Part VI

COSMIC CENSORSHIP

Cosmic Censorship

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1. Introduction

In 1978 Roger Penrose opined that cosmic censorship was "possibly the most important unsolved problem in classical general relativity theory" (1978, p. 230). Today the problem remains unsolved, but in the intervening years Penrose's sentiment about the importance of the problem has been shared by many leading researchers in relativistic gravitation (see, for example, Eardley 1987, Israel 1984, and Wald 1984a). This sentiment can be traced to several considerations. First, if a suitable form of cosmic censorship obtains, then one can appeal to various "no hair" theorems for black holes to obtain a characterization of the final state of gravitationally collapsed bodies. The now standard black hole thermodynamics makes use of Hawking's area theorems, which in turn presuppose a form of cosmic censorship. Second, the proof of the positivity of total mass of an isolated gravitating system (the so-called ADM mass) presupposes censorship of naked singularities, at least on the initial hypersurface. A third group of considerations stems from the fact that cosmic censorship would lead to a breakdown in predictability and determinism.[1] Because of its connection with traditional philosophical concerns, it is this worry which will receive the most attention in this paper.

A series of theorems due primarily to Stephen Hawking and Roger Penrose show that the laws of the general theory of relativity (GTR) entail that, under very general conditions, spacetime singularities can be expected to develop in gravitational collapse and cosmology (see Wald 1984a, Ch. 9). The hope of cosmic censorship is that these same laws preclude singularities of the naked variety, for otherwise these laws would perversely undermine themselves. Whatever their precise technical characterization, spacetime singularities signal the breakdown of classical relativistic spacetime structure and, thus, of the laws that presuppose that structure. If such a breakdown were naked in the sense of being visible to external observers, then those observers could be sprayed by unpredictable influences emerging from the singularities. Of course, quantum mechanics (QM) has accustomed us to unpredictability and indeterminism, but not of the anything-goes variety. Perhaps it will turn out that naked singularities are governed by some yet to be discovered regularities that restore a semblance of lawful predictability, perhaps of a statistical form to which QM has accustomed us; but if so, classical GTR gives no clue as to what these regularities are. Rather than having to appeal to something beyond the theory to clean up the mess that the theory has created, it would be much neater if the theory could be shown to have built in

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mechanisms to prevent the mess from occurring in the first place. Before we can begin to investigate the prospects of such a neat resolution, we need a better fix on exactly what goes into the mess of a naked singularity and how the mess might be censored.

2. What is to be censored?

The first step towards formulating a cosmic censorship hypothesis is to identify the kind of behavior that needs censoring. One approach would be to fashion a definition that says when a spacetime is nakedly singular as viewed from a given point. Letting N stand for the collection of all such points, we could then say that the spacetime satisfies strong cosmic censorship just in case $N = \varnothing$. Geroch and Horowitz (1979) have suggested a definition along the following lines: M, g_{ab} is *nakedly singular* as viewed from $p \in M$ iff $J^-(p)$ contains a timelike curve γ without any future endpoint.($J^-(p)$ stands for the *causal past* of p, i.e. the set of all points q such that there is a (possibly trivial) causal curve from q to p.) How can it be that γ, which is maximally extended in the future direction, fails to escape $J^-(p)$? Intuitively, the failure results from the fact that γ runs into a singularity. And since this fact is observable from p, the spacetime is therefore nakedly singular from that vantage point. Note that the standard Friedman-Walker-Robertson big bang models are *not* counted as nakedly singular by this definition even though the initial singularity is highly visible.

There can be little doubt that $N = \varnothing$ is sufficient for cosmic censorship/no naked singularities; in particular, on the proposed definition of N, $N = \varnothing$ entails that the spacetime possesses a Cauchy surface. (A *Cauchy surface* for a spacetime M, g_{ab} is a spacelike hypersurface $S \subset M$ such that every maximally extended causal curve intersects S exactly once. Such a hypersurface is the appropriate launching pad for Laplacian determinism. Of course, the existence of such a launching pad does not by itself show that global determinism holds, but it does show that a breakdown in determinism is not due to some pathology in the spacetime.) If strong cosmic censorship fails, a weaker form may still hold if each of the points in N is confined to the interior of a black hole.[2] For observers outside of the event horizon of the black hole, physics goes on as usual.

But the following examples raise doubts about whether $N = \varnothing$ (or the weaker condition that N is contained in black holes) is necessary for the censorship of naked singularities. *Ex. 1*. Let C be a compact ball about the origin of Minkowski spacetime. Choose a scalar field Ω which blows up as C is approached, and define a spacetime metric $g_{ab} \equiv \Omega^2 \eta_{ab}$, where η_{ab} is the Minkowski metric. The new spacetime M ($= \Re^4 - C$), g_{ab}, which contains an "internal infinity," is counted as nakedly singular by the definition under discussion; in fact, N contains all points p such that in Minkowski spacetime $C \subset J^-(p)$. *Ex. 2*. For the covering space of anti-de Sitter spacetime, N is the entire spacetime, not because there are internal infinities but because for each p, $J^-(p)$ contains timelike curves that accelerate off to spatial infinity. In both examples there is a breakdown in predictability and determinism. And yet both spacetimes are arguably non-singular: there is no "blow up" of curvature, and both spacetimes are geodesically complete (indicating that there are no "missing points" corresponding to singularities that have been cut out of the manifold). Here one simply has to make a choice: if a breakdown in predictability and determinism is the main concern behind cosmic censorship, then the behavior illustrated in Exs. 1 and 2 should be censored; but if one's concern is with, say, curvature singularities that are visible to external observers, then the definition of N has to be strengthened in some appropriate way.

Another approach to cosmic censorship/naked singularities is motivated by the hope that in physically reasonable cases of gravitational collapse, singularities will not develop from regular initial data. Thus, negative mass Schwarzschild spacetime, which contains a naked singularity on any reasonable definition of that term, would not be counted as a violation of cosmic censorship since the singularity has existed for all past times. To explore this approach, a definition is needed to identify the cases of spacetimes where a singularity develops from the data posed on some initial value hypersurface. Towards this end, define a *time slice* (aka *partial Cauchy surface*) as a spacelike hypersurface that is achronal and edgeless. The future Cauchy horizon $H^+(S)$ of such an S is the future boundary of the future domain of dependence $D^+(S)$ of S, the region of spacetime where one can reasonably hope to determine the state of things from initial data on S. (More precisely, for a spacetime M, g_{ab} the *future domain of dependence* of $S \subset M$ consists of all $p \in M$ such that every causal curve which passes through p and which has no past endpoint meets S. The *past domain of dependence* $D^-(S)$ of S is defined analogously. S is a Cauchy surface for M, g_{ab} iff $M = (D^+(S) \cup D^-(S))$.) Then following ideas of Geroch and Horowitz (1979) and Horowitz (1979) we could try: M, g_{ab} is *future nakedly singular* (FNS) with respect to the partial Cauchy surface $S \subset M$ iff $\exists p \in H^+(S)$ such that the closure of $J^-(p) \cap S$ is compact. If $H^+(S) \neq \varnothing$, prediction from initial data on S eventually breaks down. But the breakdown may not be due to any pathology in the spacetime but to a bad choice of S, as illustrated in the example of a spacelike but asymptotically null surface in Minkowski spacetime (see Fig. 1). However, the proposed definition does not count Minkowski spacetime as FNS since in the illustration the closure of $J^-(p) \cap S$ is non-compact.

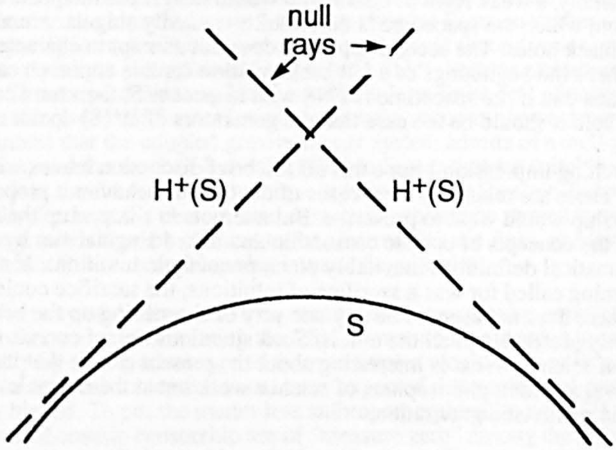

Figure 1

The first thing that needs to be shown in order to establish that this definition is doing its job is that if M, g_{ab} is FNS with respect to S, then there is a pathology in the spacetime structure to the future of S. This can be done in the minimal sense that it can be shown that $J^+(S)$ is not globally hyperbolic.[3] (The proof proceeds by way of contradiction. Supposing that the spacetime is FNS with respect to S and that $J^+(S)$ is globally hyperbolic implies that $\exists p \in H^+(S)$ such that $J^+(J^-(p) \cap S) \cap J^-(p)$ is compact. This compact set past traps the null geodesic which is the generator of $H^+(S)$ through p, which contradicts global hyperbolicity. The reader can fill in the details using the results in Hawking and Ellis 1973, Ch. 6.) But note that failure of global hyperbolicity may be due the development of acausal features, such as closed causal curves, as illustrated by Taub-NUT spacetime where $H^+(S)$ for a time slice S of the Taub portion of the spacetime is a null surface ruled by closed null geodesics. If it is naked curvature singularities that one wants to censor rather than acausal behavior, then a clause could be added to the definition of FNS, requiring that one of the generators of $H^+(S)$ (which are null geodesics) is past incomplete (see Newman 1986). In this case an observer who crosses over $H^+(S)$ could look back and see the singularity.

The real concern about the proposed definition focuses on the converse direction: Does the fact that M, g_{ab} is not FNS imply that the spacetime is free of naked singularities in the intended sense? Cases have to be divided. If M, g_{ab} is not FNS with respect to a compact S, that is because S is a Cauchy surface, so the spacetime is surely singularity free. However, if S is non-compact the story is quite different. Reissner-Nordstrom spacetime is not FNS with respect to a maximal slice S (see Fig. 12.4 of Wald 1984) even though there is a curvature singularity to the future of S which an observer can see when she crosses $H^+(S)$. Of course, this example can be dismissed as "physically unreasonable." For any $p \in H^+(S)$, $J^-(p) \cap S$ fails to have compact closure because $J^-(p) \cap S$ extends to spatial infinity; and because p can be influenced by an infinite range of initial data on S, it is not surprising that small changes in the initial conditions can lead to divergences on $H^+(S)$ ("blue shift instability"). But to defend the proposed definition in this way is to treat it as doing two jobs at once: defining singular behavior that develops from regular initial data and simultaneously excluding cases of physically unreasonable behavior. I for one would prefer a definition that does the first job and leaves the second to a further discussion.

Another potential defect of this second approach emerges in comparison with the first. The first approach provided for a fall-back position should strong cosmic censorship fail; namely, a weak form of censorship would hold if the members of the set N of points from which the spacetime is detectability nakedly singular are all contained within black holes. The second approach does not attempt to characterize the set N; but perhaps the beginnings of a fall back position for this approach can be provided by the idea that if the spacetime is FNS with respect to S, then for weak cosmic censorship to hold it should be the case that the generators of $H^+(S)$ do not reach ℓ^+.

The overarching impression I hope this all too brief discussion leaves with the reader is this. There are relatively clear cases of the type of behavior a proponent of cosmic censorship would want to proscribe. But attempts to sharpen up the fuzzy boundaries of the concepts of cosmic censorship and naked singularities by means of precise mathematical definitions inevitably pinch preanalytic intuitions. If all the process of sharpening called for was a sacrifice of intuitions, the sacrifice could be made happily; but there does not seem to be any one way of sharpening up the boundaries that is obviously preferable to all the others. Such situations are, of course, familiar to philosophers of science. What is interesting about the present case is that it occurs not at the meta-level at which philosophers of science work but at the object level of active research in relativistic gravitation.

3. The cosmic censorship hypothesis

Supposing that we have settled on a specific content for the concept of naked singularity, the cosmic censorship hypothesis (CSH) becomes the claim that GTR has a built in mechanism for preventing the development of such singularities. If the CSH is to be part of physics, as opposed to the philosophy of physics, then this vague claim has to be replaced by a claim about solutions to Einstein's field equations (EFE) that is precise enough that it lends itself to proof or, alternatively, to refutation by counterexample. Finding such a statement of cosmic censorship that fits this bill while escaping obvious counterexamples has proven difficult.

To illustrate the difficulty, begin with the observation that it is easy to produce solutions to EFE that contain naked singularities on any reasonable definition of that term. But many of the known examples can be dismissed as being physically uninteresting, either because of the nature of the singularity itself or because the presence of the singularity can be chalked up as an artifact of idealizations. Consider, for example, dust solutions to EFE. (Here the stress-energy tensor takes the form $T^{ab} = \mu V^a V^b$ where μ is the mass-energy density and V^a is the normed four-velocity of the dust.) It is possible to arrange spherically symmetric collapse of a dust cloud so that the outer shells fall inward faster than the inner shells, leading to the development of infinite density singularities that are visible to external observers. Such "shell crossing singularities," however, are relatively mild, and the solution can (perhaps) be extended through the singularity in some physically reasonable way. This caveat can be removed by exhibiting dust solutions that develop much stronger and, hence, irremovable "shell focusing singularities." But another caveat immediately arises to take the place of the first: by definition, dust models neglect pressure, and it is not initially implausible to think that if pressure p is incorporated into the description and if the equation of state $p = p(\mu)$ specifies that p diverges as μ becomes infinite, then the singularity can be avoided. This qualm can be overcome by exhibiting nakedly singular solutions to EFE for a perfect fluid source ($T^{ab} = (\mu + p)V^aV^b + pg^{ab}$) with the equation of state $p = a\mu$, $a = \text{constant} > 0$ (see Ori and Piran 1990). One can still worry that the naked singularity is a consequence of the perfect fluid idealization. But if this worry is allowed to become too obsessive it can undercut any potential counterexample to the CSH since all physics involves some idealizations. What is needed, therefore, is a criterion for when the level of description has become fundamental enough that potential counterexamples cannot be dismissed as artifacts of the analysis.

Wald (1984a, p. 303) proposed that a necessary condition for the matter fields to be fundamental in the intended sense is that the coupled Einstein-matter equations can be put in a special form (technically, quasilinear diagonal second-order hyperbolic) that holds for paradigm examples of fundamental fields, such as electromagnetism and gravity. The coupled Einstein-Euler equations for a finite ball of perfect fluid cannot be put in this preferred form. However, the main virtue of the preferred form is that it guarantees that the coupled gravity-matter system admits of a well-posed initial value problem; and the work of Rendall (1992) indicates that for suitable equations of state, initial data should determine a unique solution for the Einstein-Euler equations, at least locally. Perhaps the perfect fluid description should be denied fundamental status on the grounds that, by contrast to the electromagnetic field, in Minkowski spacetime such a fluid can develop shock wave singularities.

Yet another way of dealing with the potential counterexamples mentioned above points to another way of formulating the CSH. Someone who wanted to violate cosmic censorship by taking advantage of these examples would have to be extremely lucky since in any realistic case it is not be to expected that perfect spherical symmetry can be achieved. To put the matter less anthropomorphically, the CSH would state that violations of cosmic censorship are of "measure zero" among the solutions to EFE. Given our presently limited knowledge of generic features of EFE, the prospects of proving such a censorship theorem (even if true) do not seem bright. But numerical simulations of generic cases of gravitational collapse can furnish evidence pro or con.

The time symmetry of EFE poses a threat to the "measure zero" version of cosmic censorship. Suppose that in a typical case of gravitational collapse only a weak form of cosmic censorship obtains; that is, suppose the associated set of points N from which the spacetime can be detected to be nakedly singular is non-empty but that N is contained within a black hole. By the time symmetry of EFE, every such solution is matched by a time reversed solution, and the latter solution represents a white hole that is as naked as can be. Here we add another verse to the refrain of the problem of the direction of time, which asks why we encounter certain solutions to the fundamental laws of physics but not their time reverses. It is tempting to sweep the present threat to cosmic censorship under the rug of this ubiquitous problem, and, in line with the second approach outlined in section 2, take cognizance only of naked singularities that arise from regular initial data.

It seems fair to say that the evidence on cosmic censorship that has been amassed to date does not point strongly in either the pro or the con direction. On the pro side, very few positive censorship theorems have been proved, and the significance of the ones that have been proven is hard to assess. For example, Newman (1984) has established a version of cosmic censorship by assuming that all incomplete null geodesics (which are taken to be indicative of the presence of a singularity) satisfy a persistent curvature condition; but the applicability of this condition to generic cases of gravitational collapse remains uncertain. Proponents of cosmic censorship can take comfort from the failure of attempts to produce counterexamples. Consider, for example, the condition that separates the black hole version of Kerr-Newman family of spacetimes from a nakedly singular version: $M^2 \geq Q^2 + J^2$, where M, Q, and J are respectively the mass, electric charge, and the angular momentum. Wald (1973) showed that attempts to violate this inequality by, for instance, injecting electric charge into the black hole cannot succeed. This confirms the stability of a generic black hole configuration but does not reach the question of whether a black hole will form in the first instance.

On the negative side of the ledger one could cite the recent numerical simulation by Shapiro and Tukolsky (1991) of the collapse of a prolate spheroid. However, in view of the investigation of Wald and Iyer (1991) it is too early to take the Shapiro-Tukolsky result as a counterexample to cosmic censorship, and the simulation would have to be continued forward in time to verify that the absence of an apparent horizon means the absence of an event horizon clothing the singularity.[4]

In sum, the strongest evidence we have in favor of the CSH is the absence of solid counterexample despite many attempts to construct them, which is not much evidence at all unless we assume that we are clever enough to have found whatever counterexamples that may exist.[5]

4. Quantum considerations

Will a quantum theory of gravity provide a more friendly or a more hostile environment for cosmic censorship? Since we can only dimly perceive the shape of a successful marriage of QM and GTR, the answers that can be provided at present are necessarily speculative and quite probably unreliable. Nevertheless, some speculation is useful in defining the issues.

Ordinary QM shows an amazing ability to smooth out singularities from classical physics. Consider, for example, Newtonian point mass particles interacting via a $1/r^2$ force. A Newtonian solution can break down either because of collision or non-collision singularities (see Earman 1986). By contrast, in the quantum version of this problem the Hamiltonian operator is (essentially) self-adjoint, with the result that the evolution operator, formed by exponentiating the Hamiltonian, is unitary and is defined for all $-\infty < t < +\infty$. The singularities have completely disappeared! Similarly, one might hope that the singularities of classical GTR will disappear in a fully quantized theory of gravity. Such a hope might be based on the attitude that the only singularities that will occur in classical gravity are those that a forced to occur by the Hawking-Penrose singularity theorems. One then points to the conjunction of two facts; viz. that these theorems rely on energy constraints on T^{ab} (such as the *null energy condition* which requires that $T^{ab}k_a k_b \geq 0$ for every null vector k^a) and that these energy constraints can be violated in relativistic quantum field theory. However, quantum field theory does satisfy an averaged null energy condition (see Wald and Yurtsever 1991), and it my be that singularity results can be generated from these weaker constraints.

If quantum gravity doesn't banish singularities altogether, then a more frightening prospect opens up, even if cosmic censorship is true at the level of classical GTR. For according to calculations from semi-classical quantum gravity, black holes will eventually evaporate due to Hawking radiation. And assuming that classical relativistic spacetime concepts can be applied to the outcome, naked singularities can be expected to emerge. To discuss the reasons for that expectation, I will review a technical result due to Geroch and Wald (see Wald 1984b).

Theorem (Geroch and Wald). Let M, g_{ab} be a time oriented spacetime, and let S_1 and S_2 be closed achronal sets with S_1 connected and S_2 edgeless. Suppose that (i) there is a point $p \in D^+(S_1)$ such that $p \notin (J^-(S_2) \cup J^+(S_2))$, and (ii) $J^+(K) \cap S_1$ has compact closure, where $K \equiv S_1 - (D^-(S_2) \cap S_1)$. Then $S_2 \not\subset D^+(S_1)$.

The Penrose diagram of (one half) of a generic black hole configuration of a gravitationally collapsed body with center of symmetry $r = 0$ is shown in Fig. 2(a). In this case $D^+(S_1)$ for the time slice S_1 includes every point to the future of S_1 so that there won't be any slice S_2 to the future such that $S_2 \not\subset D^+(S_1)$. Here the conclusion of the Theorem fails because neither of the conditions (i) or (ii) is applicable. Fig. 2(b) pictures the evaporation of a black hole by means of a catastrophic burst of electromagnetic radiation. No naked singularity develops since again $D^+(S_1)$ includes everything to the future of S_1. The Theorem fails to apply since although condition (i) holds, (ii) fails. Presumably, however, Hawking radiation does not produce the catastrophic evaporation of Fig. 2(b) but something more akin to that of Fig. 2(c) where conditions (i) and (ii) both apply. The Theorem can then be invoked to conclude that there will be a violation of cosmic censorship in the form of a breakdown in predictability.

What I would like to briefly explore is the prospect of proving that black hole evaporation will produce a violation of cosmic censorship in the stronger sense of a singularity visible from ℓ^+, as Fig. 2(c) would suggest. The Theorem already tells us that the future boundary $H^+(S_1)$ of $D^+(S_1)$ is non-empty. The generators of $H^+(S_1)$ are null geodesics. I will simply assume that these generators extend to ℓ^+. It may also be assumed without loss of generality that S_1 is a partial Cauchy surface and also that $S_1 \not\subset J^-(\ell^+)$, for S_1 is supposed to correspond to a time before the black hole evaporates. Because S_1 is edgeless, the generators of $H^+(S_1)$ are past endless and, thus, past inextendible (Hawking and Ellis 1973, Prop. 6.5.3). There are now two main possibilities to consider. (1) Some of the generators of $H^+(S_1)$ are totally or partially past imprisoned in a compact set of the spacetime. This possibility can be ruled out by imposing the requirement of strong causality which says intuitively that there do not exist any almost closed causal curves (Hawking and Ellis 1973, Prop. 6.4.7). (2) With (1) ruled out, the generators of $H^+(S_1)$ must in some sense "run off the edge" of spacetime in the past direction. There are two main sub-cases to consider. (a) Some of the generators run into a singularity, i.e. are past incomplete. Since, by assumption, the generators extend to ℓ^+, we have our naked singularity. To rest content with this sub-case, we need to rule out the other. (b) The generators are all past complete. Again we have two sub-cases to consider. (i) The generators run into an internal infinity, such as illustrated in Ex. 1 of section 2. (This notion can perhaps be captured by the following definition. M, g_{ab} possess an *internal infinity* iff there is a neighborhood $N \subset M$ with compact boundary and a geodesic half-curve $\gamma \subset N$ of infinite affine length.[6]) Imposing a suitable condition to rule out such pathologies we should arrive at the second sub-case. (ii) The generators of $H^+(S_1)$ all run to ℓ^-. But in this case we should be able to show that $S_1 \subset J^-(\ell^+)$, which is contrary to assumption. In sum let me emphasize that there is no pretense of a proof here. But the considerations reviewed do seem to me to lend credence to the notion that, excluding the process of Fig. 2(b), physically reasonable cases of black hole evaporation can be expected to produce singularities, in the sense of geodesic incompleteness, that are visible from ℓ^+.

The upshot is fairly disturbing. If we believe the classical GTR, it is likely that black holes have formed throughout the universe. If semi-classical quantum gravity has any validity, then these black hole are evaporating. And the above discussion indicates that the evaporation will eventuate in naked singularities at least in the sense of a breakdown in predictability and quite possibly a stronger sense as well. Perhaps we are saved by the fact that evaporation time for black holes as massive as our sun is very long indeed.

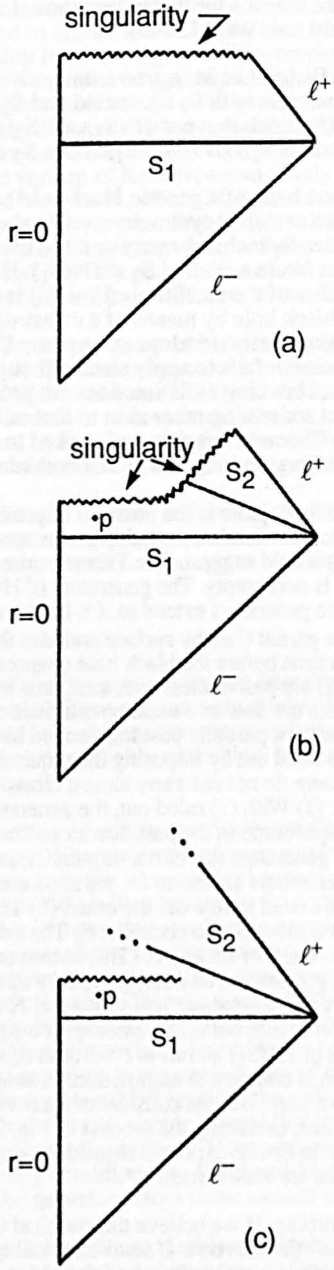

Figure 2

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5. Conclusion

On the physics side, the issue of cosmic censorship connects directly to current research in relativistic gravitation and to some of the deepest foundations in general relativity. On the philosophy side, it connects directly to issues of prediction and determinism and to the nature and function of scientific theories. It is therefore surprising that cosmic censorship has received so little attention in the philosophy of science literature. My hope in arranging the symposium on this topic is that enough interest will be stimulated in our Association to end the neglect.

Notes

[1] In his presentation to the Cosmic Censorship Symposium, Bob Geroch noted that, in a certain sense, predictability is impossible in most relativistic spacetimes; for typically an observer will be able to obtain enough information to make a deterministic prediction of an event only after the event has already occurred, in which case the prediction isn't really *pre*diction (see Earman 1986). (An exception occurs in general relativistic spacetimes which possess compact Cauchy surfaces; see below for definitions of the relevant concepts.) Nevertheless, in classical relativistic physics, determinism is thought to hold at least locally. Whether it breaks down in the large because of the development of pathologies in the spacetime structure is part and parcel of the problem of cosmic censorship.

[2] The notion of a black hole is well-defined for asymptotically flat spacetimes that permit the construction of *future null infinity* ℓ^+, which is intuitively the terminus of outgoing light rays. The interior of the black hole is then that part of spacetime that is not visible from ℓ^+, i.e. the complement of $J^-(\ell^+)$. The boundary between the interior and exterior regions of the black hole, called the (absolute) *event horizon*, is a one-way causal membrane that shields external observers from whatever craziness might on inside the black hole.

[3] For a technical definition of global hyperbolicity, see Hawking and Ellis (1973, pp. 206ff). Global hyperbolicity is equivalent to the existence of a Cauchy surface.

[4] The absence of an event horizon entails the absence of an apparent horizon but not conversely. Shapiro and Tukolsky's simulation shows the absence of an apparent horizon. Wald and Iyer (1991) show that in Schwarzschild spacetime there is a slicing that passes as near to the black hole singularity as you like for which there is no apparent horizon although, of course, there is an event horizon.

[5] See Earman (1993) for a more detailed discussion of the evidence pro and con.

[6] This definition was suggested to me by Al Janis.

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