Diana Yun-dee Wei, On the concepts of torsion and divisibility for general rings. McGill University, April 1967. (Supervisor: J. Lambek).

In a first attempt to generalize the concept of torsion, an element of a right R-module is called a torsion element if it annihilates a dense right ideal of R, that is, a right ideal which admits no non-zero homomorphisms into the injective hull of the right R-module R. Consequences of this definition are explored, but it does not readily dualize.

Following a different approach, a right R-module M is called a torsion module if $\mathrm{Hom}_R(M, Q) = 0$, where Q is Utumi's ring of right quotients of R. Dually, M is called reduced if $\mathrm{Hom}_R(Q, M) = 0$. A module is then called torsion-free if no non-zero submodule is torsion. Dually, a module is called divisible if no non-zero factor module is reduced. Many of the usual theorems are shown to be preserved, and the new definitions are compared with earlier ones by L. Levy, A. Hattori, E. Matlis, and others.

The new definition of torsion does indeed generalize the classical one for integral domains. The same is true for divisibility, provided the integral domain R has a quotient field Q such that $\mathrm{Hom}_R(Q, D) \neq 0$ for every classically divisible R-module D. The question of when this condition holds is only partially answered.

Dana I. Schlomiuk, A characterization of the category of topological spaces. McGill University, April, 1967. (Supervisor: J. Lambek).

F. W. Lawvere in "An elementary theory of the category of set" has shown how to characterize the category of sets as a complete category satisfying a finite set of elementary axioms. The same is done here for the category of topological spaces and continuous mappings.

First, the notions of discrete space, subspace, and open subspace are introduced in categorical language, and elementary axioms concerning these concepts are stated. An important role in the definition of "open" is played by an axiom which implies the existence of an object with three endomorphisms. This object represents the topological space with two points and three open subspaces.

The main metatheorem asserts that any complete category satisfying the given system of elementary axioms is equivalent to the category of topological spaces. The proof also shows that the traditional construction of this category and its translation into Lawvere's system are essentially the same.

The last of the elementary axioms is more complicated than the others. Intuitively speaking, it says that any set with a suitable closure operation determines an object of the category. When this axiom is deleted from the elementary system, a full embedding into the category of topological spaces is still obtained.