tenjes, 1985
M 104	Sa	-22.3	20.	2.05	Tenjes, 1985
M 81	Sab	-20.2	3.3	7.0	Einasto, Tenjes, Zasov, Barabanov, 1980
M 31	Sb	-20.8	0.69	6.2	Einasto, Tenjes, Zasov, Barabanov, 1980
NGC 4565	Sb	-21.7	10.	5.1	Tenjes, 1985
NGC 4321	Sbc	-20.7	20.	2.0	Tenjes, 1985

## 5. LOCAL GROUP

The Local Group of galaxies presents an unique case where the total mass of the system can be derived from the internal dynamics of both sub-systems as well as from the external dynamics by calculating the mutual orbit of its main concentration centres, M31 and the Galaxy.

The external method to calculate the mass of the Local system was first applied by Kahn and Woltjer (1959), followed by Gunn (1974), Lynden-Bell and Lin (1977) and others. A basic uncertainty of the method lies in the value of the circular velocity of the Galaxy at the Sun which must be used to correct the observed motion of Andromeda to the Galactic centre. Another uncertainty is the determination of the motion of our Galaxy in respect to the barycentre of the Local Group.

Einasto and Lynden-Bell (1982) redetermined the orbit of the Galaxy in the Local Group. They used recent determinations of the circular velocity of the Sun, which yield  $V_0 = 220$  km/s. To calculate the motion of the Galaxy in the Group only independent members of the Local Group were used, i.e. galaxies not located in one of its main concentration centres around Galaxy and Andromeda. The total mass of the Local Group depends on the age adopted. Adopting on the basis of nucleocosmochronology and globular cluster ages an interval of 13-18 billion years, they found for the mass  $M_{LG} = 5 \pm 1 \times 10^{12} M_{\odot}$ .

The masses of both subgroups in the Local Group can be derived individually from their internal kinematics. The level of the constant circular velocity in the Galaxy and Andromeda is well determined. The extent of the dark corona can be estimated from the extent of the respective satellite system. For the Andromeda subgroup Einasto and Lynden-Bell found the mass  $2.3 \times 10^{12} M_{\odot}$ . For our subgroup much more data are available, in particular the high velocity clouds of neutral hydrogen and Magellanic Stream yield valuable information, as well as distant globular clusters. Using all these data Einasto with collaborators (Einasto et al. 1975, 1976, 1978) found values between  $1.2-2.0 \times 10^{12} M_{\odot}$ .

We come to the conclusion that both internal and external data yield for the Local Group and its both subgroups masses in good mutual agreement. This suggests that the most uncertain element in the chain of the mass calculations, the radius of the dark corona of the Galaxy and Andromeda, cannot be considerably in error.

Adopting the above total mass and the total luminosity from Vennik (1985) one has for the total mass-to-light ratio of the Local Group  $M/L_B = 90$  and  $M_{tot}/M_{lum} = 20$ .

## 6. DOUBLE GALAXIES

Karachentsev (1980, 1981a, b, c, d) has observed and analyzed the majority of double galaxies from his list of relatively isolated pairs (Karachentsev 1972). At the time of the analysis redshifts were available for 440 pairs. Of this sample 17 galaxies were singles (a star was included as a galaxy in the catalog), and 59 were considered as optical pairs on the basis of the calculated mass-to-luminosity ratios. Karachentsev adopted a rather strict criterion  $M/L < 100$  in solar units



for physical pairs (photographic magnitudes and the Hubble constant  $H = 75 \text{ km/s/Mpc}$  were used). For mean mass-to-luminosity ratio Karachentsev obtained  $M/L_B = 10.8$ , and considered this low value as an indication against the presence of dark matter around galaxies.

This results seems to contradict other evidence and needs some comments. A weak point in calculating the mean mass-to-luminosity ratio is the exclusion of all pairs with  $M/L > 100$ . The velocity difference distribution shown in Fig. 6 demonstrates that pairs with  $M/L > 100$  and with the velocity difference  $\Delta V < 1000 \text{ km/s}$  form a natural tail of the distribution. The study of spatial distribution of galaxies has shown (Joeveer, Einasto, Tago 1978, Einasto et al. 1980, 1984, Tago et al. 1984) that practically all galaxies are located in systems which form thin filaments separated by huge empty voids. Thus optical pairs are usually separated by  $\Delta V \gg 1000 \text{ km/s}$ . Only in cases when we look along a filament, smaller velocity differences in optical pairs are expected. In this case other galaxies at the same Hubble velocity should be observed, since galaxies are not isolated.

To have a rough estimate of this selection effect we calculated the mean  $M/L$  for the sample with  $M/L > 100$  but  $\Delta V < 800 \text{ km/s}$ , there were 24 pairs in this additional sample. For the combined sample we have  $M/L_B = 23.7$  for  $H = 75 \text{ km/s/Mpc}$  or  $M/L_B = 16$  for  $H = 50 \text{ km/s/Mpc}$ . The mean separation of pairs is 60 kpc ( $H = 50$ ) and the  $M/L_B$  quoted refers to this distance.

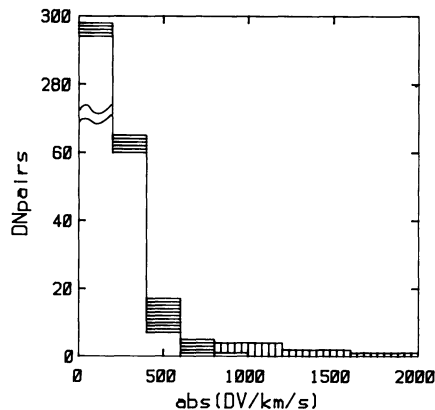


Figure 6. Distribution of velocity differences for the Karachentsev sample of double galaxies. Open areas - physical pairs by Karachentsev, horizontal shading - pairs added in present paper, vertical shading - pairs considered optical both by us and Karachentsev.

## 7. GROUPS OF GALAXIES

Recently Vennik (1984) compiled a new group catalog. Only galaxies in the Northern Hemisphere far from the galactic belt ( $b > 30^\circ$ ) were considered for group membership, and the velocity limit  $V < 3200 \text{ km/s}$  was used, which corresponds to the limit of the Fisher-Tully (1981) HI survey of nearby dwarf galaxies. The majority of galaxies in the sample

not observed by Fisher and Tully have new accurate redshifts from the CfA optical study. Groups were selected using a hierarchical clustering algorithm (Materne 1978, Tully 1980). The density level at which a group was selected was taken as a function of the mean spatial density of galaxies in a particular field. Using this adaptive cut-off it was possible to separate density enhancements in large clouds of galaxies which in earlier studies were considered as single groups.

The catalog has been studied for dynamics of groups by Vennik (1985). Groups were combined on the basis of similarity of morphological types of main galaxies. The dynamics of groups is determined by the dark component, thus all galaxies were considered as test particles and entered into dynamical calculations with equal weights. The extent of synthesized groups has been estimated from the distribution of number density of all member galaxies.

TABLE 3. MASS-TO-LUMINOSITY RATIOS IN SYSTEMS OF GALAXIES

	R Mpc	M/L <sub>B</sub>	M/M <sub>lum</sub>	Reference
Galaxies	0.02	4	1	Table 2
Double gal.	0.06	16	4	Karachentsev 1980, 1981
Local Group	0.7	90	20	Einasto, Lynden-Bell 1982
Groups Sc-Irr	1.5	37	9	Vennik 1985
Sa-Sbc	1.5	61	14	Vennik 1985
E-So/a	1.5	95	20	Vennik 1985
Clusters	3	150	20:	Faber, Gallagher 1979
Closure		460	100	Felten 1985

The results obtained demonstrate that mass-to-luminosity ratio in groups is slightly lower than thought earlier. The difference is caused by a more strict definition of groups and more accurate velocities. However, the principal conclusion is the same: M/L in groups exceeds respective values in visible populations of galaxies by a factor of 10-20. It is also important to note that the mass-to-luminosity ratio decreases considerably with increasing de Vaucouleurs morphological type of main galaxies of groups.

## 8. LARGE-SCALE DISTRIBUTION OF DARK MATTER

In Fig.7 we plot mass-to-luminosity ratios of various systems as a function of the extent of the system. Data on clusters of galaxies were taken from Faber and Gallagher (1979), data corresponding to the critical cosmological density from Felten (1985). All data are given in the B system for  $H = 50$  km/s/Mpc.

Practically all galaxies are located in systems: groups, clouds, clusters and superclusters. The percentage of galaxies situated in rich clusters is rather small. The most common form of galaxy environment is a group, a cloud or a loose cluster. If one adopts for these systems

mass-to-luminosity ratios given above one can argue that the majority of galaxies are located in systems which comprise about 20 per cent of the cosmological critical density. All visible systems of galaxies fill a volume of several per cent of the total volume of space, the rest being void of visible galaxies (Einasto et al. 1980).

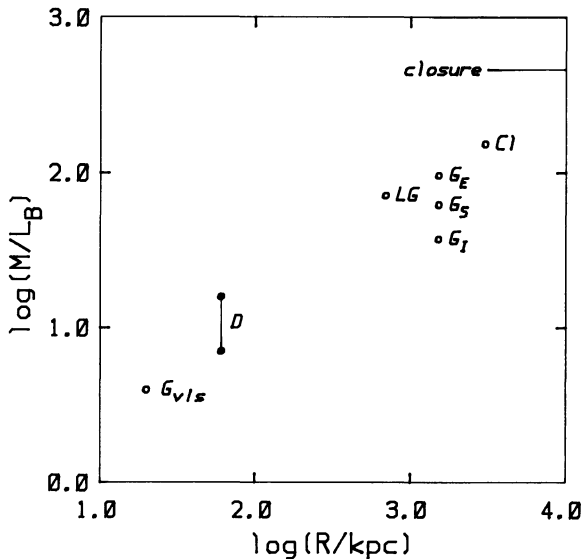


Figure 7. Mass-to-luminosity ratios for systems of different scale  $R$ . For double galaxies a range from the Karachentsev original determination to our present estimate is shown.  $G_{\text{vis}}$  denotes the normal mass-to-luminosity ratio of stars,  $D$  - double galaxies,  $LG$  - the Local Group,  $G_E$ ,  $G_S$  and  $G_I$  - groups with the main galaxy of elliptical, spiral or irregular morphology, respectively, and  $Cl$  stands for rich clusters of galaxies.

Voids cannot be entirely empty. The smoothness of the microwave radiation background demonstrates that at the epoch of decoupling of matter from radiation the density distribution was rather uniform. The only force which can evacuate voids is gravitation. Calculations demonstrate that the available cosmological time is insufficient to evacuate voids completely, there should be primeval matter there, both the primordial gas and the dark matter (Zeldovich, Einasto, Shandarin 1982).

Astronomical determinations of the total density of matter in the Universe are rather uncertain. But if one accepts the inflationary scenario of the creation of the observed Universe, its total density should be equal to the critical density. In this case about 80 per cent of matter should be located in voids.

Physical properties of gas in cosmic voids have been studied by Ozernoy and his coworkers (Ozernoy and Chernomordik 1985, Chernomordik and Ozernoy 1983 and references therein), using UV spectral data by Brosh and Gondhalekar (1984).

## 9. STRUCTURE OF THE UNIVERSE: CORRELATION FUNCTION

In order to test various scenarios of the formation of the structure of the Universe a number of quantitative tests have been proposed, such as the correlation analysis, cluster analysis, etc. By far the most popular method used is the study of the correlation function.

The correlation function,  $\xi(r)$ , is defined as the excess of the observed number of pairs of galaxies at a given distance,  $N_0(r)$ , over the respective number of pairs in a random catalog,  $N_p(r)$ ,

$$1 + \xi(r) = N_0(r)/N_p(r). \quad (9)$$

All studies carried out so far indicate that the correlation function at small distances can be represented by a power law

$$\xi(r) = (r/r_0)^{-\gamma}, \quad (10)$$

where  $r_0$  is the correlation length. For galaxies  $r_0 = 5 \text{ h}^{-1}\text{Mpc}$  (Peebles 1980 and references therein), whereas for clusters it is  $25 \text{ h}^{-1}\text{Mpc}$  (Klypin and Kopylov 1983, Bahcall and Soneira 1983).

To clarify the reason of this discrepancy Einasto, Klypin and Saar (1985) determined the correlation function for a number of observational samples of galaxies and clusters of galaxies of various depth using Huchra's (1983) compilation of redshifts which includes the CfA survey. To avoid incompleteness of data, conical samples were taken with boundaries used in the CfA redshift survey. Samples were cut off at certain  $V_0$  and an absolute magnitude  $M_0$  corresponding to the apparent magnitude limit of the CfA survey, 14.5, at the limiting redshift  $V_0$ . Redshift limits from 1000 to 10000 km/s were used. The use of the same absolute magnitude limit over the whole particular sample makes samples homogeneous, and no magnitude dependent weighting is necessary.

The correlation functions found for the northern galactic hemisphere are plotted in Fig. 8. One can see that with increasing sample depth the curves are shifted up and right. Thus respective correlation length also grows with the sample depth (Fig. 9). For samples containing only a part of a supercluster, the length  $r_0$  is much smaller than the conventional value,  $5 \text{ h}^{-1}\text{Mpc}$ ; for samples containing a whole supercluster, the length is approximately equal to the conventional value, and for samples containing several superclusters, the correlation length is about twice the conventional value. This result is not due to differences in absolute magnitude cutoff or mean spatial density of objects, as suggested by Szalay and Schramm (1985). Samples picked up from the same space with different absolute magnitude limit and mean density have practically identical correlation functions.

Einasto, Klypin and Saar determined also the correlation function for theoretical samples, calculated from N-body experiments. Adiabatic and isothermal scenarios were used, both either with all test particles included, or with particles only from high-density regions included (biased galaxy formation). In one case, the adiabatic scenario with biased galaxy formation, theoretical samples behave as observational ones: the correlation length increases with sample volume (Fig. 10). This

model shares one important feature with observations - it contains large empty regions between systems of test particles. In all other cases either the regions between clusters are filled with a rarefied population of particles (both scenarios, all test particles included) or empty regions are rather small, of size of systems of particles (I-scenario with biased galaxy formation).

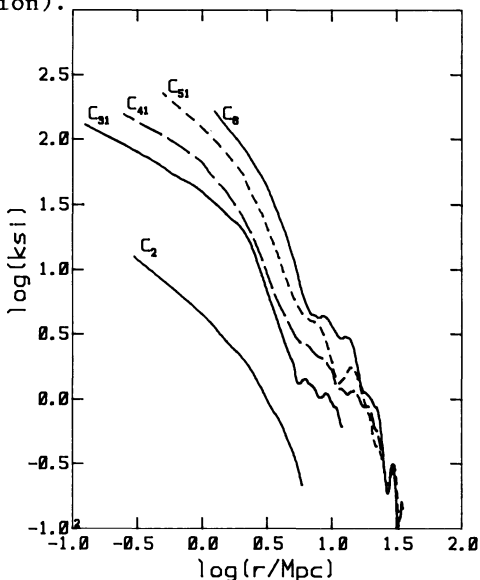


Figure 8. Correlation functions for samples in the Northern Galactic Hemisphere in the direction of the Coma supercluster. Samples C<sub>2</sub>, C<sub>31</sub>, C<sub>41</sub>, C<sub>51</sub> and C<sub>6</sub> have limiting redshifts 1200, 2000, 4000, 8000 and 10000 km/s, respectively.

This test indicates that the volume dependence of the correlation length is crucially dependent on the presence of large empty voids. Actually, the dependence is due to differences in the normalizing factor,  $N_p(r)$ , of formula (9). For small radii this factor is proportional to the relative volume of a spherical shell of radius  $r$

$$N_p(r) = 2\pi r^2 dr N^2 / V, \tag{11}$$

where  $N$  is the total number of particles in the Poisson sample and  $V$  is the volume of the sample. Let us compare two identical samples of galaxies, surrounded by empty regions of different volume,  $V_1$  and  $V_2$ .  $N_0(r)$  in (9) is identical for both samples, but  $N_p(r)$  is not, due to differences in volume  $V_i$ . Let  $V_0$  be the volume filled with systems of galaxies,  $C_i = V_0/V_i$  - the filling factor of the sample, and we get for correlation functions of our two samples

$$1 + \xi_1(r) = C_2/C_1 (1 + \xi_2(r)). \tag{12}$$

Within superclusters the filling factor is about 0.1 (Einasto et al. 1984), but in large volumes it is only about 0.01, thus the difference

in filling factors explains the difference in correlation functions. Since the correlation length is defined as the value of the argument at which  $\xi(r) = 1$ , it also increases with decreasing filling factor.

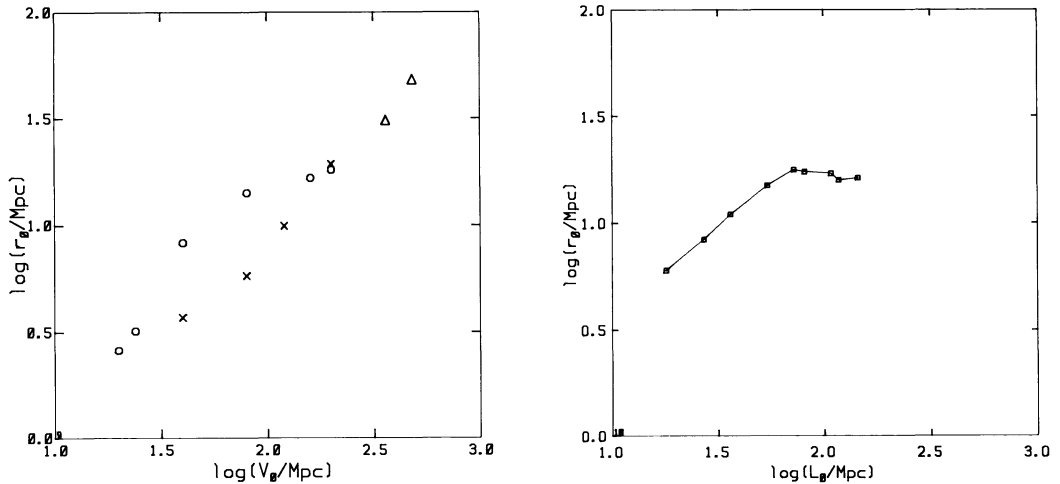


Figure 9.(left). Plot of the correlation length,  $r_0$ , versus limiting redshift,  $V_0$ , in Megaparsecs ( $H = 50 \text{ km/s/Mpc}$ ). Open circles and crosses designate samples in the Northern and Southern Galactic Hemisphere, respectively, triangles - Abell clusters of galaxies.

Figure 10.(right). The correlation length versus cube size plot for theoretical samples (A-scenario, biased galaxy formation: points from low density regions removed). For comparison with Fig. 9  $r_0$  and  $L$  are given in Megaparsecs.

Model calculations demonstrate that for samples exceeding in volume the characteristic volume of a void between superclusters, the correlation length reaches its global value (see Fig. 10). On the other hand, observed samples show no trend to converge at large sample sizes to a constant correlation length (see Fig. 9). Even clusters of galaxies sampling much larger distances than galaxies continue the same  $r_0$  versus  $V_0$  trend. Thus we come to the conclusion that presently available samples are not deep enough to derive the global value of the correlation length. The conventional value of the correlation length is definitely too low, available deepest galaxy samples indicate a value  $r_0 = 10 h^{-1}\text{Mpc}$ .

## 10. STRUCTURE OF THE UNIVERSE: MORPHOLOGY OF GALAXIES

The morphology of galaxies enters as an important parameter of the formation and evolution of galaxies. In earlier studies Dressler (1980) and Postman and Geller (1984), among others, studied the morphology of clusters and groups of galaxies of various mean densities. A pronounced

variation of the morphology with density was found. This dependence is most probably due to evolutionary effects. Galaxies in low-density regions, in particular, isolated galaxies, do not change their morphology. For this reason the study of isolated galaxies is of great interest for theories of galaxy formation.

Einasto and Einasto (1985) used CfA redshift survey, supplemented with other available data, to study morphology of galaxies both in systems of galaxies as well as for isolated galaxies. Galaxies were attributed to systems of various richness (isolated galaxies, small systems, large systems) using cluster analysis. The neighborhood radius at which galaxies were included into various systems was varied by a factor of ten, which corresponds to a variation of three orders of magnitude in density enhancement. Three areas on the sky were studied which correspond to the Local Supercluster, and to the Coma and Perseus superclusters. Fig. 11 plots the number of galaxies in systems of different richness as a function of the neighborhood radius for the Local supercluster, Fig. 12 gives the relative distribution of morphological types. Other superclusters have similar distributions.

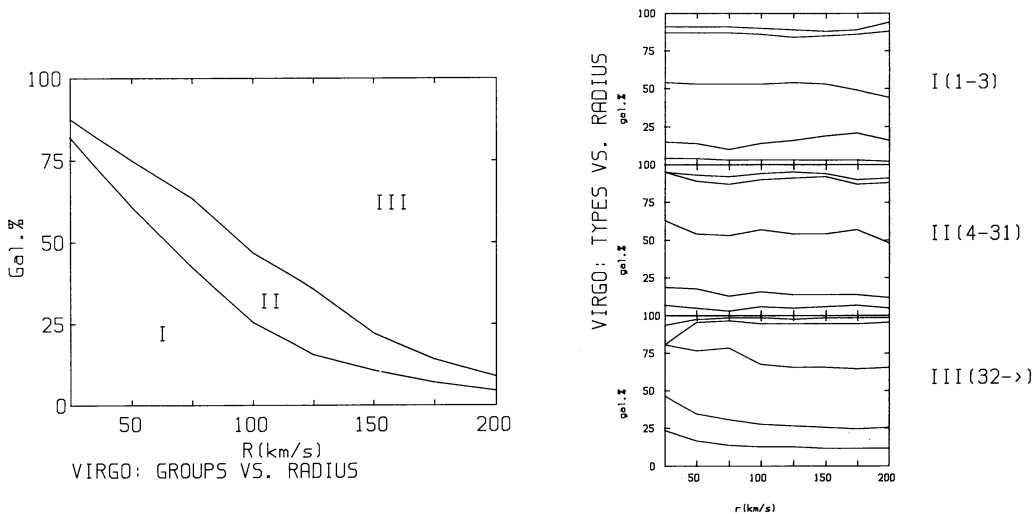


Figure 11.(left). Plot of relative number of members of systems of galaxies of different richness as a function of neighborhood radius  $R$  in the Virgo supercluster. Isolated galaxies and small systems (number of galaxies in the system  $n \leq 3$ ) are designated as field I, intermediate systems ( $4 \leq n \leq 31$ ) as II, and large systems ( $n \geq 32$ ) as III. Neighborhood radius is expressed in Hubble expansion velocity units.

Figure 12.(right). Plot of relative numbers of galaxies of various morphological types in the Virgo supercluster as a function of neighborhood radius. Systems of various richness are plotted in panels I, II, and III. In all panels six areas are given, representing (from bottom to top) the following morphological types: E, S0, Sa - Sd, Irr, Pec, unclassified.

As expected (Dressler 1980, Postman and Geller 1984) in systems of galaxies (groups and clusters) morphology changes with neighborhood radius (i.e. density). However for isolated galaxies the morphological distribution is remarkably stable and has little, if any, change with the degree of isolation. This distribution should reflect the formation conditions of galaxies.

## 11. CONCLUSIONS

The principal results of the studies reviewed can be summarized as follows:

1) There is yet no firm evidence for the presence of large amounts of invisible matter in the solar vicinity and in dwarf spheroidal galaxies.

2) The total amount of matter in visible galaxies, around galaxies and clusters, and smoothly distributed in voids relates as 1:20:80.

3) The correlation function of galaxies strongly depends on the presence of large voids in the sample. In largest observed samples the correlation length is  $10 h^{-1} \text{Mpc}$ , about twice the conventional value.

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## DISCUSSION

PEEBLES: The deep galaxy surveys in narrow fields, the Durham and Edinburgh deep angular distributions, the Jagellonian field and Lick sample, and the redshift sample of Kirshner, Oemler, Schechter and Shectman all indicate  $r_0 \sim 5h^{-1}$  Mpc, consistent with the shallow CfA result. How are these results to be reconciled with your proposal that the effective value of  $r_0$  increases with increasing depth? Are you proposing that the galaxy distribution is not a stationary random process?

EINASTO: The correlation length determined from a particular sample is inversely proportional to the filling factor of this sample. The deep samples you mentioned cover small areas on the sky; thus voids in these samples are essentially one-dimensional, and the filling factor is relatively large. It is possible that this effect is responsible for the disagreement quoted. We have used large areas on the sky; thus voids enter as three-dimensional objects and suppress the filling factor. In two-dimensional distributions (the Lick sample and Jagellonian field counts), information on the presence of voids is incomplete. To transform two-dimensional correlation functions into three-dimensional ones, additional information on voids must be used.

Our data indeed indicate that on scales smaller than the characteristic diameter of voids, the galaxy distribution is not a stationary random process. Available galaxy samples have depth smaller than this characteristic diameter.

DEKEL: Like Peebles, I am worried about the growth of  $\xi(r)$  with the depth of the sample. This contradicts all previous results. In particular, I believe that the sample you use is incomplete beyond  $10,000 \text{ km s}^{-1}$ , which may be the reason for the effect you find. Could you elaborate on how you weight the galaxies at different redshifts, and how you deal with the boundaries of the volume sampled?

EINASTO: The growth of  $\xi(r)$  with sample depth is clearly seen in the whole interval of depths of subsamples. Thus the result is not dependent on the deepest samples. The absolute magnitude cutoff for each subsample of different depth is taken equal to the absolute magnitude corresponding to the CfA apparent magnitude limit at the cutoff redshift of the subsample. Thus the absolute magnitude cutoff within all subsamples is redshift independent and no weighting of

galaxies at different redshifts is needed. Sample volume boundaries have been taken into account in generating the Monte-Carlo catalogue of test particles used in the calculation of the respective number of pairs.

DRESSLER: Although my study of galaxy morphology vs. environmental density did not extend to regions as sparse as you discussed, Postman and Geller used the CfA redshift survey to show that a correlation persisted down to very weak enhancements. The work you described showed no such effect, although the parameterization seemed somewhat different. Is there a contradiction between these two studies and, if so, have you any explanations?

CHINCARINI: In support of Dressler's question, I refer to work on Perseus-Pisces by Giovanelli, Haynes and Chincarini (preprint). We find, in agreement with Postman and Geller, that the autocorrelation function depends on morphological type. We also find a strong dependence between type percentage and density. The same density dependence is detected in a catalogue complete to 14.5 mag by DeSouza, Vettolani and Chincarini (preprint). We find that the relation between type and density depends also on the luminosity function of each type.

EINASTO: Our results are consistent with the other studies mentioned for high-density systems (clusters and groups). In low-density regions our results seem to be in contradiction with others. One possible reason may be the method of analysis used. The correlation technique used in some of the quoted studies is volume dependent whereas the clustering method is not. A more detailed comparison of both sets of results is needed to clarify the situation.

J. BAHCALL: You have raised the interesting question of why the Soviet studies of the local missing mass have given a different answer than those made in the West. I believe that the primary reason for this discrepancy is the incorrect assumption by Kuzmin that the gravitational potential in the  $z$  direction is quadratic. If one wants to limit the error in the potential caused by this approximation to less than 10%, then one must use stars at no more than 18 pc ( $\sigma/4$  km s<sup>-1</sup>) above the plane. No population that I know about satisfies this condition. In addition, some of the samples you mentioned are not sufficiently pure or homogeneous to use in this context (e.g., bright stars or B stars).

EINASTO: The population used in Kuzmin's study and in later work has a  $V_z$  dispersion of about 7 km s<sup>-1</sup> and a  $z$  coordinate dispersion of  $\sim 100$  pc. In this  $z$  interval the inaccuracy of the quadratic assumption can hardly explain the factor of two difference in results. I agree that Kuzmin's original sample was inhomogeneous, but in recent work modern data have been used, so the inhomogeneity, if present, is small. Finally, a completely independent method which uses the ages of stars also supports Kuzmin's original results. Presently it is difficult to see where the weak points of density determinations are. So I come to the conclusion that further detailed work is needed to find a better value of the local density.