caught off-guard by its successor. In the years between the two wars, people had begun to realise that mathematical research could have significant relevance to wartime needs, and as a result, even before the US officially entered the war, mathematicians had formed a War Preparedness Committee, which Parshall discusses at some length. Post-war events in the development of mathematics in the US included a 1946 conference in Princeton on mathematical problems (emulating the famous Hilbert speech at the turn of the century) and Congressional action which ultimately resulted in the creation of the National Science Foundation (which recognised mathematics from the outset). And then, in 1950, the International Congress of Mathematicians was held, for the first time, in the United States (at Harvard). An American ICM had been scheduled for 1940, but had to be cancelled because of the War. These events are also discussed in the book.

This is not a book that will be used much for casual reading, or even as a classroom text. It is dense with details and footnotes, and seems primarily intended as a scholarly reference. The bibliography alone is fifty pages of small type. But as a reference it is outstanding. It certainly belongs in any university library, and on the shelves of any mathematician with an interest in the history of the subject.

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**All the mathematics you missed (but needed to know for graduate school)** (second edition) by Thomas A. Garrity, pp. 388, £22.99 (paper), ISBN 978-1-00900-919-5, Cambridge University Press (2021)

This book is aimed as a reasonably niche audience, but I think it merits a wider appeal. It is written for those attending graduate school in the US, which for us in the UK translates to those studying beyond their first degree. There are many higher degrees needing a level of mathematics perhaps beyond that which the student studied in their undergraduate course, or students starting a higher degree after some time has passed since their first degree so that some or much of what was learned has become rather hazy.

Garrity takes the reader through twenty key mathematical areas, ranging from linear algebra, Euclidean geometry, Stokes's theorems and probability theory to more specialised topics such as the Zariski topology of commutative rings.

The introduction deals with some hard truths for the reader—some encouragements that maths is exciting but coupled with the warning that it is hard and needs hard work and self-discipline. This reinforces the challenges that all of us face from time to time at all levels but it should serve as a warning and a comfort to those wishing to study at the highest of levels.

Garrity spells out definitions fully and explains examples clearly for each topic. He emphasises that this is not a rigorous exposition; rather, he aims to be intentionally non-rigorous, to be deliberately loose in style, to present the material in a more conversational than academic style.

I remember finding Fourier series rather a challenge during my degree, so I turned to that chapter to see what I can remember. The initial definitions and discussion are very clear and give a good overall explanation. I am clearer in a couple of pages as to the big picture—the *why* of Fourier series—and indeed far clearer after a few minutes reading than I remember feeling during my degree. There are clear definitions and examples to explain inner products, induced norms, Lebesgue integrable functions and so on.

I am sure this book would be very useful for anyone turning back to study mathematics with any kind of break since their first degree. But I think it would also be useful for undergraduates who are finding themselves rather overwhelmed by their courses, and who perhaps would benefit from the overviews, definitions and examples given here, rather than the greater detail demanded of their studies.

Many of my A-level students do continue to study Mathematics at an undergraduate level. This would be an interesting parting gift for them, on the lines of "one day, all this will be clear", and I am sure this would be a useful reference book for them.

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**Algebra, notes from the underground** by Paolo Aluffi, pp. 488, £29.99 (paper), ISBN 978-1-108-95823-3, Cambridge University Press (2021)

Despite the Dostoevskian title, this is nothing more sinister than a rigorous undergraduate textbook whose subversiveness consists in presenting rings before groups. The author, who is based at Florida State University, argues that a readership without significant experience in abstract algebra will find it more natural to deal with, say, the ring of integers by not having to ignore one of the two natural operations, and claims that, as groups require fewer axioms than rings, groups represent greater generalisation and hence greater difficulty. He puts much emphasis on modules, quoting Atiyah and Macdonald who wrote in 1970 that 'following the modern trend, we put more emphasis on modules ...', and commenting that such a trend has not reached introductory algebra teaching in 2020.

Aluffi's style is rigorous but also chatty. (An extreme example: 'Wait ... this seems to depend on the choice of representative *a* for the class. Don't we have to check that it is well-defined, that is, independent of the representative? You bet we do. Check it now. OK, OK, let's check it together.'). He starts with two chapters of number theory. To him the Well-Ordering Principle is crucial; he claims that proofs based on induction are better dealt with through the Principle (provided you grant its truth), not liking the use of 'and so on forever' in ordinary mathematical induction. He gives a careful justification of the Euclidean Algorithm, but refers to 'irreducible' numbers rather than primes. Shortly he says that this is why $\mathbb{Z}/0\mathbb{Z}$ is an integral domain but not a field ('0 is a prime number but is not irreducible'); I would think this sort of argument might look to beginners like begging the question. Quotient sets are defined early and clearly, but the author's injunction to 'resist with all your might the temptation to think of '$\mathbb{Z}/n\mathbb{Z}$ as a subset of $\mathbb{Z}$' comes rather too early, I think, to be very meaningful to the beginning reader. There is also a section on RSA encryption.

By the end of Chapter 3 we have seen that $\mathbb{Z}/n\mathbb{Z}$ is a field if, and only if, $n$ is irreducible, and that a finite integral domain is a field. Chapter 4 introduces Cartesian products and ring homomorphisms; I was glad to see Aluffi explaining that isomorphic rings were 'just different manifestations of the "same" concept.' In chapter 5 we meet a ('canonical') decomposition of functions, obviously dear to the author's heart, into a composition of an injective function, a bijective function and a surjective function. This motivates the introduction of kernels and ideals ('$\mathbb{Z}$ is not an ideal in $\mathbb{Q}$'); typical of Aluffi's clarity is his demonstration that the quotient ring $\mathbb{R}[x]/(x^2 + 1)$ is isomorphic to $\mathbb{C}$. He also proves the Chinese Remainder Theorem–that is, the theorem itself and not the algorithm derived from it to which its name is usually given. Chapter 6 is on integral domains, including fields of fractions, and the last chapter in Part I is on polynomial rings and factorisation.