Bull. London Math. Soc. 36 (2004) 841–853
DOI: 10.1112/S0024609304003662

OBITUARY

FRANK SMITHIES 1912–2002

Frank Smithies was born in Edinburgh, on 10 March 1912. His father, also Frank Smithies, and his mother, Mary (née Blakemore) had met through their involvement in the socialist movement. They had two children, Frank and his sister Violet, some four years younger.

Frank Smithies (senior) had been born in West Yorkshire in 1880, but his family moved to London shortly thereafter. He was apprenticed as a mechanical engineer, worked for a time on the construction of part of what is now the Central Line, and moved to Edinburgh about 1907. He was a self-educated man, a convinced secularist, and an active participant in socialist and trade union matters. Mary had been born in West Lothian in 1878; her parents were in domestic service, as butler and housekeeper, and Mary herself trained as a cook, after leaving school at about fourteen. Like Frank (senior), she had socialist leanings. They met through their membership of the Clarionettes, a socialist cycling group of the Edwardian era, associated with the weekly newspaper, *The Clarion*.

Edinburgh: 1912–1931

At an early stage, partly through the influence of his father, Frank (junior) developed a wide range of interests, for example in botany and in geography (the latter leading to a lifelong passion for maps). By the time he started school in 1917, he was already reading. With a mental age far ahead of his actual years, however, he was rather a misfit in school; he did not easily get on with other children, and matters were not helped by frequent, and at times prolonged, absences due to poor health. The situation was much improved in 1921, when he joined a small 'school' (having never more than six pupils) organised on very informal lines by W. H. Roberts, a retired wool merchant with a passionate enthusiasm for teaching. Roberts was interested in Frank's case – he liked the challenge of teaching boys who did not fit easily into the conventional school system – and he took him on without charging any fee. This was just as well, for Frank's parents could not have afforded to pay one; it was around this time that Frank's father was black-listed by the engineering firms, because of his trade union activities, and had subsequently to undertake many and varied forms of work in order to support the family finances.

Frank flourished under Roberts's tutelage; with a regime involving a good deal of interesting outdoor activity, there was a marked improvement in his health, and he learned much about the topography and history of the Old Town in Edinburgh. At times, the teaching was centred outside the city, with periods based in Galashiels and Port Seton. By 1926, Frank was judged to be almost ready for the Edinburgh University entrance examination; after a couple of terms in a cramming establishment (arranged by Roberts), he passed the examination in March 1927, and entered the university in October of that year.

The four-year undergraduate course at Edinburgh, leading to an honours degree in Mathematics and Natural Philosophy, covered pure and applied mathematics and physics (including practical work for two years). There was also a requirement to study two additional subjects, for one year each; Smithies chose French for the first year, Philosophy for the second. Financial pressures were eased when he was awarded the Spence Bursary (open to first-year mathematics students from Edinburgh or St Andrews) in 1928. Courses that he found of particular interest were Theory of Knowledge (Kemp Smith), History of Mathematics (E. T. Whittaker) and Partial Differential Equations (W. H. McCrea). Through attending lectures on applied mathematics, he learned of Courant and Hilbert's book, *Methoden der mathematischen Physik*, which he started reading in his final year after taking a course on scientific German. This lead him to a detailed study of Lovitt's *Linear integral equations*. In his final examination in 1931, he was first in his year, and was awarded the Napier medal, the Gadgil Prize, and a John Edward Baxter scholarship, available for two further years of study.

Cambridge and Princeton: 1931–1940

It was traditional at that time for the best students completing the Edinburgh mathematics course to continue with two years as affiliated students in Cambridge. Smithies had already taken the entrance scholarship examination, and had been awarded a scholarship at St John's College in 1930. He went up to St John's in October 1931, and was highly placed in the Preliminary Examination the next year; he took Part II of the Mathematical Tripos in 1933, emerging as a Wrangler and with a distinction in Schedule B (equivalent to the later Part III). Courses that he found of particular interest were on Fourier analysis (G. H. Hardy), mechanics (E. Cunningham) and integral equations (J. M. Whittaker). A talk by S. W. P. Steen, at the college mathematical society, aroused his interest in functional analysis.

During the academic years 1933–1936, Smithies worked on integral equations, under the guidance of G. H. Hardy, whom he found to be a superb research supervisor. He retained an interest in functional analysis, although this did not directly impinge on his work at that time. He was encouraged by Hardy (despite the latter's well-known aversion to the growth of abstraction in mathematics) to read the (then) recently published books by S. Banach on linear operators and M. H. Stone on Hilbert space theory. He attended lectures by R. Courant, who was visiting Cambridge during the year 1933–1934, and consulted him about possible problems on integral equations; this led to some work, concerning expansions in eigenfunctions and singular functions, that formed part of his PhD thesis. In 1935, he attended lectures by another visitor to Cambridge, J. von Neumann, and formed the ambition of working with him in Princeton. He was awarded a Rayleigh Prize in 1935, for an essay on differential equations of fractional order. His PhD thesis, submitted in 1936, was approved after a very informal oral examination, conducted by Hardy and J. M. Whittaker in the grounds of Trinity College.

Throughout his student days in Cambridge, Smithies found time for broader interests, for example in philosophy, in student politics (left of centre, but not the far left), and in exploration on foot of the surrounding countryside. Financial circumstances were still constrained; he was dependent on his scholarship income, and his father was unemployed for much of the time. Among those he met during this period was M. V. Wilkes, later to be best man at his wedding, and a lifelong friend.

Supported by a Carnegie Fellowship and a studentship from St John's College, Smithies spent the academic year 1936–1937 at the Institute for Advanced Study in Princeton; he was able to remain there for a further year, following his election (at the same time as R. A. Lyttleton and A. C. Offord) to a Research Fellowship at St John's. In the stimulating atmosphere at Fine Hall, among mathematicians from both the Institute and Princeton University, he continued his research on integral equations, published two notes on completely continuous (in today's terminology, 'compact') linear operators, and did some work with von Neumann on the relation between continuous geometries and rings of operators (the latter subsequently renamed 'von Neumann algebras'). Collaboration with R. P. Boas led to the publication of an article on Fourier transforms and a (less serious, but probably more widely read) paper on mathematical methods of big game hunting. Boas became a lifelong friend.

Upon returning to St John's College in 1938, Smithies undertook some college teaching and lecturing,along with his continuing research. By 1940, although it would probably not have seemed likely at that time, his most substantial original work in mathematics (described in the next section) was already completed. In the Easter term of that year, however, he gave a course of lectures on Banach spaces, thus beginning an activity that would later lead on to perhaps his greatest mathematical achievement: the fostering of functional analysis, as a subject for study and research, within the British mathematical community. That, however, was still in the future; against the background of World War II, academic life had to be put aside for a few years.

Mathematical work: integral equations and operator theory

For Smithies, the years 1933–1940, two spent in Princeton and the rest in Cambridge, provided a fertile period for mathematical research. The most substantial papers [**1, 2, 6, 7**] are all concerned with the theory of linear integral equations. The joint paper [**5**], with R. P. Boas, deals with the relationship between local properties (in particular, the existence of points of non-analyticity) of a distribution function and the order of magnitude at infinity of its Fourier transform. In [**1, 2, 5, 6**], both the methods used and the results obtained lie squarely within the framework of classical analysis. The two brief notes [**3**] and [**4**], however, deal with more abstract, operator-theoretic, situations. In [**7**], the proofs are set out in terms of Hilbert space operator theory, and the results are initially expressed in that context, with subsequent application to integral equations.

The main purpose of [**1**] is to extend the theory of inhomogeneous linear integral equations

$$\varphi(x) = f(x) + \lambda \int_a^x K(x,t)\varphi(t)\,dt \qquad (a \leqslant t \leqslant b) \tag{1}$$

of Volterra type, to the case in which the kernel is Lebesgue measurable and satisfies a condition

$$\int_a^b dx \left\{ \int_a^x |K(x,t)|^p \,dt \right\}^{q/p} < \infty,$$

where $1 < p < \infty$ and q is the conjugate index. The given function f, and the unknown function φ to be found, are both required to be of class $L^q(a,b)$. With $f_0, f_1, f_2, \ldots$ defined by

$$f_0 = f, \qquad f_{n+1}(x) = \int_a^x K(x,t) f_n(t)\,dt \quad (a \leqslant x \leqslant b),$$

repeated application of Hölder's inequality is used to obtain estimates for f_n, essentially that

$$|f_n(x)| = O\big([(n-1)!]^{-1/q}\big).$$

These are used to show that the Neumann series

$$f_0 + \lambda f_1 + \lambda^2 f_2 + \ldots$$

converges (pointwise almost everywhere, and in mean of order q), for all values of the parameter λ, and that its sum is the unique solution φ of (1). With the usual interpretations, the results remain valid with $p = \infty$, but not (without restriction on the magnitude of $|\lambda|$) in the case $p = 1$. There is also a brief discussion of Fredholm integral equations with kernels satisfying conditions analogous to those set out above.

In [**2**], Smithies considers linear integral equations in which the kernel $K(s,t)$ is real-valued and of class L^2 on a square $a \leqslant s,\ t \leqslant b$. For symmetric kernels, his focus is on *eigenvalues* (necessarily real) and the corresponding (normalised) *eigenfunctions*; but note that λ is said to be an eigenvalue in [**2**] when λ^{-1} is an eigenvalue in current parlance. For general kernels, which may have no eigenvalues, the interest is in *singular values* λ (greater than 0) and adjoint pairs (φ, ψ) of (normalised) *singular functions* corresponding to λ; here, the requirements

https://doi.org/10.1112/S0024609304003662 Published online by Cambridge University Press

on λ, φ and ψ are that

$$\begin{cases} \varphi(s) = \lambda \int_a^b K(s,t)\psi(t)\,dt; \\ \psi(s) = \lambda \int_a^b K(t,s)\varphi(t)\,dt; \\ \int_a^b |\varphi(s)|^2\,ds = \int_a^b |\psi(s)|^2\,ds = 1. \end{cases} \tag{2}$$

In the first part of [**2**] , Smithies shows that the classical theory of singular values and singular functions, first developed by E. Schmidt under more restrictive conditions, remains valid for square-summable kernels. His main concern, however, is to give estimates for the order of magnitude of λ_n, where $\{\lambda_1, \lambda_2, \lambda_3, \ldots\}$ is the sequence of all singular values (in the symmetric case, eigenvalues), arranged in order of increasing magnitude and counted according to their multiplicities. Under the assumption that the kernel satisfies a generalised Lipschitz condition

$$\int_a^b dt \left\{ \int_a^b |K(s+\theta,t) - K(s-\theta,t)|^p ds \right\}^{2/p} \leqslant A\theta^\alpha, \tag{3}$$

where A is a constant, $1 < p \leqslant 2$ and $\alpha > p^{-1} - \frac{1}{2}$, it is shown that $|\lambda_n|^{-1} = O(n^{-\beta})$, where $\beta = \alpha + 1 - p^{-1}$; moreover,

$$K(s,t) = \sum \lambda_n^{-1} \varphi_n(s)\psi_n(t) \tag{4}$$

(in the sense of convergence in mean-square, and also pointwise almost everywhere), where $\{\varphi_n\}$ and $\{\psi_n\}$ are orthonormal systems such that (2) is satisfied with λ_n, φ_n and ψ_n in place of λ, φ and ψ, respectively. If $\alpha > p^{-1}$, then the series (4) is absolutely convergent almost everywhere.

Suppose next that $1 < p \leqslant 2$, $\alpha > 0$ and r is a positive integer. Under slightly more elaborate conditions, involving the existence of the first r partial derivatives with respect to s of the kernel $K(s,t)$, and with the last of these derivatives satisfying the generalized Lipschitz condition (3), it is again shown that the series (4) is absolutely convergent almost everywhere, and the estimate for $|\lambda_n|^{-1}$ is sharpened to $O(n^{-\beta-r})$.

The results in [**2**] concerning the order of magnitude of $|\lambda_n|^{-1}$ complement (and, for symmetric kernels, improve upon) earlier work by Hille and Tarmarkin. The proofs involve intricate use of the theory of Fourier series.

An extensive study of integral equations of the form

$$f(x) = \int K(x-y)f(y)\,dy \tag{5}$$

is undertaken in [**6**]. Three main cases are considered.

In the *Volterra case*, the function K is defined on the non-negative real axis, the range of integration in (5) is the interval $(-\infty, x)$, and the equation is to be satisfied for all real x. It is assumed that K can be expressed as the sum of two functions, one of bounded variation on $[0,\infty)$, the other in $L(0,\infty)$. A *characteristic function* $\kappa(s)$ can be defined on the open half-plane $\{s : \operatorname{Re} s > 0\}$ (and is analytic there) by

$$\kappa(s) = \int_0^\infty K(x)e^{-sx}dx,$$

and $s\kappa(s)$ extends continuously to the closed half-plane $\{s : \operatorname{Re} s \geqslant 0\}$. With

https://doi.org/10.1112/S0024609304003662 Published online by Cambridge University Press

the additional assumption that $\kappa(s) - 1$ does not vanish anywhere on the (non-zero) imaginary axis and is bounded away from zero on some neighbourhood of 0, the *characteristic equation* $\kappa(s) = 1$ has only finitely many roots (say $s_1, \ldots, s_n$, with multiplicities $a_1, \ldots, a_n$, respectively) in the open half-plane. In these circumstances, it is shown that each of the functions

$$x^r \exp(s_j x) \qquad (r = 0, 1, \ldots, a_j - 1;\ j = 1, \ldots, n) \tag{6}$$

is a solution of the Volterra equation (5); moreover if f is any solution of this equation, and $f \in L(-\infty, X)$ for all real X, then f is a linear combination of the (finitely many) functions listed in (6). The same conclusion is drawn, under more stringent (and more elaborate) restrictions on the kernel K, for solutions f required to satisfy only the weaker condition that $(1 + |x|^k)^{-1} f(x) \in L(-\infty, X)$ for all real X, where k is a positive integer. The proofs of these results make use of the Volterra equation (5) to establish certain general properties of the Laplace transforms

$$\chi_X(s) = \int_{-\infty}^{X} f(x) e^{-sx} dx$$

(of truncations of f), thereafter applying complex variable techniques to identify each χ_X, and so to determine f.

In the *Fredholm case*, the function K is defined on the real line, the range of integration in (5) is $(-\infty, \infty)$, and the equation is to be satisfied for all real x. The methods, and results, are broadly analogous to those of the Volterra case. The initial assumption is that $K(x)\exp(|x|)$ can be expressed as the sum of two functions, one of bounded variation on $(-\infty, \infty)$, the other in $L(-\infty, \infty)$. The characteristic function $\kappa(s)$ is in this case defined by

$$\kappa(s) = \int_{-\infty}^{\infty} K(x) e^{-sx} dx;$$

it is analytic on the open strip $\{s : -1 < \operatorname{Re} s < 1\}$, and $(s^2 - 1)\kappa(s)$ extends continuously to the closed strip $\{s : -1 \leqslant \operatorname{Re} s \leqslant 1\}$. With the additional assumption that $\kappa(s) - 1$ does not vanish (where defined) on the boundary of the strip, and is bounded away from zero near the exceptional points ± 1, the characteristic equation $\kappa(s) = 1$ has only finitely many roots (say $s_1, \ldots, s_n$, with multiplicities $a_1, \ldots, a_n$, respectively) in the open strip. Each of the functions listed in (6) is then a solution of the Fredholm equation (5); moreover, any solution f of that equation, such that $f(x)\exp(|x|) \in L(-\infty, \infty)$, is a linear combination of those functions. The same conclusion is drawn, under the weaker assumption that

$$f(x)\exp(|x|)(1 + |x|^k)^{-1} \in L(-\infty, \infty),$$

subject to more stringent conditions on the kernel K.

In the *Wiener–Hopf* case, the function K is again defined on the real line, the range of integration in (5) is the interval $(0, \infty)$, and the equation is to be satisfied for all non-negative x. The initial conditions imposed on K, and the definition of the characteristic function κ, are the same as in the Fredholm case. The assumptions made about κ are those of the Fredholm case, together with the further requirement that

$$\int_{-\infty}^{\infty} \frac{|\kappa(\sigma + it)|}{1 + |t|} dt < \infty \qquad (-1 < \sigma < 1).$$

https://doi.org/10.1112/S0024609304003662 Published online by Cambridge University Press

The solutions f of the Wiener–Hopf equation (5), subject to the condition that $f(x)\exp(-x) \in L(0,\infty)$, are once again closely related to the roots of the characteristic equation $\kappa(s) = 1$ in the strip $\{s : -1 < \operatorname{Re} s < 1\}$, but in this case, they are not expressible simply in terms of the roots and their multiplicities as in (6). Nevertheless, the solutions again form a finite-dimensional linear space, and the analysis shows how it is possible (at least in principle) to determine the form of the Laplace transform of a solution f, and hence of f. As before, the same results can be obtained with weaker assumptions on the solutions f, provided that the kernel K satisfies some additional conditions.

The general results described above are applied to determine the solutions of a number of specific integral equations, including some that can be viewed as differential equations of fractional order, and also classical equations associated with the names of Lalesco, Picard and Milne.

In [**7**], Smithies develops the theory of Hilbert–Schmidt operators (he uses the term 'operator of finite norm') acting on a Hilbert space. He then establishes a Fredholm-type theory for a Hilbert–Schmidt operator K, constructing a (scalar-valued) entire function

$$\delta(\lambda) = 1 + \lambda\delta_1 + \lambda^2\delta_2 + \dots \tag{7}$$

(the 'Fredholm determinant' of K), and an (operator-valued) entire function

$$\Delta(\lambda) = I + \lambda\Delta_1 + \lambda^2\Delta_2 + \dots \tag{8}$$

(the 'first Fredholm minor' of K), with the property that $I - \lambda K$ has an inverse if and only if $\delta(\lambda) \neq 0$, the inverse then being equal to $[\delta(\lambda)]^{-1}\Delta(\lambda)$. As an application, he provides a new proof of results due to Carleman, in which the classical Fredholm theory for an integral equation

$$x(s) = y(s) + \lambda \int k(s,t)x(t)\,dt$$

(with all the functions required to be continuous) is amended, and extended to the case in which the functions are required only to be square-integrable.

In developing this Fredholm theory, Smithies starts by considering an operator K of finite rank, and uses elementary linear algebra to express the inverse of $I - \lambda K$ in the form

$$(I - \lambda K)^{-1} = [d(\lambda)]^{-1}D(\lambda)$$

(provided that $d(\lambda) \neq 0$), where $d(\lambda)$ is the polynomial $\det(I - \lambda K)$ and $D(\lambda)$ is an operator-valued polynomial. Upon writing $d(\lambda)$ and $D(\lambda)$ as (formally, infinite) power series

$$d(\lambda) = \sum_0^\infty \lambda^n d_n,$$

$$D(\lambda) = \sum_0^\infty \lambda^n D_n,$$

and with $\sigma_1, \sigma_2, \sigma_3, \dots$ denoting the traces of the operators $K, K^2, K^3, \dots$, he obtains specific formulae for d_n (as a determinant involving $\sigma_1, \dots, \sigma_n$) and for D_n (as a determinant involving $\sigma_1, \dots, \sigma_n$ and $K, K^2 \dots, K^n$). Before there is any possibility of extending the theory to the case of a general Hilbert–Schmidt operator K, it is necessary to find a formulation that does not involve σ_1 (because, while $K^2, K^3, \dots$

are trace-class operators, K itself may not be in the trace class). To this end, the (finite-rank) theory is modified by the introduction of the functions

$$\delta(\lambda) = e^{\sigma_1 \lambda} d(\lambda) = \sum_0^\infty \lambda^n \delta_n;$$
$$\Delta(\lambda) = e^{\sigma_1 \lambda} D(\lambda) = \sum_0^\infty \lambda^n \Delta_n.$$

Clearly, $I - \lambda K$ has an inverse if and only if $\delta(\lambda) \neq 0$, the inverse then being $[\delta(\lambda)]^{-1}\Delta(\lambda)$; and it turns out that there are formulae for δ_n and Δ_n, obtained from those for d_n and D_n respectively, upon replacing σ_1 by 0.

For a general Hilbert–Schmidt operator K, the formulae just mentioned can be used to *define* coefficients $\delta_0, \delta_1, \delta_2, \dots$ and $\Delta_0, \Delta_1, \Delta_2, \dots$, and the power series for $\delta(\lambda)$ and $\Delta(\lambda)$. A careful process of approximation to K by operators of finite rank then leads to a proof that $\delta(\lambda)$ and $\Delta(\lambda)$ are entire functions with the requisite properties.

A square-integrable kernel $k(s,t)$ gives rise to a linear (integral) operator K, of Hilbert–Schmidt class, acting on the appropriate L^2 space. By applying to K the operator version of Fredholm theory, Smithies recovers Carleman's adaptation of the classical Fredholm theory of linear integral equations.

The approach used in [**7**] to establish a Fredholm theory for a Hilbert–Schmidt operator can easily be amended so as to apply to an operator in any of the von Neumann–Schatten classes. In the case of a trace-class operator, one can proceed with the functions $d(\lambda)$ and $D(\lambda)$, and the introduction of $\delta(\lambda)$ and $\Delta(\lambda)$ is not necessary.

The war years

From July 1940 until September 1945, Smithies worked on war-related matters at the Directorate of Scientific Research in the Ministry of Supply. At first concerned with the development of electrical predictors for anti-aircraft gunnery, he became secretary to the Gun Design Committee in May 1941. In this position, he had a coordinating role, as the committee initiated numerous theoretical and experimental investigations, the central theme being the yield strength of gun barrels. His responsibilities also included 'miscellaneous mathematical problems'; matters drawn to his attention under this heading ranged widely, from serious questions about particular differential equations, to a breathless telephone enquiry concerning the formula for $\sin(A + B)$.

Smithies became interested in statistical quality control, having been introduced to the basic principles of the subject through numerous discussions with J. R. Womersley, who was using statistical methods in connection with his work for the Woolwich Research Department. The two had met after Womersley had asked the Ministry to arrange for additional support with statistical computations. At the same time, through transatlantic contacts, the Ministry was receiving quantities of literature on the subject, and was not sure what to do with it; Smithies was asked to deal with the matter. He began by forwarding material to departments that he believed should find it useful. At first, those who responded with requests for help were referred, unofficially, to Womersley. When the demand for such services increased to the point where both another ministry and a Parliamentary Select Committee were adding to the pressure, Smithies and Womersley had already prepared a blueprint for an advisory service, which their masters accepted with alacrity. With Womersley in charge, the Advisory Service on Statistical Quality Control was established in 1942, as a branch of the Ministry of Supply. In retrospect, Smithies believed that the part he had played in setting up this service was probably his most useful piece of work at the Ministry.

Cambridge: 1945–1979

Smithies returned to St John's College, Cambridge, in September 1945. With effect from the beginning of the next month, he became a college lecturer and a teaching fellow, also an Assistant Lecturer in the Faculty of Mathematics. He was promoted to a full University Lectureship in 1947, and to a Readership in 1962.

During his London years, Frank had met Nora Arone, who had joined the Ministry of Supply around 1942, after horticultural studies at the University of Reading. He and Nora were married in December 1945, with Maurice Wilkes as best man. They set up home in Cambridge, at first in a college flat in Bridge Street, later moving to a house in Huntingdon Road where they were to remain for the rest of their lives. Together, they made their home a hospitable place. Nora undertook a great deal of work for the National Council for Women, and was also interested in mental healthcare. She was much involved in setting up a 'halfway house' for mental patients released from hospital but not yet ready for full independence; she served on its House Committee, as secretary, and later as chairman. Sadly, she and Frank had no children. After a long period of ill-health, during which Frank cared for her with devotion, she died in 1987. By this time, both his parents had already died (his father in 1944, his mother in 1957). His sister Violet died two years later, having retired in 1976 after a career in the civil service. Throughout her life, she and Frank remained very close, keeping in touch by frequent telephone conversations.

In his teaching at all levels, Smithies was conscientious and meticulous. Throughout the period 1945–1979, he undertook a full load of undergraduate supervision (in small groups) at St John's College, where he was Director of Studies in Mathematics from 1953 until 1971. His lectures, at both undergraduate and postgraduate level, were a source of inspiration – not, perhaps, in the conventional sense as a result of charismatic presentation, but rather because his excellent judgement as to choice of material, combined with exceptional clarity of exposition, allowed the subject matter to speak for itself. Through his undergraduate courses on linear algebra, the unexpected power and elegance of simple mathematical ideas were made apparent. His presentation of various aspects of functional analysis (and its applications) in more advanced lectures was a significant factor in attracting aspiring Cambridge mathematicians to pursue the subject at research level.

Other would-be functional analysts came to Cambridge in considerable numbers after undergraduate studies elsewhere, usually overseas. Through his success in generating so much interest in the field, Smithies created very heavy demands on his own time, for during the early postwar years, probably as late as the early 1960s, it was he himself who had to undertake nearly all the supervision of Cambridge research students of functional analysis. Some of his earlier students were extremely fortunate in being guided towards promising areas of research that arose naturally from his own pre-war work; these involved matters that he could well have followed up himself, but he chose instead to use them as a means of nurturing a new generation of research workers.

With growing numbers of research students, Smithies obtained a room in Laundress Lane for their use. He organised a weekly research seminar, in which the talks were given usually by the students themselves, but occasionally by distinguished visitors from overseas. In this way, his students acquired a sense of community and a broadened mathematical experience; they also had the opportunity to strike sparks off one another, and to develop at least rudimentary expository skills. Although Smithies was doing little original work of his own during this period, he kept abreast of the literature on a wide range of subjects, always on the look-out for directions that appeared promising for further research. It seems that his approach in supervising postgraduate work varied widely from one student to another. While this may have arisen, in part, as a reaction to the differing expectations and work patterns of the students, it appears likely also that he made his own assessment of the needs of the individual, and adapted his approach accordingly. In some cases, original work written up by the student would be read in detail, and returned promptly with voluminous written comments on possible improvements and suggested directions for further development (this being similar to the practice that Hardy had followed in supervising Smithies). For other students, there might be just the occasional suggestion of a recent paper worth reading, or of an area in which significant further progress might reasonably be expected, with encouragement to persist. When Smithies judged that one of his students had produced a suitable body of original work, he was likely to suggest that it be written up at once, with a view to publication; from the student viewpoint, this was very encouraging, as was doubtless the intention.

By the middle 1960s, there were other functional analysts in academic posts around the country, including one or two with Smithies in Cambridge; a large proportion of them were his former students, and several of them were taking on research students of their own. While quite a lot of Cambridge undergraduates were still being attracted to research in functional analysis, those remaining in Cambridge could be spread amongst two or three supervisors, and a few chose to spend their research student years elsewhere. While these factors to some extent relieved the pressure on Smithies, he nevertheless continued to take on quite large numbers of PhD students, still including a good sprinkling from abroad. Over the years, he had no less than fifty-three research students.

The influence that Smithies exerted on the growth of functional analysis within the British mathematical community is already apparent from the preceding paragraphs. That influence resulted not only from his teaching and research supervision in Cambridge, but also from lectures given elsewhere. One of the first British research schools in the subject, outside Cambridge, was established in Newcastle upon Tyne under the leadership of F. F. Bonsall, who wrote (in relation to a talk at the 1950 British Mathematical Colloquium), 'A brilliant lecture on commutative Banach algebras by Frank Smithies in 1950 stimulated my interest in abstract analysis ...'. It was no exaggeration when Paul Halmos described Smithies as 'the father (or grandfather) of functional analysis in Great Britain'.

Along with his teaching and postgraduate supervision, Smithies completed the book on integral equations that Hardy had asked him to write for the Cambridge Mathematical Tracts [**11**]. From time to time, he thought about some of the major unsolved problems in functional analysis, questions that interested him, as possible topics for his own research. Such was their intractability that neither he nor anybody else at that time could solve them, though some have been settled since.

https://doi.org/10.1112/S0024609304003662 Published online by Cambridge University Press

Smithies was a very well-organised person, a meticulous record-keeper, and an effective administrator, qualities that were reflected at both a personal and a professional level. His files of correspondence, and his collections of lecture notes, covered the entire period from the late 1920s onward. Throughout his adult life, he kept a daily diary, which not only gave an indication of his activities, but also bore evidence of his enduring political commitment, with records of Labour Party gains and losses at by-elections and general elections. He served on a wide range of Faculty committees, sometimes as chairman; he was one of the editors of Hardy's collected papers, and was an editor of the Cambridge Mathematical Tracts from 1953 until 1979. He was a Syndic of the Cambridge University Press from 1958 until 1972, and was particularly glad to be able to arrange contact between the Press and the School Mathematics Project; all the SMP textbooks were published by CUP.

Smithies was secretary to the International Congress of Mathematicians, held in Edinburgh in 1958; this was a major administrative responsibility that took up a great deal of time during the preceding three years. He was active in both the Cambridge Philosophical Society and the London Mathematical Society, in the latter case serving as one of the Secretaries (1948–1951) and as Vice President (1951–1952). He was elected to a Fellowship of the Royal Society of Edinburgh in 1961.

Smithies was interested both in the exposition of advanced mathematics to a wider audience, and also in mathematical education at school (as well as university) level. This is exemplified in his work in the Mathematical Association, especially on its Universities and Schools Committee (which he chaired for a time), and also in some of his publications in the Mathematical Gazette and elsewhere [**9, 15, 17-19**]. During his postwar years in the Cambridge Faculty of Mathematics, Smithies took periods of sabbatical leave in France (University of Nancy), in Uganda (Makerere College), and twice at the Australian National University in Canberra. Shorter visits took him to many other destinations, in Australia, Canada, Iran, New Zealand, and the USA, and in various European countries. Lectures that he gave on these occasions might cover not only his mathematical interests, but also topics in mathematical education and the history of mathematics.

When he retired in 1979, a new chapter opened.

Retirement: Cambridge, 1979–2002

Smithies had been interested in the history of mathematics ever since attending lectures by E. T. Whittaker during his undergraduate years. Throughout his career, it had been his habit to look into the historical background of any mathematical topic currently occupying his attention, and he had occasionally given lectures on such matters. He had joined the British Society for the History of Mathematics in 1972, later serving on its committee, and as its President from 1982 until 1985.

In each Easter term, during a few years leading up to his retirement, he had given a short (non-examinable) course of lectures on some aspect of the history of analysis, usually from the nineteenth century. In preparing these lectures, he had seen discrepancies between the secondary literature (on which he had at first relied) and some of the original papers. In the case of complex function theory, he had found the secondary literature confused and inconsistent. This had led him to formulate a programme of research into Cauchy's early work on the subject.

In retirement, Smithies found sufficient time, at last, to concentrate on historical matters; he later had some regrets about not having taken up serious historical work at an earlier stage. He was interested in the beginnings of functional analysis, and his lecture on that subject, at a joint meeting in 1995 of the British Society for the History of Mathematics and the London Mathematical Society, gave rise to a subsequent publication [**31**]. The most major, and most sustained, theme of his historical work, however, concerned Cauchy and the origins of complex function theory. He produced one or two papers along the way, and spoke in 1989 at the Cauchy bi-centenary colloquium in Paris; the whole programme reached its culmination with the publication in 1997 of his authoritative monograph [**32**].

Historical research brought Smithies into contact with numerous other workers in the field; he particularly valued the friendship of Professor H. S. Tropp of Humboldt State University in California, with whom his work on Cauchy was discussed in detail over many years. He took on just one research student of the history of mathematics, Piers Bursill-Hall, who writes:

> "Frank sorted out the development of Cauchy's thinking on foundations of analysis and complex analysis in a way that nobody else has come even near to doing, and more important, he did it by understanding exactly what and how Cauchy thought *in Cauchy's own terms* without 'reconstructing' it into modern mathematical terms or concepts. The technical mastery of Cauchy's work is something that only a very patient, competent, and very painstaking mathematician could do: it required unlearning what we have subsequently learned and sorting out Cauchy's own attack on problems and study of structures using limited and crude tools compared to those of the modern analyst. The combination of attention to detail and brute bibliographical patience, historical sensitivity and mathematical sensitivity, is not so much a new standard of rigour as a remarkable combination of historical and mathematical skills, and one that we have seen in few other modern historians of mathematics."

In his last published work [**34**], Smithies discussed an early paper that provides a (flawed) proof of the 'fundamental theorem of algebra'. He showed how the argument set out in that paper by its author, James Wood, could have been augmented so as to provide a valid proof of the theorem a year before the first generally accepted proof was given by Gauss. This work involved both historical discussion and some new mathematical arguments.

Upon looking back over his career, Smithies felt that he had been fortunate to be on the mathematical scene at the time that functional analysis was becoming established as a subject in its own right, and so to have had the opportunity to encourage interest and stimulate research in the field. He found that the achievements that gave him most pleasure, in retrospect, seemed to have arisen from situations in which he had been able to act as a catalyst. Within this category, he included his role during the war years in setting up the Advisory Service for Statistical Quality Control, the part he played in bringing together the School Mathematics Project and the Cambridge University Press, and of course, his teaching and research supervision. Although physically rather frail during his last few years, Smithies remained mentally alert until the end. He died, after a very brief illness, on 16 November 2002. Once again, we quote the words of Piers Bursill-Hall:

> "Frank's mathematical capacity and his mental sharpness seems not to have diminished one iota in his later years; he had a certain amount of ill health and certainly he was physically less robust in his last years, and deafness isolated him ... but I never found his memory or his intellectual powers to be in any way dimmed by the passing years, nor his sense of humour diminished. What a lucky man!"

Acknowledgements. In preparing this obituary notice, I have been greatly assisted by access to some biographical notes that Frank Smithies left among his papers. I am grateful for comments and information received from F. F. Bonsall, A. L. Brown, P. Bursill-Hall, D. J. H. Garling and T. T. West. All have had an influence on one or another part of what I have written, but I alone am responsible for any errors.

Publications of Frank Smithies

1. 'On the theory of linear integral equations', *Proc. Cambridge Philos. Soc.* 31 (1935) 76–84.
2. 'The eigen-values and singular values of integral equations', *Proc. London Math. Soc.* (2) 43 (1937) 255–279.
3. 'A note on completely continuous transformations', *Ann. of Math.* (2) 38 (1937) 626–630.
4. 'Completely continuous transformations in Hilbert space', *Bull. Amer. Math. Soc.* 44 (1938) 835–836.
5. (with R. P. Boas Jr.) 'On the characterization of a distribution function by its Fourier transform', *Amer. J. Math.* 60 (1938) 523–531.
6. 'Singular integral equations', *Proc. London Math. Soc.* (2) 46 (1940) 409–466.
7. 'The Fredholm theory of integral equations', *Duke Math. J.* 8 (1941) 107–130.
8. 'Some aspects of Hardy's mathematical work: (2) integral equations', *J. London Math. Soc.* 25 (1950) 107–109.
9. 'Abstract analysis', *Math. Gazette* 35 (1951) 2–7.
10. 'Professor F. Riesz', *Nature* 177 (1956) 870.
11. *Integral equations*, Cambridge Math. Tracts 49 (Cambridge Univ. Press, 1958).
12. 'John Ronald Womersley', *J. London Math. Soc.* 34 (1959) 370–372.
13. 'John von Neumann', *J. London Math. Soc.* 34 (1959) 373–384.
14. 'Extensions of ideals in commutative algebras', *Proc. Cambridge Philos. Soc.* 55 (1959) 277–281.
15. 'What is modern mathematics?', *Math. Gazette* 47 (1963) 278–298.
16. (with S. J. Bernau) 'A note on normal operators', *Proc. Cambridge Philos. Soc.* 59 (1963) 727–729.
17. 'Continuous geometry', *J. Math. Assoc. Nigeria* 4 (1965) 62–68.
18. 'Recent changes in education in the United Kingdom', *Canadian Mathematical Congress* (1965) 720–735.
19. 'What makes a mathematician?', *Math. Gazette* 54 (1970) 224–227.
20. 'Two remarks on a note by Mordell', *Math. Gazette* 54 (1970) 260–261.
21. (ed.) *Collected papers of G. H. Hardy*, vol. 5 (Oxford Univ. Press, 1972).
22. 'Inspiration or perspiration?', St John's College (Cambridge) lecture (University of Hull, 1974).
23. 'Weierstrass's theory of the real numbers', *Bull. Inst. Math. Appl.* 11 (1975) 24–27.
24. 'René Maurice Fréchet', *Year Book, Royal Society of Edinburgh* (1975) 31–33.
25. 'The background to Cauchy's definition of the integral', *From A to Z* (Leiden, 1982) 93–100.
26. 'Maxwell Herman Alexander Newman', *The Eagle* (St John's College magazine) 69 (1984) 51–53.
27. 'Cauchy's conception of rigour in analysis', *Arch. Hist. Exact Sci.* 36 (1986) 41–62.
28. 'Augustin-Louis Cauchy', *Math. Spectrum* 22 (1989) 1–6.
29. 'Sir Harold Jeffreys', *The Eagle* (St John's College magazine) 72 (1990) 29–33.
30. 'Reminiscences of Ralph Boas', *Lion-hunting and other mathematical pursuits* (Math. Assoc. America, 1995) 25–31.
31. 'The shaping of functional analysis', *Bull. London Math. Soc.* 29 (1997) 129–138.
32. *Cauchy and the creation of complex function theory* (Cambridge Univ. Press, 1997).
33. 'The development of a historian', *Newsletter BHSM* 35 (1997) 29–31.
34. 'A forgotten paper on the fundamental theorem of algebra', *Notes and Records of the Royal Society of London* 54 (2000) 333–341.

6 Polwarth Road
Gosforth
Newcastle upon Tyne NE3 5ND
United Kingdom

J. R. Ringrose